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Properties of Baryons in the Chiral Quark Model

TOMMY OHLSSON

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Abstract

In this thesis, several properties of baryons are studied using the chiral quark model. The chiral quark model is a theory which can be used to describe low energy phenomena of baryons.

In Paper 1, the chiral quark model is studied using wave functions with configuration mixing. This study is motivated by the fact that the chiral quark model cannot otherwise break the Coleman–Glashow sum-rule for the magnetic moments of the octet baryons, which is experimentally broken by about ten standard deviations. Configuration mixing with quark-diquark components is also able to reproduce the octet baryon magnetic moments very accurately.

In Paper 2, the chiral quark model is used to calculate the decuplet baryon magnetic moments. The values for the magnetic moments of the Δ^{++} and Ω^- are in good agreement with the experimental results. The total quark spin polarizations are also calculated and are found to be significantly smaller than the non-relativistic quark model results.

In Paper 3, the weak form factors for semileptonic octet baryon decays are studied in the chiral quark model. The “weak magnetism” form factors are found to be consistent with the conserved vector current (CVC) results and the induced pseudotensor form factors, which seem to be model independent, are small. The results obtained are in general agreement with experiments and are also compared with other model calculations.

Key words: Chiral quark model, baryons, configuration mixing, magnetic moments, Coleman–Glashow sum-rule, spin structure, spin polarizations, weak form factors, semileptonic decays.

Preface

This thesis is a result of my research work at the Division of Mathematical Physics, Theoretical Physics, Department of Physics, Royal Institute of Technology in Stockholm during the years 1996-1997.

The thesis consists of two parts. The first part is a summary of the theory of the field of my research. The summary is intended to introduce the reader to the field and to put the research into its context. The second part contains a collection of my papers (*i.e.* scientific articles) in which my research work and the results thereof are presented.

List of Papers

The papers in this thesis are:

1. Johan Linde, Tommy Ohlsson, and Håkan Snellman
Octet Baryon Magnetic Moments in the Chiral Quark Model with Configuration Mixing
Phys. Rev. D **57**, ? (1998).
Published electronically November 10, 1997 in Physical Review D online¹.
hep-ph/9709353²
2. Johan Linde, Tommy Ohlsson, and Håkan Snellman
Decuplet Baryon Magnetic Moments in the Chiral Quark Model
Submitted to Phys. Rev. D
hep-ph/9709468²
3. Tommy Ohlsson and Håkan Snellman
Weak Form Factors for Semileptonic Octet Baryon Decays in the Chiral Quark Model
Submitted to Phys. Rev. D

¹Physical Review D online: <http://publish.aps.org/PRDO/>

²Available from the Los Alamos National Laboratory e-Print archive: <http://xxx.lanl.gov/>

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Then I want to thank all of the other graduate students and the rest of the staff at Theoretical Physics for their companionship.

All my friends should of course not be forgotten, so this line is entirely for all of you.

Finally, and absolutely not last, I want to thank my family for always being there for me.

Stockholm, November 1997

A handwritten signature in cursive script, reading "Taimy Oksanen".

Contents

| | |
|--|------------|
| Abstract | iii |
| Preface | iv |
| List of Papers | iv |
| Acknowledgements | v |
| Contents | vii |
| 1 Introduction | 1 |
| 2 A Survey of Baryons | 3 |
| 2.1 The Non-Relativistic Quark Model | 5 |
| 3 Properties of Baryons | 11 |
| 3.1 Spin Structure and Spin Polarizations | 11 |
| 3.2 Magnetic Moments | 13 |
| 3.3 Weak Axial-Vector Form Factors | 14 |
| 3.4 The \bar{u} - \bar{d} Asymmetry | 17 |
| 3.4.1 Deep Inelastic Scattering | 18 |
| 3.4.2 The Gottfried Sum-Rule | 20 |
| 3.4.3 Drell-Yan Processes | 22 |
| 4 The Chiral Quark Model | 25 |
| 4.1 The Chiral Quark Model – The Theory | 26 |
| 4.1.1 The QCD Lagrangian | 26 |
| 4.1.2 Chiral Symmetry | 28 |
| 4.1.3 The Chiral Quark Model Lagrangian | 29 |
| 4.2 The Chiral Quark Model – The Recipe | 32 |
| 4.2.1 Flavor Content Calculation | 32 |
| 4.2.2 Spin Content Calculation | 35 |
| 4.2.3 The χ QM with U(3) Symmetry Breaking | 37 |
| 4.2.4 The χ QM with SU(3) Symmetry Breaking | 38 |

| | | |
|----------|--|-----------|
| 4.2.5 | The Probability Parameter a | 40 |
| 4.3 | The Coleman–Glashow Sum-Rule | 41 |
| 4.4 | The Chiral Quark Model with Configuration Mixing | 42 |
| 4.4.1 | Quark-Gluon Configuration Mixing | 42 |
| 4.4.2 | Diquarks | 43 |
| 4.4.3 | Quark-Diquark Configuration Mixing | 43 |
| 4.5 | Weak Form Factors in the χ QM | 44 |
| 5 | Conclusions | 47 |
| | References | 49 |
| | Papers | 53 |

Chapter 1

Introduction

Quantum chromodynamics (QCD) is the present theory of the strong interaction. The particles of QCD, quarks and gluons, are interacting with each other with a strength denoted α_s , the strong coupling constant. The coupling constant α_s is, however, not a constant, but a “running” coupling constant, that varies with energy, see Fig. 1.1.

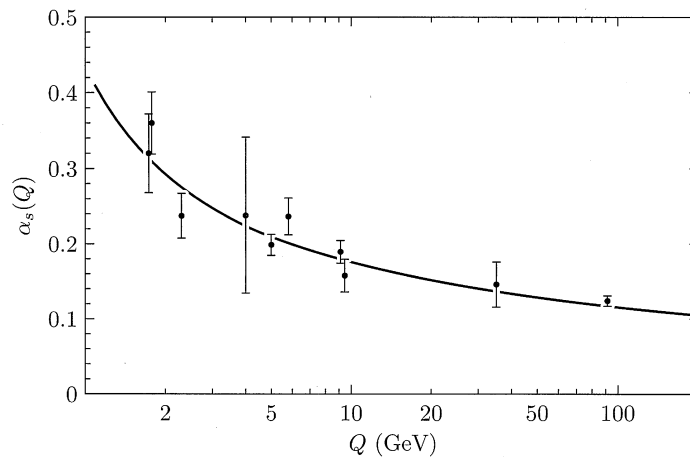


Figure 1.1: The running coupling constant α_s . The plot shows measurements of α_s , plotted against the momentum scale Q at which the measurements were made. The figure has been obtained from the book by M. E. Peskin and D. V. Schroeder [1].

At high energies, when α_s is small, QCD can be used perturbatively, but at low energies, when α_s becomes large, this is no longer possible.

Since perturbative QCD breaks down at low energies, one has to use other methods, such as QCD lattice techniques or effective Lagrangian models, to describe physics in this energy region.

The non-relativistic quark model (NQM) is a simple model for describing properties of hadrons (particles which are bound states of quarks) in the low energy region. The NQM gives very good results for the masses of the hadrons, but only moderate results for the magnetic moments and the weak form factors, and fails to describe the spin structure of baryons.

When describing properties of hadrons at low energies, it has been known since long that it is important to incorporate spontaneous chiral symmetry breaking in the theory. One model for describing such a property is the chiral quark model (χ QM).

The χ QM is an effective chiral field theory approach for describing structure and properties of hadrons and especially baryons at low energies. In this model the appropriate degrees of freedom are quarks, gluons, and Goldstone bosons.

The χ QM has been suggested to replace the NQM, which is a too simple model. We are therefore going to describe and discuss the χ QM and various extensions of this model, which can better account for properties of baryons. Properties that can be calculated with the χ QM are *e.g.* magnetic moments, spin polarizations, form factors, and mass spectra.

The background material for my research is presented in the following chapters and the research results can be found in the three papers at the end of this thesis.

Chapter 2

A Survey of Baryons

This chapter is a survey of the particles having strong interaction. These particles are called *hadrons*. The hadrons are divided into *mesons*, which are bosons (integer spin particles) with baryon number 0, and *baryons*, which are fermions ($\frac{1}{2}$ -integer spin particles) with baryon number different from 0. Especially the baryons will be discussed in some detail, since the physical properties of the baryons are one of the main topics in this thesis.

The hadrons that are known all fall into multiplets that reflect underlying internal symmetries. To account for this in a simple way, it was suggested that hadrons are composed of more elementary particles with certain basic symmetries, called *quarks* [2, 3]. The quarks are fermions with spin $\frac{1}{2}$. There are at present believed to be six different kinds of quarks or *flavors*. These are denoted u ('up'), d ('down'), s ('strange'), c ('charm'), b ('bottom'), and t ('top'). The u , c , and t have charge $\frac{2}{3}$ and the d , s , and b have charge $-\frac{1}{3}$. The first three quarks u , d , and s are called light quarks and in what follows of this thesis only these quarks will be discussed.

The actual existence of quarks has only been indirectly confirmed by experiments that probe hadronic structure by means of electromagnetic and weak interactions. The reason why quarks cannot be studied directly is due to quark confinement.

Internal symmetries refer to the fact that particles come in families, called multiplets, which have degenerate or nearly degenerate masses. Each multiplet is looked upon as a realization of an irreducible representation of an internal symmetry group. Such groups are identified by the patterns of experimentally observed multiplets. If the masses in a multiplet are not exactly equal to each other, then the associated symmetry is said to be only an approximate symmetry. Among the hadrons which consist of the light quarks, the internal symmetries Q (charge), I (isospin), B (baryon number), and S (strangeness)

are since long recognized. These symmetries have varying degrees of exactness in nature.

The hypercharge is defined by

$$Y \equiv B + S, \quad (2.1)$$

and I_3 , the third component of isospin I , is related to the electric charge Q by the empirical relation

$$Q = I_3 + \frac{Y}{2}, \quad (2.2)$$

called the Gell-Mann–Nishijima relation.

In addition to already mentioned quantum numbers, quarks also possess the quantum number *color*. Experiments indicate that there should be three different colors. These are denoted by r ('red'), g ('green'), and b ('blue'). Any physical state must be a color singlet, since color is confined. This immediately implies that the number of quarks making up a baryon must be three or divisible by three.

Three quarks q_1 , q_2 , and q_3 can bind together to form a baryon ($q_1 q_2 q_3$). Similarly, a quark q_1 and an antiquark \bar{q}_2 can bind together to form a meson ($q_1 \bar{q}_2$).

Since quarks have spin one-half, the baryon and meson ground state configurations can carry the spin quantum numbers $J = \frac{1}{2}, \frac{3}{2}$ and $J = 0, 1$, respectively. There are also excited states of baryons and mesons which have other spin quantum numbers.

If the mass difference between strange and non-strange quarks are neglected, then in the language of group theory, the three light quarks (u, d, s) belong to the fundamental representation (denoted $\mathbf{3}$) of $SU(3)$, sometimes denoted $SU(3)_{\text{flavor}}$, whereas the antiquarks belong to the conjugate representation (denoted $\mathbf{3}^*$). The qqq and $q\bar{q}$ constructions then involve the following group representation products

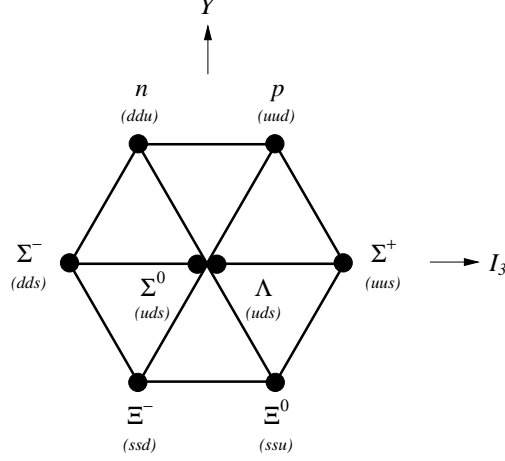
$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}, \quad (2.3)$$

$$\mathbf{3} \otimes \mathbf{3}^* = \mathbf{8} \oplus \mathbf{1}. \quad (2.4)$$

Thus baryons appear as decuplets, octets, and singlets, whereas mesons appear as octets and singlets.

Finally, quarks and antiquarks transform as triplets and antitriplets of the color $SU(3)$ gauge group, sometimes denoted $SU(3)_{\text{color}}$, and all baryons and mesons are color singlets.

The octet baryons have spin $\frac{1}{2}$ and the decuplet baryons have spin $\frac{3}{2}$. The weight diagrams for the baryon octet and the baryon decuplet are shown in Figs. 2.1 and 2.2, respectively.

Figure 2.1: The $J^P = \frac{1}{2}^+$ octet of baryons.

In this thesis only baryons which consist of the light quarks (u, d, s) will be discussed. In Papers 1 and 3 we study the octet baryons and in Paper 2 we study the decuplet baryons.

2.1 The Non-Relativistic Quark Model

The non-relativistic quark model (NQM) attempts to describe the properties of light hadrons as a composite system of the u , d , and s valence quarks.

Let us first consider the masses of the octet and decuplet baryons. The degenerate isospin multiplet masses of the baryons are presented in Table 2.1.

We observe that there is a mass difference between the Λ and Σ^0 , although they are built up from the same quark flavors (see Fig. 2.1). This has to do with the fact that Λ is an isosinglet and Σ^0 is a member of an isotriplet.

In the NQM the baryon masses can be described by the following simple mass formula with a hyperfine coupling term [5, 6]

$$m(B(q_1 q_2 q_3)) = m_{q_1} + m_{q_2} + m_{q_3} + h \left(\frac{\mathbf{s}_{q_1} \cdot \mathbf{s}_{q_2}}{m_{q_1} m_{q_2}} + \frac{\mathbf{s}_{q_2} \cdot \mathbf{s}_{q_3}}{m_{q_2} m_{q_3}} + \frac{\mathbf{s}_{q_3} \cdot \mathbf{s}_{q_1}}{m_{q_3} m_{q_1}} \right), \quad (2.5)$$

where m_{q_i} and \mathbf{s}_{q_i} are the (constituent) mass and spin of the quark q_i , respectively. The hyperfine term accounts for the difference in quark spin structure between Λ and Σ , thereby giving them different masses.

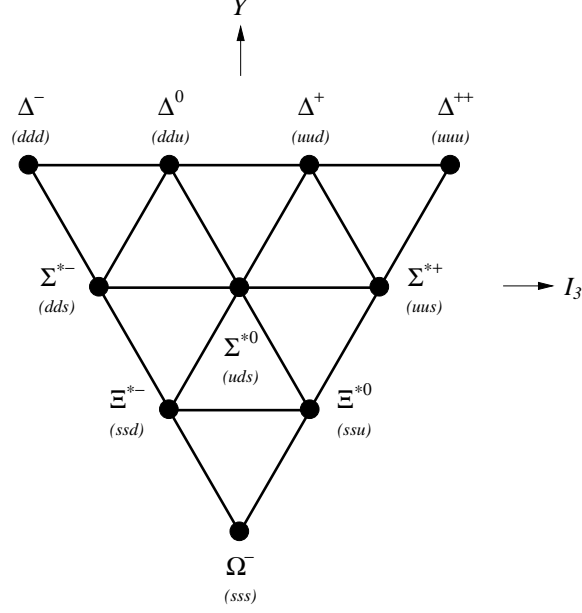


Figure 2.2: The $J^P = \frac{3}{2}^+$ decuplet of baryons.

We will here assume that the u and d quarks have the same mass. The u and d quarks are sometimes called non-strange quarks. The mass of the non-strange quarks is m and the mass of the strange quark s is m_s . The parameter h is a measure of the hyperfine coupling strength.

By fitting the quark masses m and m_s and the hyperfine parameter h to the baryon masses, one can obtain excellent results (the masses are within about 1 %) [7]. See, *e.g.* Refs. [5, 6, 8] for results.

The parameter values from the fit are

$$m \approx 363 \text{ MeV}, \quad (2.6)$$

$$m_s \approx 538 \text{ MeV}, \quad (2.7)$$

and

$$\frac{h}{4m^2} \approx 50 \text{ MeV}. \quad (2.8)$$

A similarly good fit can also be obtained for the mesons.

Despite the success with the NQM for calculating the baryon masses, one is not able to obtain the magnetic moments and the weak axial-vector form

| Particle | Mass (MeV) |
|---|------------|
| N (p and n) | 939 |
| Λ | 1116 |
| Σ (Σ^+ , Σ^0 , and Σ^-) | 1193 |
| Ξ (Ξ^0 and Ξ^-) | 1318 |
| Δ (Δ^{++} , Δ^+ , Δ^0 , and Δ^-) | 1232 |
| Σ^* (Σ^{*+} , Σ^{*0} , and Σ^{*-}) | 1385 |
| Ξ^* (Ξ^{*0} and Ξ^{*-}) | 1533 |
| Ω | 1672 |

Table 2.1: The degenerate isospin multiplet masses of the baryons. Data have been obtained from Ref. [4].

factors with the same accuracy.

For baryons, flavor and spin may be combined in an approximate flavor-spin SU(6) symmetry, in which the six basic quark states are u^\uparrow , u^\downarrow , d^\uparrow , d^\downarrow , s^\uparrow , and s^\downarrow . Consider now the proton SU(6) wave function [9]:

$$|p^\uparrow\rangle = \frac{1}{\sqrt{18}} \epsilon_{ijk} \left(u_i^\downarrow d_j^\uparrow - u_i^\uparrow d_j^\downarrow \right) u_k^\uparrow |0\rangle. \quad (2.9)$$

When calculating physical quantities, many terms in this wave function yield the same contributions. Hence one can often use the following simplified SU(6) wave function for the proton

$$|p^\uparrow\rangle = \frac{1}{\sqrt{6}} \left(2|u^\uparrow u^\uparrow d^\downarrow\rangle - |u^\uparrow u^\downarrow d^\uparrow\rangle - |u^\downarrow u^\uparrow d^\uparrow\rangle \right), \quad (2.10)$$

where color and permutations in flavor have been suppressed. From this wave function one can count the number of quark flavors with spin parallel and antiparallel to the total spin of the proton. The result of the counting is

$$n_{u^\uparrow}(p) = \frac{5}{3}, \quad n_{u^\downarrow}(p) = \frac{1}{3}, \quad n_{d^\uparrow}(p) = \frac{1}{3}, \quad n_{d^\downarrow}(p) = \frac{2}{3}. \quad (2.11)$$

These numbers are the eigenvalues of the quark number operator with respect to helicity and they also sum up to two u quarks and one d quark. From the differences, which are exactly the expectation values of twice the quark spin operator, one obtains the contribution by each of the quark flavors to the proton helicity. The differences are

$$\Delta u^P = \frac{4}{3}, \quad \Delta d^P = -\frac{1}{3}, \quad \Delta s^P = 0. \quad (2.12)$$

The sum of these differences is $\Delta\Sigma^p = \Delta u^p + \Delta d^p + \Delta s^p = 1$. (Sometimes also the notations Δu , Δd , Δs , and $\Delta\Sigma$ will be used for the proton.) In the next chapter we will discuss experimental results for these differences. We will see that they differ a lot from the above NQM values.

The experimentally measured magnetic moments of the octet baryons are presented in Table 2.2.

| Particle | Magnetic moment (μ_N) |
|------------|-----------------------------|
| p | 2.79 ± 0.00 |
| n | -1.91 ± 0.00 |
| Λ | -0.61 ± 0.01 |
| Σ^+ | 2.46 ± 0.02 |
| Σ^0 | - |
| Σ^- | -1.16 ± 0.03 |
| Ξ^0 | -1.25 ± 0.02 |
| Ξ^- | -0.65 ± 0.01 |

Table 2.2: The magnetic moments of the octet baryons. Data have been obtained from Ref. [4] and are given in units of the nuclear magneton, μ_N .

In the NQM, the magnetic moments of the octet baryons can be parameterized as [5, 8]

$$\mu(p) = \Delta u \mu_u + \Delta d \mu_d + \Delta s \mu_s, \quad (2.13)$$

$$\mu(n) = \Delta u \mu_d + \Delta d \mu_u + \Delta s \mu_s, \quad (2.14)$$

$$\mu(\Sigma^+) = \Delta u \mu_u + \Delta d \mu_s + \Delta s \mu_d, \quad (2.15)$$

$$\mu(\Sigma^-) = \Delta u \mu_d + \Delta d \mu_s + \Delta s \mu_u, \quad (2.16)$$

$$\mu(\Xi^0) = \Delta u \mu_s + \Delta d \mu_u + \Delta s \mu_d, \quad (2.17)$$

$$\mu(\Xi^-) = \Delta u \mu_s + \Delta d \mu_d + \Delta s \mu_u, \quad (2.18)$$

$$\mu(\Lambda) = \mu_s, \quad (2.19)$$

$$\mu(\Sigma^0) = \frac{1}{2} (\mu(\Sigma^+) + \mu(\Sigma^-)), \quad (2.20)$$

where in the first six formulas the factors $\Delta u = \frac{4}{3}$, $\Delta d = -\frac{1}{3}$, and $\Delta s = 0$ are the differences in Eq. (2.12) and μ_u , μ_d , and μ_s are the quark magnetic moments.

Because of the assumption of isospin symmetry $m_u = m_d$, one has $\mu_u = -2\mu_d$. Using this, the proton and neutron magnetic moments are reduced to

$\mu(p) = -3\mu_d$ and $\mu(d) = 2\mu_d$, and thus the ratio between them is

$$\frac{\mu(p)}{\mu(n)} = -1.5, \quad (2.21)$$

which is close to the experimental value of -1.46 .

Furthermore, we have seen from the fit to the baryon masses that m_s is about $3/2$ times heavier than m , *i.e.* $\frac{m_s}{m} \simeq \frac{3}{2}$. Thus, we can make the approximation $\mu_s = \frac{2}{3}\mu_d$. This means that the formulas for the magnetic moments of the octet baryons can all be expressed in terms of the magnetic moment of the d quark, μ_d .

However, when we fit the NQM formulas for the octet baryon magnetic moments to the experimental data, we obtain results which are of the right order, but still far from good (the magnetic moments are within about 25 %). See, *e.g.* Ref. [8] for results. The parameter value for the magnetic moment of the d quark from the fit is

$$\mu_d \approx -0.9\mu_N, \quad (2.22)$$

where $\mu_N \equiv \frac{1}{2M_p}$ is the nuclear magneton (M_p is the mass of the proton). The magnetic moments of the quarks can be introduced as [6]

$$\mu_u = \frac{2}{3} \frac{1}{2m}, \quad \mu_d = -\frac{1}{3} \frac{1}{2m}, \quad \text{and} \quad \mu_s = -\frac{1}{3} \frac{1}{2m_s}. \quad (2.23)$$

Using these magnetic moments, we obtain the following masses

$$m \approx \frac{M_p}{3 \cdot 0.9} \approx 348 \text{ MeV} \quad \text{and} \quad m_s = \frac{3}{2}m \approx 522 \text{ MeV}, \quad (2.24)$$

which are slightly smaller, but entirely compatible with the constituent quark mass values obtained by fitting the baryon masses.

Let us next discuss the weak axial-vector form factor for the neutron-proton transition, which is the matrix element of the quark axial-vector current operator. This matrix element, denoted g_A^{np} , is essentially the matrix element of σ_z between the neutron and the proton. Experimentally, this is $g_A^{np\text{exp}} = 1.2601 \pm 0.0025$. With the NQM, the value is $g_A^{np} = \Delta u^p - \Delta d^p = \frac{5}{3} \approx 1.67$. As we see, the NQM value is far from the experimental value. This is yet another indication that the NQM does not provide a good description of physics.

It seems that we have to abandon the NQM for a more realistic model.

In Chapter 4, we are going to discuss the χ QM. With this model we are able to obtain magnetic moments and weak axial-vector form factors, which are very accurate. But first, in the next chapter, we will set up a more general framework for calculating different properties of the baryons.

Chapter 3

Properties of Baryons

In this chapter some of the most important properties of baryons are discussed. Here the discussion will be more general than in the previous one for the NQM, so that we will be able to apply this on the χ QM in the next chapter. In the last section of this chapter we will discuss the concept of \bar{u} - \bar{d} asymmetry, which is not present at all in the NQM, but as we will see in the χ QM. Definitions for the properties that have been used in the scientific articles are also given here.

3.1 Spin Structure and Spin Polarizations

The NQM can be successfully used to predict the masses of hadrons and especially the baryons, but it fails to give a good description of the quark spin structure of these particles.

In the NQM, we have the following quark spin polarizations for the proton

$$\Delta u = \frac{4}{3}, \quad \Delta d = -\frac{1}{3}, \quad \Delta s = 0, \quad \text{and} \quad \Delta \Sigma = 1. \quad (3.1)$$

In 1988 the European Muon Collaboration (EMC) [10,11] announced that it had measured the fraction of the proton spin carried by the quarks $\Delta \Sigma$ to be $\Delta \Sigma \approx 0$, and also $\Delta s \neq 0$. This was the beginning of what was to be called the “nucleon spin crisis”. A lot of activity has been going on since then in this field, and the present experimental results may be summarized as [12]

$$\Delta u = 0.83 \pm 0.03, \quad (3.2)$$

$$\Delta d = -0.43 \pm 0.03, \quad (3.3)$$

$$\Delta s = -0.10 \pm 0.03, \quad (3.4)$$

$$\Delta \Sigma = 0.31 \pm 0.07. \quad (3.5)$$

The E143 Collaboration [13] has found the total contribution from all quarks to be $\Delta\Sigma = 0.30 \pm 0.06$, and the contribution from strange quarks and antistrange quarks to be $\Delta s = -0.09 \pm 0.02$. They are the most precise determinations to date and are consistent with the earlier results [12].

Note that $\Delta\Sigma = 1$ means that the spin is carried by the quarks alone. If $\Delta\Sigma < 1$, then the remainder of the spin must be carried by something else. The remaining spin could be built up from other components, such as orbital motion of the quarks, so called collective effects, and, if in the relevant energy region, also gluons.

Let us now define the spin structure and the quark spin polarizations generally for a baryon.

The spin structure of a baryon B is described by the function \hat{B} , which is defined by

$$\hat{B} \equiv \langle B^\uparrow | \mathcal{N} | B^\uparrow \rangle, \quad (3.6)$$

where $|B^\uparrow\rangle$ is the wave function and \mathcal{N} is the number operator

$$\mathcal{N} = N_{u\uparrow}\hat{u}^\uparrow + N_{u\downarrow}\hat{u}^\downarrow + N_{d\uparrow}\hat{d}^\uparrow + N_{d\downarrow}\hat{d}^\downarrow + N_{s\uparrow}\hat{s}^\uparrow + N_{s\downarrow}\hat{s}^\downarrow.$$

Using the definition (3.6) together with the number operator, it follows that

$$\hat{B} = n_{x\uparrow}(B)\hat{x}^\uparrow + n_{x\downarrow}(B)\hat{x}^\downarrow + n_{y\uparrow}(B)\hat{y}^\uparrow + n_{y\downarrow}(B)\hat{y}^\downarrow + n_{z\uparrow}(B)\hat{z}^\uparrow + n_{z\downarrow}(B)\hat{z}^\downarrow, \quad (3.7)$$

where the coefficient $n_{q\uparrow\downarrow}(B)$ of each symbol $\hat{q}^{\uparrow\downarrow}$ should be interpreted as the number of $q^{\uparrow\downarrow}$ quarks.

The quark spin polarization, Δq^B , where $q = u, d, s$, is defined as

$$\Delta q^B \equiv \langle B^\uparrow | \sigma_z^q | B^\uparrow \rangle = n_{q\uparrow}(B) - n_{q\downarrow}(B), \quad (3.8)$$

where the σ_z^q is the Pauli spin matrix of the quark q . If there are antiquarks in the baryons, then one has to redefine the quark spin polarization as

$$\Delta q^B \equiv \Delta_q^B + \Delta_{\bar{q}}^B, \quad (3.9)$$

where $\Delta_q^B = n_{q\uparrow}(B) - n_{q\downarrow}(B)$ and $\Delta_{\bar{q}}^B = n_{\bar{q}\uparrow}(B) - n_{\bar{q}\downarrow}(B)$. Thus the quark spin polarizations are the sum of the quark and antiquark polarizations.

The total spin polarization of a baryon B (the spin fraction carried by the quarks in the baryon) is given by

$$\Delta\Sigma^B = \Delta u^B + \Delta d^B + \Delta s^B. \quad (3.10)$$

In Papers 1 and 2 we investigate the quark spin polarizations for different types of baryons. In Paper 1 we study the octet baryons, and in Paper 2 the decuplet baryons.

3.2 Magnetic Moments

The magnetic moment for a baryon B is defined as the expectation value of the z component of the magnetic moment operator with maximal spin projection along the z axis

$$\mu(B) \equiv \langle B; J, J_z = J | \mu_z | B; J, J_z = J \rangle, \quad (3.11)$$

where μ_z is the z component of the magnetic moment operator. Eq. (3.11) can also be written as

$$\mu(B) = \sum_{q=u,d,s} \langle B | \mu_q \sigma_z^q | B \rangle, \quad (3.12)$$

where

$$\mu_q = \frac{e_q g_q}{4m_q}$$

is the magnetic moment of the quark q and σ_z^q is the Pauli spin matrix of the quark q . The parameters e_q , g_q , and m_q are the electric charge, the gyromagnetic ratio, and the mass for the quark q , respectively. For the light quarks $e_u = \frac{2}{3}$ and $e_d = e_s = -\frac{1}{3}$. The gyromagnetic ratio for a quark is about 2, since a quark has spin $\frac{1}{2}$. We will use $g_q = 2$, for $q = u, d, s$, which means that

$$\mu_q = e_q \frac{1}{2m_q}. \quad (3.13)$$

Using the right hand side of Eq. (3.12), the magnetic moment of a baryon B can be expressed by

$$\mu(B) = \sum_{q=u,d,s} \Delta q^B \mu_q = \Delta u^B \mu_u + \Delta d^B \mu_d + \Delta s^B \mu_s. \quad (3.14)$$

If there are antiquarks in the baryon, then one has to rewrite Eq. (3.14) as

$$\mu(B) = \sum_{q=u,d,s} (\Delta_q^B \mu_q + \Delta_{\bar{q}}^B \mu_{\bar{q}}) = \sum_{q=u,d,s} (\Delta_q^B - \Delta_{\bar{q}}^B) \mu_q \equiv \sum_{q=u,d,s} \widetilde{\Delta}_q^B \mu_q, \quad (3.15)$$

since $\mu_{\bar{q}} = -\mu_q$. Note that if $\Delta_{\bar{q}}^B = 0$, then $\widetilde{\Delta}_q^B = \Delta_q^B = \Delta_q^B$.

Using the magnetic moment formulas for the octet baryons p , n , Σ^- , Σ^+ , Ξ^0 , and Ξ^- , one can show the so called Coleman–Glashow sum-rule for the octet baryon magnetic moments

$$\mu(p) - \mu(n) + \mu(\Sigma^-) - \mu(\Sigma^+) + \mu(\Xi^0) - \mu(\Xi^-) = 0. \quad (3.16)$$

In Section 4.3 we will discuss this sum-rule further in the framework of the χ QM.

In Paper 1 we study the magnetic moments of the octet baryons in the χ QM. In both Paper 2 and 3 we use various quantities obtained in Paper 1. In Paper 2 we extend our model for the octet baryon magnetic moments to the decuplet baryon magnetic moments. As input in Paper 2 we use the parameters obtained in different fits in Paper 1. We obtain magnetic moments for the Δ^{++} and Ω^- , which are in good agreement with experimental data. In Paper 3 we calculate weak form factors for semileptonic octet baryon decays. These form factors will be discussed in the next section.

3.3 Weak Axial-Vector Form Factors

The weak axial-vector form factors of the semileptonic baryon decays are an important set of parameters for the investigation of the quark spin structure of the baryons.

The matrix element of the weak V-A current, $J_{\text{weak}} = J_V - J_A$, for the semileptonic decays $B \rightarrow B' + l^- + \bar{\nu}_l$ can be written to lowest order as

$$\langle B' | J_{\text{weak}}^\mu | B \rangle = \bar{u}' \left(f_1^{\text{QM}} \gamma^\mu - g_1^{\text{QM}} \gamma^\mu \gamma^5 \right) u, \quad (3.17)$$

where u (u') and $|B\rangle$ ($|B'\rangle$) are the Dirac spinor and external baryon state of the initial (final) baryon B (B'), respectively. The f_1^{QM} is the SU(6) quark model vector form factor and the g_1^{QM} is the SU(6) quark model axial-vector form factor.

The weak axial-vector form factor G_A is defined by

$$G_A \equiv \frac{g_1^{\text{QM}}}{f_1^{\text{QM}}}. \quad (3.18)$$

A matrix element between two octet baryon states, B and B' , of the weak hadronic current j , that transforms as an octet under SU(3), has the general form

$$\langle B'_i | j_j | B_k \rangle = i f_{ijk} F + d_{ijk} D, \quad (3.19)$$

where f_{ijk} are the totally antisymmetric structure constants of SU(3) and d_{ijk} are the corresponding constants which are totally symmetric. The axial-vector parameters F and D are the only parameters needed to express all such matrix elements. This means that all G_A 's can be expressed in terms of the two axial-vector parameters F and D [14].

Expressed in the parameters F and D they are [15]

$$G_A^{np} = F + D \quad (3.20)$$

$$G_A^{\Sigma^- \Sigma^0} = F \quad (3.21)$$

$$g_1^{\text{QM}\Sigma^\pm\Lambda} = \sqrt{\frac{2}{3}}D \quad (3.22)$$

$$G_A^{\Xi^-\Xi^0} = F - D \quad (3.23)$$

for the $\Delta S = 0$ decays and

$$G_A^{\Sigma^-n} = F - D \quad (3.24)$$

$$G_A^{\Xi^-\Sigma^0} = F + D \quad (3.25)$$

$$G_A^{\Xi^-\Lambda} = F - \frac{D}{3} \quad (3.26)$$

$$G_A^{\Lambda p} = F + \frac{D}{3} \quad (3.27)$$

$$G_A^{\Xi^0\Sigma^+} = F + D \quad (3.28)$$

for the $\Delta S = 1$ decays. The $\Sigma^0 \rightarrow \Sigma^+ + l^- + \bar{\nu}_l$ and $\Sigma^0 \rightarrow p + l^- + \bar{\nu}_l$ decays cannot be observed, since the electromagnetic decay $\Sigma^0 \rightarrow \Lambda + \gamma$ is predominant. The corresponding G_A 's are therefore not listed above. See Fig. 3.1 for all possible semileptonic octet baryon decays.

The axial-vector parameters F and D are related to the quark spin polarizations of the proton by the following definitions

$$\Delta u - \Delta d \equiv F + D, \quad (3.29)$$

$$\Delta u + \Delta d - 2\Delta s \equiv 3F - D, \quad (3.30)$$

where as before the quantities Δu , Δd , and Δs are the proton quark spin polarizations. These relations can be found in *e.g.* Refs. [16, 17]. Solving Eqs. (3.29) and (3.30) for the parameters F and D expressed in Δu , Δd , and Δs , one obtains

$$F = \frac{1}{2}(\Delta u - \Delta s), \quad (3.31)$$

$$D = \frac{1}{2}(\Delta u - 2\Delta d + \Delta s). \quad (3.32)$$

Using the relations (3.31) and (3.32), the G_A 's can be rewritten as

$$G_A^{np} = \Delta u - \Delta d \quad (3.33)$$

$$G_A^{\Sigma^-\Sigma^0} = \frac{1}{2}(\Delta u - \Delta s) \quad (3.34)$$

$$g_1^{\text{QM}\Sigma^\pm\Lambda} = \frac{1}{\sqrt{6}}(\Delta u - 2\Delta d + \Delta s) \quad (3.35)$$

$$G_A^{\Xi^-\Xi^0} = \Delta d - \Delta s \quad (3.36)$$


$$G_4^{\Xi^0 \Sigma^+} = \Delta u - \Delta d \quad (3.41)$$

We observe that the G_A 's are not within the experimental errors, except

| Decay | Experiment | NQM |
|-----------------------------|------------------|------------------------------|
| $n \rightarrow p$ | 1.26 ± 0.01 | $\frac{5}{3} \approx 1.67$ |
| $\Sigma^- \rightarrow n$ | -0.34 ± 0.02 | $-\frac{1}{3} \approx -0.33$ |
| $\Xi^- \rightarrow \Lambda$ | 0.25 ± 0.05 | $\frac{1}{3} \approx 0.33$ |
| $\Lambda \rightarrow p$ | 0.72 ± 0.02 | 1 |

Table 3.1: The weak axial-vector form factors $G_A^{BB'}$. Experimental data have been obtained from Ref. [4].

for $G_A^{\Sigma^- n}$.

In Section 4.5, we discuss the G_A 's further in the spirit of the χ QM and we will see that we are able to improve on the NQM results.

3.4 The \bar{u} - \bar{d} Asymmetry

Several experiments during the years have shown that there is an asymmetry in the light antiquark distributions in the nucleon.

The \bar{u} - \bar{d} asymmetry in the nucleon, sometimes also called the antiquark flavor asymmetry, is described by the difference and ratio between antiquarks in the proton $\bar{u} - \bar{d}$ and \bar{u}/\bar{d} , respectively. If we have \bar{u} - \bar{d} symmetry, then $\bar{u} - \bar{d} = 0$ and $\bar{u}/\bar{d} = 1$. In the case when both \bar{u} and \bar{d} are equal to zero, then the ratio \bar{u}/\bar{d} is undefined.

In 1975 data from the Stanford Linear Accelerator Center (SLAC) [19] indicated a significant deviation from the expected vanishing value for the \bar{u} - \bar{d} difference, $\bar{u} - \bar{d} \neq 0$. This was the first experiment measuring the \bar{u} - \bar{d} asymmetry via the Gottfried sum-rule. The next experiment, which showed the flavor asymmetry, was the E288 Collaboration with the Drell-Yan experiment at Fermilab in 1981 [20]. For a recent compilation of experiments which have measured the \bar{u} - \bar{d} asymmetry, see Ref. [21].

In Subsections 3.4.1 - 3.4.3 we will discuss the physical processes, deep inelastic scattering (DIS) and Drell-Yan processes, from which we can obtain the \bar{u} - \bar{d} quantities and the experimental results thereof. Later on in Chapter 4 we are going to derive the theoretical expressions for these quantities in the χ QM.

There are several theoretical models for magnetic moments in which this measured \bar{u} - \bar{d} asymmetry is not present. Among them are the NQM, the purely phenomenological SU(3) parameterization by Bos *et al.* [22, 23], the model by

Buck and Perez [24], which is a model where they have added a configuration mixing term to the usual SU(6) spin function, and the model suggested by Casu and Sehgal [25].

3.4.1 Deep Inelastic Scattering

In deep inelastic lepton-nucleon scattering (DIS), to lowest order, the virtual photon γ^* from the lepton l interacts with the nucleon target N . The reaction is illustrated in Fig. 3.2. The cross section of the reaction is related to

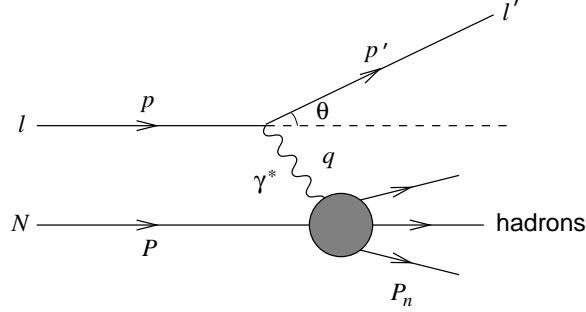


Figure 3.2: The kinematics in deep inelastic lepton-nucleon scattering $l + N \rightarrow l' + \text{hadrons}$.

two structure functions F_1 and F_2 depending on transverse and longitudinal reactions of the virtual photon.

The momentum of the in-going lepton is p and the momentum of the out-going lepton is p' . According to the kinematics in Fig. 3.2, the momentum transfer in the reaction must be $q = p - p'$. The momentum of the nucleon is P , whereas the momentum of the out-going hadron system must be $P_n = P + q$. Introduce the notations

$$\nu \equiv p^0 - p'^0 = E - E' \quad (3.42)$$

$$q^2 \equiv (p - p')^2 = m_l^2 + m_{l'}^2 - 2(E E' - \mathbf{p} \cdot \mathbf{p}') \approx -4 E E' \sin^2 \frac{\theta}{2} \quad (3.43)$$

$$W^2 \equiv (P + q)^2 = M^2 + 2M\nu - Q^2, \quad (3.44)$$

where $Q^2 \equiv -q^2$ and $P \cdot q \equiv M\nu$ (M is the nucleon mass).

The parameter ν describes the energy loss of the lepton. The square of the momentum transfer q^2 is always negative, *i.e.* $q^2 < 0$, since the photon is

virtual. A Lorentz invariant expression for ν is

$$\nu = \frac{P \cdot q}{M}. \quad (3.45)$$

Introduce also the variable

$$x \equiv \frac{Q^2}{2M\nu}, \quad (3.46)$$

where $0 \leq x \leq 1$.

Assuming that the one-photon exchange process in Fig. 3.2, the cross section of unpolarized lepton (electron or muon) DIS is calculated to be

$$d\sigma = \frac{2M}{s - M^2} \frac{\alpha^2}{Q^4} L^{\mu\nu} W_{\mu\nu} \frac{d^3 p'}{E'}, \quad (3.47)$$

where α is the fine structure constant, s is one of the Mandelstam variables and is given by $s = (p + P)^2$. The lepton tensor can be calculated in the unpolarized case to be

$$L^{\mu\nu} = 2 (p^\mu p'^\nu + p'^\mu p^\nu - p \cdot p' g^{\mu\nu}), \quad (3.48)$$

where $g^{\mu\nu}$ is the metric tensor. The hadron tensor in terms of two structure functions W_1 and W_2 is given by

$$W_{\mu\nu} = -W_1 \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + W_2 \frac{1}{M^2} \left(P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \left(P_\nu - \frac{P \cdot q}{q^2} q_\nu \right). \quad (3.49)$$

Using Eqs. (3.47) - (3.49), the cross section becomes

$$\frac{d\sigma}{d\Omega dE'} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left[2W_1(\nu, Q^2) \sin^2 \frac{\theta}{2} + W_2(\nu, Q^2) \cos^2 \frac{\theta}{2} \right], \quad (3.50)$$

where $d\Omega$ is the solid angle in which the final lepton emerges.

The structure functions F_1 and F_2 are defined in terms of W_1 and W_2 as

$$F_1 \equiv MW_1, \quad (3.51)$$

$$F_2 \equiv \nu W_2. \quad (3.52)$$

The F_1 is associated with the transverse cross section, and the F_2 with both the transverse and longitudinal ones.

In the Bjorken limit ($Q^2 \rightarrow \infty$ with finite and fixed x), the two structure functions F_1 and F_2 are related to each other by the Callan-Gross relation

$$2xF_1 = F_2. \quad (3.53)$$

3.4.2 The Gottfried Sum-Rule

Experiments on DIS of leptons and nucleons have revealed evidence that the leptons scatter, not from the whole nucleon, but from a (almost free) pointlike spin $\frac{1}{2}$ constituent of it, called a *parton*. It is natural to identify the partons as quarks. This is called Feynman's parton model [26, 27].

The Gottfried sum-rule is associated with the difference between the proton and neutron F_2 structure functions measured in unpolarized electron and muon DIS.

Since there is no fixed neutron target, the deuteron is usually used for obtaining the neutron F_2 by subtracting out the proton part with nuclear corrections [21].

In the parton model, the structure functions F_2 and F_1 are given by [28, 29]

$$F_2(x) = x \sum_{q=u,d,s} e_q^2 [q(x) + \bar{q}(x)], \quad (3.54)$$

$$F_1(x) = \frac{1}{2x} F_2(x), \quad (3.55)$$

where e_q is the charge of the quark q . Here $e_u = \frac{2}{3}$ and $e_d = e_s = -\frac{1}{3}$. The structure function F_2 for the proton and neutron are

$$F_2^p(x) = \frac{4}{9}x(u^p(x) + \bar{u}^p(x)) + \frac{1}{9}x(d^p(x) + \bar{d}^p(x) + s^p(x) + \bar{s}^p(x)) \quad (3.56)$$

and

$$F_2^n(x) = \frac{4}{9}x(u^n(x) + \bar{u}^n(x)) + \frac{1}{9}x(d^n(x) + \bar{d}^n(x) + s^n(x) + \bar{s}^n(x)), \quad (3.57)$$

respectively. With the assumption of isospin symmetry in the nucleon, the parton distributions in the neutron can be related to those in the proton. The relations are

$$\begin{aligned} u^n(x) &= d^p(x), & \bar{u}^n(x) &= \bar{d}^p(x), \\ d^n(x) &= u^p(x), & \bar{d}^n(x) &= \bar{u}^p(x), \\ s^n(x) &= s^p(x), & \bar{s}^n(x) &= \bar{s}^p(x). \end{aligned}$$

Using these relations, one obtains the difference between the F_2 for the proton and neutron as

$$F_2^p(x) - F_2^n(x) = \frac{1}{3}x(u(x) - d(x)) + \frac{1}{3}x(\bar{u}(x) - \bar{d}(x)), \quad (3.58)$$

where $u(x) = u^p(x)$, $d(x) = d^p(x)$, $\bar{u}(x) = \bar{u}^p(x)$, and $\bar{d}(x) = \bar{d}^p(x)$. Introducing the valence quark probability distributions $u_v(x) = u(x) - \bar{u}(x)$ and

$d_v(x) = d(x) - \bar{d}(x)$, this difference can be written as

$$\frac{F_2^p(x) - F_2^n(x)}{x} = \frac{1}{3} [u_v(x) - d_v(x) + 2(\bar{u}(x) - \bar{d}(x))]. \quad (3.59)$$

The valence quark distributions should satisfy

$$\int_0^1 u_v(x) dx = 2 \quad \text{and} \quad \int_0^1 d_v(x) dx = 1, \quad (3.60)$$

due to the fact that there are two u valence quarks and one d valence quark in the proton. Integrating Eq. (3.59) over the variable x , one obtains the result

$$\int_0^1 \frac{F_2^p(x) - F_2^n(x)}{x} dx = \frac{1}{3} + \frac{2}{3} \int_0^1 [\bar{u}(x) - \bar{d}(x)] dx. \quad (3.61)$$

If the quark sea is flavor symmetric, *i.e.* $\bar{u}(x) = \bar{d}(x) \forall x$, then

$$\int_0^1 \frac{F_2^p(x) - F_2^n(x)}{x} dx = \frac{1}{3}, \quad (3.62)$$

which is called the Gottfried sum-rule [30]. Introducing the definitions

$$I_G \equiv \int_0^1 \frac{F_2^p(x) - F_2^n(x)}{x} dx, \quad (3.63)$$

$$\bar{u} \equiv \int_0^1 \bar{u}(x) dx, \quad \text{and} \quad \bar{d} \equiv \int_0^1 \bar{d}(x) dx, \quad (3.64)$$

Eq. (3.61) can be rewritten as

$$I_G = \frac{1}{3} + \frac{2}{3} (\bar{u} - \bar{d}). \quad (3.65)$$

Experimentally, the New Muon Collaboration (NMC) [31, 32] at CERN found that, with a reasonable extrapolation in the very small x region, the integral I_G deviated significantly from $\frac{1}{3}$:

$$I_G = 0.235 \pm 0.026.$$

Solving Eq. (3.65) for $\bar{u} - \bar{d}$, one finds

$$\bar{u} - \bar{d} = \frac{3}{2} \left(I_G - \frac{1}{3} \right). \quad (3.66)$$

This leads to the result that, in the proton quark sea, there are more \bar{d} quarks as compared to \bar{u} quarks

$$\bar{u} - \bar{d} = -0.148 \pm 0.039.$$

3.4.3 Drell-Yan Processes

The Drell-Yan process in proton-nucleon collisions, $p + N \rightarrow l^+ + l^- + X$, is illustrated in Fig. 3.3.

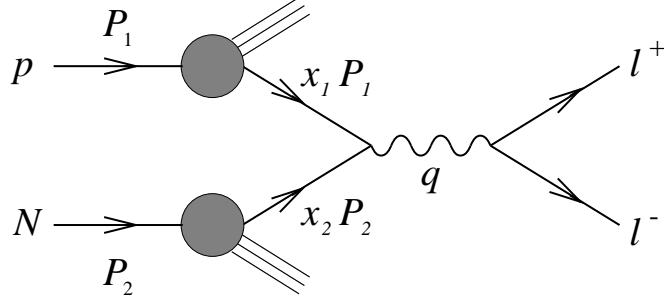


Figure 3.3: The Drell-Yan process $p + N \rightarrow l^+ + l^- + X$. P_1 is the momentum of the proton p and P_2 is the momentum of the nucleon N . x_i , where $i = 1, 2$, is the momentum fraction of the i -parton. q is the momentum of the virtual photon.

A measurement of the difference of the Drell-Yan process of proton on proton and proton on neutron can detect the antiquark ratio, because in such processes the massive $l^+ l^-$ pair is produced by $q\bar{q}$ annihilations [33]. The Drell-Yan cross section in proton-nucleon collisions is [33]

$$\begin{aligned} s \frac{d^2 \sigma^{pN}}{d\sqrt{\tau} dy} &\equiv s \frac{d^2 \sigma(p + N \rightarrow l^+ + l^- + X)}{d\sqrt{\tau} dy} \\ &= \frac{8\pi\alpha^2}{9\sqrt{\tau}} \sum_{q=u,d,s} e_q^2 (q^p(x_1) \bar{q}^N(x_2) + \bar{q}^p(x_1) q^N(x_2)), \end{aligned} \quad (3.67)$$

where $\tau \equiv \frac{M^2}{s}$, $x_1 \equiv \sqrt{\tau} e^y$, and $x_2 \equiv \sqrt{\tau} e^{-y}$. Furthermore, \sqrt{s} is the center of mass collision energy, M is the invariant mass of the lepton pair, and y is the rapidity. Neglecting the sea-sea contributions to the cross sections, one obtains

$$\sigma^{pp} \equiv s \frac{d^2 \sigma^{pp}}{d\sqrt{\tau} dy} \Big|_{y=0} = \frac{8\pi\alpha^2}{9\sqrt{\tau}} \left(\frac{8}{9} u_v \bar{u} + \frac{2}{9} d_v \bar{d} \right), \quad (3.68)$$

$$\begin{aligned} \sigma^{pn} &\equiv s \frac{d^2 \sigma^{pn}}{d\sqrt{\tau} dy} \Big|_{y=0} = \frac{8\pi\alpha^2}{9\sqrt{\tau}} \left(\frac{4}{9} (u_v \bar{d} + d_v \bar{u}) + \frac{1}{9} (d_v \bar{u} + u_v \bar{d}) \right) \\ &= \frac{8\pi\alpha^2}{9\sqrt{\tau}} \frac{5}{9} (u_v \bar{d} + d_v \bar{u}), \end{aligned} \quad (3.69)$$

where all quark distributions are evaluated at $x = \sqrt{\tau}$, since $x = x_1 = x_2$ when $y = 0$. The Drell-Yan (DY) cross section asymmetry is defined as

$$A_{\text{DY}} \equiv \frac{\sigma^{pp} - \sigma^{pn}}{\sigma^{pp} + \sigma^{pn}}. \quad (3.70)$$

Inserting Eqs. (3.68) and (3.69) into Eq. (3.70) gives

$$A_{\text{DY}} = \frac{(4u_v - d_v)(\bar{u} - \bar{d}) + (u_v - d_v)(4\bar{u} - \bar{d})}{(4u_v + d_v)(\bar{u} + \bar{d}) + (u_v + d_v)(4\bar{u} + \bar{d})}, \quad (3.71)$$

where again all quark distributions are evaluated at $x = \sqrt{\tau}$. Introducing the notations

$$\lambda(x) \equiv \frac{u_v(x)}{d_v(x)} \quad \text{and} \quad \bar{\lambda}(x) \equiv \frac{\bar{u}(x)}{\bar{d}(x)}, \quad (3.72)$$

Eq. (3.71) can be expressed as

$$A_{\text{DY}} = \frac{(4 - \lambda(x))(\bar{\lambda}(x) - 1) + (1 - \lambda(x))(4 - \bar{\lambda}(x))}{(4 + \lambda(x))(\bar{\lambda}(x) + 1) + (1 + \lambda(x))(4 + \bar{\lambda}(x))}. \quad (3.73)$$

Thus with experimental measurement of $A_{\text{DY}} = -0.09 \pm 0.02(\text{stat.}) \pm 0.025(\text{syst.})$ and data fit for λ in the range of $[2.00, 2.70]$, the NA51 Collaboration [34] at CERN obtained, at the kinematic point of $y = 0$ and $x = x_1 = x_2 = 0.18$, the following ratio of the antiquark distributions

$$\bar{u}/\bar{d} = 0.51 \pm 0.04(\text{stat.}) \pm 0.05(\text{syst.}).$$

This result is again a clear indication of the excess of \bar{d} quarks over \bar{u} quarks in the nucleon.

In the next chapter we are going to discuss the χQM . With this model we are able to obtain a \bar{u} - \bar{d} asymmetry, which is not present in the NQM and the other models discussed in the beginning of this section.

Chapter 4

The Chiral Quark Model

The chiral quark model (χ QM) was originated by Weinberg [35] and developed by Manohar and Georgi [36], in order to improve the NQM. The chiral quark idea is that there is a set of internal Goldstone bosons (GBs), which couple directly to the constituent (valence) quarks in the interior of the hadrons and particularly the baryons, but not at so small distances that perturbative QCD is applicable.

Although we cannot use QCD perturbatively in the low energy region, it is generally assumed that it has the features of

1. *confinement*: Asymptotic freedom, *i.e.* $\lim_{Q \rightarrow \infty} \alpha_s(Q) = 0$, suggests that the running coupling constant α_s increases at low momentum transfer and long distance, the reason for the binding of quarks and gluons into hadrons. Experimental data indicate that the confinement scale is $\Lambda_{\text{QCD}} = (100 \rightarrow 300) \text{ MeV}$.
2. *chiral symmetry breaking* (χ SB): If $m_q = 0$, $q = u, d, s$, then the QCD Lagrangian is invariant under the independent $\text{SU}(3)$ transformations of the left-handed and right-handed light quark fields, *i.e.* the QCD Lagrangian has a global $\text{SU}(3)_L \times \text{SU}(3)_R$ symmetry. This can be realized in
 - (a) *Wigner mode*: One expects a chirally degenerate particle spectrum: an octet of scalar mesons which have approximately the same masses as the octet of pseudoscalar mesons and a spin $\frac{1}{2}^-$ octet baryons degenerate with the spin $\frac{1}{2}^+$ octet baryons, etc.
 - (b) *Goldstone mode*: The QCD vacuum is not a chiral singlet, but it possesses a set of quark condensates $\langle 0 | \bar{q}q | 0 \rangle \neq 0$, where $q = u, d, s$.

Thus, the symmetry is spontaneously broken as

$$\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \rightarrow \mathrm{SU}(3)_{L+R} = \mathrm{SU}(3)_V,$$

which gives rise to an octet of approximately massless pseudoscalar mesons. These have been tentatively identified with the observed 0^- mesons, *i.e.* the π , K , and η mesons, although the precise relation between the internal and physical GBs is not yet completely understood.

The absence in nature of the degeneracy described in the Wigner mode suggests that the symmetry must be realized in the Goldstone mode.

One of the basic ideas of the χ QM is that the non-perturbative QCD phenomenon of chiral symmetry breaking will take place at an energy scale much higher than that of QCD confinement. It was suggested by Manohar and Georgi [36] that the chiral symmetry breaking will manifest in the Goldstone mode at the scale

$$\Lambda_{\chi\mathrm{SB}} = 4\pi f_\pi, \quad (4.1)$$

where $f_\pi \approx 93 \text{ MeV}$ is the pseudoscalar (pion) decay constant, so $\Lambda_{\chi\mathrm{SB}} \approx 1169 \text{ MeV} \simeq 1 \text{ GeV}$. Thus on scales between Λ_{QCD} and $\Lambda_{\chi\mathrm{SB}}$ there are internal GBs of pseudoscalar nature, massive quarks, and gluons. These are all coupled rather weakly to each other in this regime. The value of α_s in the effective theory region (the χ QM region) can be estimated to $\alpha_s(\Lambda_{\mathrm{QCD}} \leq Q \leq \Lambda_{\chi\mathrm{SB}}) \simeq 0.3$ [36].

4.1 The Chiral Quark Model – The Theory

4.1.1 The QCD Lagrangian

The strong interactions of the light quarks u , d , and s are described by the QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4}G^{\mu\nu,a}G_{\mu\nu}^a + \bar{\Psi}(i\not{D} - M)\Psi, \quad (4.2)$$

where

$$G^{\mu\nu,a} = \partial^\mu G^{\nu,a} - \partial^\nu G^{\mu,a} - gf^{abc}G^{\mu,b}G^{\nu,c}$$

is the gluonic gauge field strength tensor,

$$\Psi \equiv \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

is the Dirac quark field,

$$D^\mu = \partial^\mu + igG^\mu$$

is the covariant derivative, and M is the color-independent quark mass matrix. The mass matrix M is a phenomenological quantity whose origin is unknown, but which can always be brought to diagonal form. Thus, the mass term in the QCD Lagrangian can be written as

$$\mathcal{L}_{\text{mass}} \equiv -\bar{\Psi} M \Psi, \quad (4.3)$$

where

$$M \equiv \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}.$$

The g is the gluon coupling constant (strong coupling constant), f^{abc} , where $a, b, c = 1, \dots, 8$, are the totally antisymmetric structure constants of $\text{SU}(3)$, $G^{\mu,a}$, where $a = 1, \dots, 8$, are the gluon fields,

$$G^\mu \equiv G^{\mu,a} \frac{\lambda^a}{2},$$

and m_q , where $q = u, d, s$, are the quark masses. The gluons are the quanta of the gauge fields of $\text{SU}(3)_{\text{color}}$ and they are massless. The λ^a , where $a = 1, \dots, 8$, are the Gell-Mann matrices that satisfy the $\text{SU}(3)$ commutation relations

$$[\lambda^a, \lambda^b] = 2if^{abc}\lambda^c \quad (4.4)$$

and the normalization condition

$$\text{tr}(\lambda^a \lambda^b) = 2\delta^{ab}. \quad (4.5)$$

If QCD leads to quark confinement, as we shall assume, then the mass parameters m_q , where $q = u, d, s$, are not directly observable quantities. However, they can be determined in terms of observable hadronic masses through current algebra methods (which we shall not go into here). These masses are called *current* quark masses, to distinguish them from the *constituent* quark masses, which will be used here. Constituent quark masses, also called effective quark masses, are parameters used in phenomenological quark models of hadronic structure, like the χQM . The constituent quark masses are in general larger than the current quark masses.

Finally, we mention that QCD is a renormalizable theory, which means that it is possible to use the theory to make finite calculations to arbitrary order in perturbation theory. For details see Refs. [37, 38].

4.1.2 Chiral Symmetry

If the left- and right-handed quark fields are introduced as

$$q_R \equiv P_R q = \frac{1}{2} (1 + \gamma^5) q = \begin{pmatrix} \psi_{q_R} \\ 0 \end{pmatrix} \quad (4.6)$$

$$q_L \equiv P_L q = \frac{1}{2} (1 - \gamma^5) q = \begin{pmatrix} 0 \\ \psi_{q_L} \end{pmatrix}, \quad (4.7)$$

where $q = u, d, s$, $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$ and ψ_{q_R} and ψ_{q_L} are two-component spinors, then

$$q = q_R + q_L = \begin{pmatrix} \psi_{q_R} \\ \psi_{q_L} \end{pmatrix}. \quad (4.8)$$

For the γ matrices we use the chiral representation convention of Ref. [39].

Using these left- and right-handed quark fields, the QCD Lagrangian can be written as

$$\mathcal{L} = -\frac{1}{4} G^{\mu\nu,a} G_{\mu\nu}^a + i\bar{\Psi}_R \not{D} \Psi_R + i\bar{\Psi}_L \not{D} \Psi_L - \bar{\Psi}_L M \Psi_R - \bar{\Psi}_R M \Psi_L, \quad (4.9)$$

where $\Psi_R \equiv \begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix}$ and $\Psi_L \equiv \begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix}$.

The transformations

$$q \rightarrow \gamma^5 q$$

are called *chiral* transformations. Under these transformations $\psi_{q_R} \rightarrow \psi_{q_R}$ and $\psi_{q_L} \rightarrow -\psi_{q_L}$, since

$$\begin{pmatrix} \psi_{q_R} \\ \psi_{q_L} \end{pmatrix} \rightarrow \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \begin{pmatrix} \psi_{q_R} \\ \psi_{q_L} \end{pmatrix} = \begin{pmatrix} \psi_{q_R} \\ -\psi_{q_L} \end{pmatrix}. \quad (4.10)$$

Then, it follows from the definitions (4.6) and (4.7) that $q_R \rightarrow q_R$ and $q_L \rightarrow -q_L$, and also that $\Psi_R \rightarrow \Psi_R$ and $\Psi_L \rightarrow -\Psi_L$. This means that the Lagrangian (4.9) is not invariant under chiral transformations, since the mass terms $-\bar{\Psi}_L M \Psi_R$ and $-\bar{\Psi}_R M \Psi_L$ change sign when chiral transformations are applied. If the mass terms are neglected, then the Lagrangian (4.9) will have an $SU(3)_L \times SU(3)_R$ chiral flavor symmetry. This symmetry is not manifest at low energies in nature, thus it must be broken. As discussed above, two non-perturbative effects are believed to occur in this theory. These are QCD confinement and chiral symmetry breaking. The $SU(3)_L \times SU(3)_R$ chiral symmetry is spontaneously broken down to an $SU(3)_V$ symmetry at an energy

scale $\Lambda_{\chi\text{SB}}$, which is about 1 GeV. This is significantly larger than the QCD confinement scale Λ_{QCD} , which is about 200 MeV. The relevant energy regions are:

- $Q > \Lambda_{\chi\text{SB}}$: In this energy scale region quarks are free and massless. Chiral symmetry (χS) is also manifest.
- $\Lambda_{\text{QCD}} \leq Q \leq \Lambda_{\chi\text{SB}}$: In this energy scale interval, the χQM works. The GBs have given mass to the quarks due to spontaneous symmetry breaking. The GBs are also massive. The degrees of freedom are quarks, gluons, and GBs.
- $Q < \Lambda_{\text{QCD}}$: In this energy scale region quarks are bound into hadrons and they are also massive.

See Fig. 4.1 for a qualitative illustration of the different energy regions.

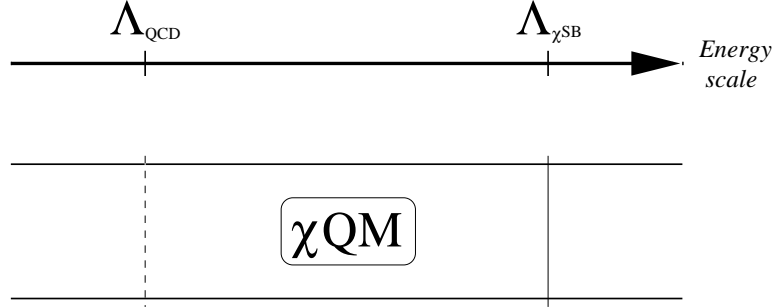


Figure 4.1: The energy scale. The QCD confinement scale (Λ_{QCD}) lies somewhere between 100 MeV and 300 MeV and the chiral symmetry breaking ($\Lambda_{\chi\text{SB}}$) takes place at about 1 GeV.

4.1.3 The Chiral Quark Model Lagrangian

The dynamics of the Goldstone bosons is usually described by a 3×3 unitary matrix field Σ , which takes values in $\text{SU}(3)$. This field is defined as

$$\Sigma \equiv e^{2i\Phi/f_\pi}, \quad (4.11)$$

where the Goldstone boson fields are

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}.$$

and f_π is the pseudoscalar (pion) decay constant measured in $\pi^+ \rightarrow l^+ + \nu_l$. Experimentally, $f_\pi \approx 93 \text{ MeV}$. For simplicity, an auxiliary field ξ is introduced as

$$\xi \equiv e^{i\Phi/f_\pi}, \quad (4.12)$$

which satisfies the relation

$$\Sigma = \xi \xi = \xi^2. \quad (4.13)$$

The effective Lagrangian above Λ_{QCD} and below $\Lambda_{\chi\text{SB}}$ includes quark, gluon, and Goldstone boson fields and can be written down as the most general Lagrangian, which conserves \mathcal{C} , \mathcal{P} , and \mathcal{T} . The conditions follow because the Lagrangian (4.2) has these three symmetries, which are not spontaneously broken by QCD. There are many equivalent such effective Lagrangians.

The first few terms of the effective Lagrangian are

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2} \text{tr} (G^{\mu\nu} G_{\mu\nu}) + \bar{\Psi} (i\not{D} + \not{V} - g_a A \gamma^5 - M) \Psi + \frac{1}{4} f_\pi^2 \text{tr} (\partial^\mu \Sigma^\dagger \partial_\mu \Sigma) + \dots, \quad (4.14)$$

where

$$V^\mu = \frac{1}{2} (\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger), \quad (4.15)$$

$$A^\mu = \frac{1}{2i} (\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger), \quad (4.16)$$

g_a is a possible quark axial-vector current coupling constant, which, however, should be equal to one in the χQM [40], and

$$G^{\mu\nu} = \partial^\mu G^\nu - \partial^\nu G^\mu + ig [G^\mu, G^\nu].$$

The lowest order terms of the effective Lagrangian are

$$\mathcal{L}_{\text{eff},0} = \bar{\Psi} (i\not{D} + \not{V} - A \gamma^5 - M) \Psi + \dots, \quad (4.17)$$

The quark-GB interaction part of the Lagrangian (4.17) is

$$\mathcal{L}_{\text{int}} = \bar{\Psi} (\not{V} - A \gamma^5) \Psi. \quad (4.18)$$

Expanding the vector and axial-vector fields (4.15) and (4.16) up to first order terms in Φ/f_π , one finds

$$V^\mu = 0 + \mathcal{O}((\Phi/f_\pi)^2), \quad (4.19)$$

$$A^\mu = \frac{1}{f_\pi} \partial^\mu \Phi + \mathcal{O}((\Phi/f_\pi)^3). \quad (4.20)$$

Inserting Eqs. (4.19) and (4.20) into the Lagrangian (4.18), one obtains

$$\mathcal{L}_{int} = -\frac{1}{f_\pi} \bar{\Psi} \partial_\mu \Phi \gamma^\mu \gamma^5 \Psi + \mathcal{O}\left((\Phi/f_\pi)^2\right) \approx -\frac{1}{f_\pi} \bar{\Psi} \partial_\mu \Phi \gamma^\mu \gamma^5 \Psi. \quad (4.21)$$

Using the Dirac equation $(i\gamma^\mu \partial_\mu - m_q)q = 0$, the Lagrangian (4.21) can be reduced to

$$\mathcal{L}_{int} \approx i \sum_{q,q'=u,d,s} \frac{m_q + m_{q'}}{f_\pi} \bar{q}' \Phi_{q'q} \gamma^5 q = i \sum_{q,q'=u,d,s} g_8^{qq'} \bar{q}' \Phi_{q'q} \gamma^5 q, \quad (4.22)$$

where $g_8^{qq'} \equiv \frac{m_q + m_{q'}}{f_\pi}$. The parameters m_q and $m_{q'}$ are quark masses.

The Lagrangian of the quark-GB interaction, ignoring space-time structure, is to lowest order

$$\mathcal{L}_I = g_8 \bar{\Psi} \Phi \Psi, \quad (4.23)$$

where g_8 is a coupling constant. This is the interaction Lagrangian of the chiral quark model (χ QM), when other interactions are small and can be ignored.

The χ QM interaction Lagrangian describes the reaction

$$q^\uparrow \rightarrow q'^\downarrow + \text{GB} \rightarrow q'^\downarrow + (q\bar{q})_0,$$

where $q, q' = u, d, s$. The initial quark has spin up, q^\uparrow , whereas the final quark has spin down, q'^\downarrow . This is due to the emission of a GB, which is of pseudoscalar nature. The reaction is called the “spin flip” process. Of course the opposite reaction could occur, *i.e.* that the initial quark has spin down and the final quark has spin up. The reaction process is shown in Fig. 4.2. Observe that

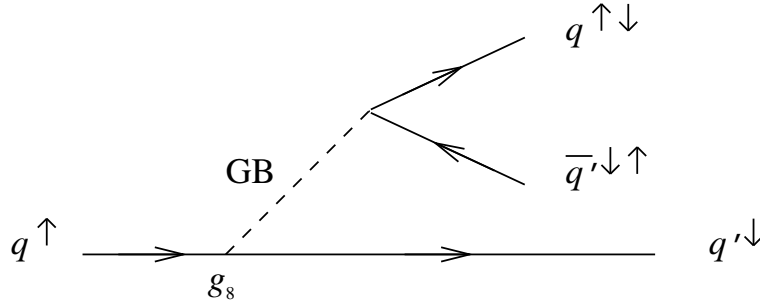


Figure 4.2: The spin flip process.

this reaction, unlike gluon emission, is also a possible flavor changing process, since $q \neq q'$ in general.

4.2 The Chiral Quark Model – The Recipe

In this section the recipe how to calculate the quark spin polarizations for a baryon in the χ QM will be described. This section is quite technical, since all formulas are derived in detail. The reason is that these are not easily accessible elsewhere in the literature.

4.2.1 Flavor Content Calculation

Let a denote the probability of emitting a GB. This GB will eventually split up into a quark-antiquark pair, see Fig. 4.2. These fluctuations of GBs should be small enough to be treated as perturbations. The probability parameter a is proportional to the square of the coupling constant g_8 , *i.e.* $a \propto |g_8|^2$. The GBs of the χ QM are pseudoscalars and will therefore be denoted by the $J^P = 0^-$ meson names π , K , η , and η' , as is usually done.

For example, the transition $u \rightarrow d + \pi^+$ (see Fig. 4.3 (c)) gives

$$|\psi(u \rightarrow d + \pi^+)|^2 = a\hat{d} + a\widehat{\pi^+} = a\hat{d} + a\hat{u} + a\hat{\bar{d}}. \quad (4.24)$$

In the following, only the transitions $u \rightarrow u, d, s + \text{GB}$ will be discussed, see Fig. 4.3 (b) - (d). A very similar discussion can of course be easily made for

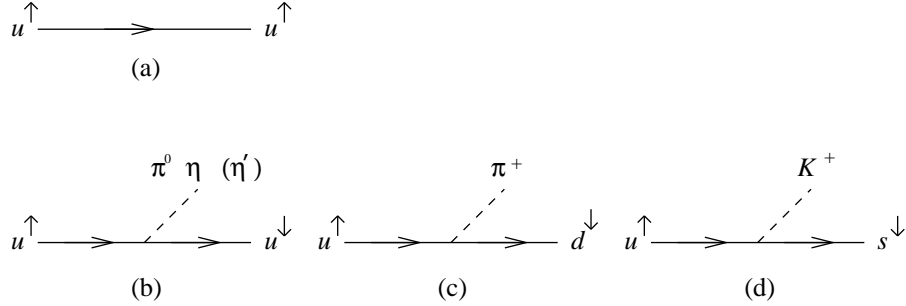


Figure 4.3: $u \rightarrow q + \text{GB}$, where $q = u, d, s$.

the $d \rightarrow u, d, s + \text{GB}$ and $s \rightarrow u, d, s + \text{GB}$ transitions.

The possible u transitions are

$$u \rightarrow (u + (\pi^0, \eta), d + \pi^+, s + K^+).$$

Expanding the Lagrangian (4.23), one obtains

$$\mathcal{L}_I = g_8 \bar{\Psi} \Phi \Psi$$

$$\begin{aligned}
&= g_8 \begin{pmatrix} \bar{u} & \bar{d} & \bar{s} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix} \\
&= g_8 \left[\bar{u} \left(\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta \right) + \bar{d}\pi^- + \bar{s}K^- \right] u + \dots
\end{aligned} \tag{4.25}$$

The first term in expansion (4.25) corresponds to the wave function

$$\psi(u) \sim u \left(\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta \right) + d\pi^+ + sK^+. \tag{4.26}$$

If the GBs are substituted with their quark contents, then

$$\begin{aligned}
\psi(u) \sim & \left(\frac{1}{2} + \frac{1}{6} \right) u(u\bar{u}) + \left(-\frac{1}{2} + \frac{1}{6} \right) u(d\bar{d}) + \left(-\frac{2}{6} \right) u(s\bar{s}) \\
& + d(u\bar{d}) + s(u\bar{s}),
\end{aligned} \tag{4.27}$$

since $\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$, $\eta = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$, $\pi^+ = u\bar{d}$, and $K^+ = u\bar{s}$. The transition probabilities of a u quark can be expressed by the function

$$\begin{aligned}
|\psi(u)|^2 &= a \left[2 \left(\frac{1}{2} + \frac{1}{6} \right)^2 + \left(-\frac{1}{2} + \frac{1}{6} \right)^2 + \left(-\frac{2}{6} \right)^2 + 1^2 + 1^2 \right] \hat{u} \\
&+ \left(\frac{1}{2} + \frac{1}{6} \right)^2 \hat{u} + \left(\left(-\frac{1}{2} + \frac{1}{6} \right)^2 + 1^2 \right) (\hat{d} + \hat{\bar{d}}) \\
&+ \left(\left(-\frac{2}{6} \right)^2 + 1^2 \right) (\hat{s} + \hat{\bar{s}}) \\
&= \frac{2a}{9} \left[14\hat{u} + 2\hat{\bar{u}} + 5 \left(\hat{d} + \hat{\bar{d}} + \hat{s} + \hat{\bar{s}} \right) \right],
\end{aligned} \tag{4.28}$$

where the coefficient of the \hat{q} (or $\hat{\bar{q}}$), where $q = u, d, s$, should be interpreted as the probability of creating this quark (or antiquark) flavor by emitting a GB from a u quark. Similarly for the d and s quarks, the transition probability functions are obtained as

$$|\psi(d)|^2 = \frac{2a}{9} \left[14\hat{d} + 2\hat{\bar{d}} + 5 \left(\hat{u} + \hat{\bar{u}} + \hat{s} + \hat{\bar{s}} \right) \right] \tag{4.29}$$

and

$$|\psi(s)|^2 = \frac{2a}{9} \left[14\hat{s} + 2\hat{\bar{s}} + 5 \left(\hat{u} + \hat{\bar{u}} + \hat{d} + \hat{\bar{d}} \right) \right]. \tag{4.30}$$

The total probabilities of emission of a GB from u , d , and s quarks are given by

$$\Sigma P_q = \frac{2a}{9} (14 + 2 + 5 \cdot 4) = \frac{8a}{3}, \tag{4.31}$$

where $q = u, d, s$.

There is also another possibility that the u quark will not emit a GB as is shown in Fig. 4.3 (a). The probability of no GB emission from the u quark is given by

$$P_u = 1 - \Sigma P_u = 1 - \frac{8a}{3}. \quad (4.32)$$

Similarly for the d and s quarks

$$P_d = P_s = 1 - \frac{8a}{3}. \quad (4.33)$$

The quark (flavor) content in the proton according to the NQM is

$$\tilde{p}_0 = 2\hat{u} + \hat{d}. \quad (4.34)$$

This means that $\bar{u} - \bar{d} = 0$ in the NQM. An emission of a GB can be represented in the χ QM by making the substitution

$$\hat{q} \rightarrow P_q \hat{q} + |\psi(q)|^2$$

in the NQM quark (flavor) content. This substitution describes one interaction. Making this substitution in the proton quark content \tilde{p}_0 , it follows that

$$\tilde{p}_1 = 2P_u \hat{u} + P_d \hat{d} + 2|\psi(u)|^2 + |\psi(d)|^2. \quad (4.35)$$

Inserting Eqs. (4.32), (4.33), (4.28), and (4.29) into Eq. (4.35) yields

$$\tilde{p}_1 = 2(1-a)\hat{u} + 2a\hat{u} + \left(1 - \frac{8a}{3}\right)\hat{d} + \frac{8a}{3}\hat{d} + \frac{10a}{3}\hat{s} + \frac{10a}{3}\hat{s}. \quad (4.36)$$

This leads to a $\bar{u} - \bar{d}$ asymmetry in the proton quark sea

$$\bar{u} - \bar{d} = 2a - \frac{8a}{3} = -\frac{2a}{3}, \quad (4.37)$$

which is not present in the NQM. Using the value $a = 0.083$ from Ref. [41], one has $\bar{u} - \bar{d} \approx -0.06$. Experimentally, this difference is $\bar{u} - \bar{d} = -0.15 \pm 0.04$ [31, 32]. Similarly for the ratio, one has

$$\bar{u}/\bar{d} = \frac{2a}{8a/3} = \frac{3}{4} = 0.75 \quad (4.38)$$

Experimentally, this ratio is $\bar{u}/\bar{d} = 0.51 \pm 0.09$ [34].

We see that we are able to obtain a $\bar{u} - \bar{d}$ asymmetry in the framework of the χ QM. The important thing here is not the exact values, which we have obtained, but the fact that we obtain values different from zero. Further improvements of the χ QM will be discussed later in this chapter. Let us first discuss the spin content of the proton in the basic χ QM.

4.2.2 Spin Content Calculation

The GB emission will flip the helicity (spin) of a quark q as is indicated in the reaction

$$q^\uparrow \rightarrow q'^\downarrow + \text{GB} \rightarrow q'^\downarrow + (q\bar{q}')_0$$

(or $q^\downarrow \rightarrow q'^\uparrow + \text{GB} \rightarrow q'^\uparrow + (q\bar{q}')_0$), while the quark-antiquark pair created by the GB is unpolarized:

$$\psi(\text{GB}) = \frac{1}{\sqrt{2}} \left[\psi(q^\uparrow)\psi(\bar{q}'^\downarrow) - \psi(q^\downarrow)\psi(\bar{q}'^\uparrow) \right]. \quad (4.39)$$

One of the first χ QM predictions about the spin structure is that (to leading order) the antiquarks are not polarized [8]:

$$\Delta_{\bar{q}} = n_{\bar{q}^\uparrow} - n_{\bar{q}^\downarrow} = 0. \quad (4.40)$$

The possible transitions for a u quark with spin up are

$$u^\uparrow \rightarrow (u^\downarrow + (\pi^0, \eta), d^\downarrow + \pi^+, s^\downarrow + K^+),$$

which corresponds to the wave function

$$\psi(u^\uparrow) \sim u^\downarrow \left(\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta \right) + d^\downarrow\pi^+ + s^\downarrow K^+. \quad (4.41)$$

Inserting the quark contents of the GBs, one obtains

$$\psi(u^\uparrow) \sim u^\downarrow \left(\frac{2}{3}u\bar{u} - \frac{1}{3}d\bar{d} - \frac{1}{3}s\bar{s} \right) + d^\downarrow(u\bar{d}) + s^\downarrow(u\bar{s}). \quad (4.42)$$

The transition probability of a u quark can be expressed by the function

$$\begin{aligned} |\psi(u^\uparrow)|^2 &= a \left[\left(\left(\frac{2}{3} \right)^2 + \left(-\frac{1}{3} \right)^2 + \left(-\frac{1}{3} \right)^2 \right) \hat{u}^\downarrow + 1^2 \hat{d}^\downarrow + 1^2 \hat{s}^\downarrow \right] \\ &= a \left(\frac{2}{3} \hat{u}^\downarrow + \hat{d}^\downarrow + \hat{s}^\downarrow \right), \end{aligned} \quad (4.43)$$

where the coefficient of the \hat{q}^\downarrow , where $q = u, d, s$, should be interpreted as the probability of creating this quark with spin down by emitting a GB from a u quark with spin up. Similarly for the d and s quarks with spin up, the transition probability functions are obtained as

$$|\psi(d^\uparrow)|^2 = a \left(\hat{u}^\downarrow + \frac{2}{3} \hat{d}^\downarrow + \hat{s}^\downarrow \right) \quad (4.44)$$

and

$$|\psi(s^\uparrow)|^2 = a \left(\hat{u}^\downarrow + \hat{d}^\downarrow + \frac{2}{3} \hat{s}^\downarrow \right). \quad (4.45)$$

The quark spin structure in the proton according to the NQM is

$$\hat{p}_0 = \frac{5}{3} \hat{u}^\uparrow + \frac{1}{3} \hat{u}^\downarrow + \frac{1}{3} \hat{d}^\uparrow + \frac{2}{3} \hat{d}^\downarrow. \quad (4.46)$$

From this spin structure one is able to obtain the quark spin polarizations in the NQM. The quark spin polarizations for the proton are

$$\Delta u^p = \frac{4}{3}, \quad \Delta d^p = -\frac{1}{3}, \quad \text{and} \quad \Delta s^p = 0. \quad (4.47)$$

This means that the total quark spin polarization $\Delta\Sigma$ for the proton is

$$\Delta\Sigma^p = 1, \quad (4.48)$$

i.e. the spin of the proton in the NQM is carried by the quarks alone.

Making the substitution

$$\hat{q}^\uparrow \rightarrow P_q \hat{q}^\downarrow + |\psi(q^\uparrow)|^2$$

in Eq. (4.46), one finds the quark spin structure of the proton after one interaction in the χ QM as

$$\hat{p}_1 = \left(\frac{5}{3} - \frac{32}{9}a \right) \hat{u}^\uparrow + \left(\frac{1}{3} + \frac{5}{9}a \right) \hat{u}^\downarrow + \left(\frac{1}{3} - \frac{1}{9}a \right) \hat{d}^\uparrow + \left(\frac{2}{3} + \frac{1}{9}a \right) \hat{d}^\downarrow + a \hat{s}^\uparrow + 2a \hat{s}^\downarrow. \quad (4.49)$$

The quark spin polarizations for the proton in the χ QM can now be calculated. They are

$$\Delta u^p = \left(\frac{5}{3} - \frac{32}{9}a \right) - \left(\frac{1}{3} + \frac{5}{9}a \right) = \frac{4}{3} - \frac{37}{9}a, \quad (4.50)$$

$$\Delta d^p = \left(\frac{1}{3} - \frac{1}{9}a \right) - \left(\frac{2}{3} + \frac{1}{9}a \right) = -\frac{1}{3} - \frac{2}{9}a, \quad (4.51)$$

$$\Delta s^p = a - 2a = -a. \quad (4.52)$$

The total quark spin polarization for the proton in the χ QM is thus given by

$$\Delta\Sigma^p = 1 - \frac{16}{3}a. \quad (4.53)$$

The quark spin polarizations in the χ QM will of course reduce to the ones in the NQM, if one puts $a = 0$.

The quark spin polarizations for the neutron can be found by using isospin symmetry. This means that $\Delta u^n = \Delta d^p$, $\Delta d^n = \Delta u^p$, and $\Delta s^n = \Delta s^p$. The quark spin polarizations for all the other baryons can of course be obtained from similar derivations as the one above for the proton.

The quark spin polarizations for the proton in the χ QM in its simplest form were first obtained by Eichten, Hinchliffe, and Quigg [41].

Using $a = 0.083$ [41], we obtain $\Delta\Sigma^p \approx 0.56$. This is much better than the NQM result $\Delta\Sigma^p = 1$. Remember that the latest experiments indicate that $\Delta\Sigma^p \approx 0.30$. Thus, the χ QM gives an explanation for why the spin carried by the quarks can be so small in the experiments.

Let us now turn to a discussion of improvements of the χ QM.

4.2.3 The χ QM with U(3) Symmetry Breaking

The χ QM discussed above has an SU(3) symmetric Lagrangian. To make the model more realistic one can add an SU(3) singlet of η' bosons to the octet of GBs. The Lagrangian is now U(3) symmetric, and such a symmetry is not observed in nature. To insure that the U(3) symmetry is broken, the η' bosons should come with a coupling constant different from the coupling constant g_8 for the other GBs. This is realized by adding the SU(3) scalar interaction $\mathcal{L}' = g_0 \bar{\Psi} \frac{\eta'}{\sqrt{3}} \Psi$, where g_0 is the coupling constant for the η' bosons, to the Lagrangian (4.23). The total interaction Lagrangian is then

$$\mathcal{L}_I = g_8 \bar{\Psi} \Phi \Psi + g_0 \bar{\Psi} \frac{\eta'}{\sqrt{3}} \Psi = g_8 \bar{\Psi} \left(\Phi + \zeta \frac{\eta'}{\sqrt{3}} I \right) \Psi, \quad (4.54)$$

where $\zeta \equiv g_0/g_8$ and I is the 3×3 identity matrix.

Adding the η' meson to the GBs, one obtains the following quark spin polarizations for the proton [42, 43]

$$\Delta u^p = \frac{4}{3} - \frac{1}{9} (8\zeta^2 + 37) a, \quad (4.55)$$

$$\Delta d^p = -\frac{1}{3} + \frac{2}{9} (\zeta^2 - 1) a, \quad (4.56)$$

$$\Delta s^p = -a. \quad (4.57)$$

Here the total quark spin polarization for the proton is given by

$$\Delta\Sigma^p = 1 - \frac{1}{3} (16 + 2\zeta^2) a \quad (4.58)$$

and the \bar{u} - \bar{d} asymmetry formulas are

$$\bar{u} - \bar{d} = \frac{2}{3} (\zeta - 1) a \quad (4.59)$$

and

$$\bar{u}/\bar{d} = \frac{6 + 2\zeta + \zeta^2}{8 + \zeta^2}. \quad (4.60)$$

Again, if $\zeta = 0$ (*i.e.* no η'), the formulas in this subsection reduce to the basic χ QM formulas discussed in Subsections 4.2.1 and 4.2.2.

Using Eq. (4.60) together with the experimental result $\bar{u}/\bar{d} = 0.51 \pm 0.09$, this implies that $-4.3 < \zeta < -0.7$, see Figure 4.4.

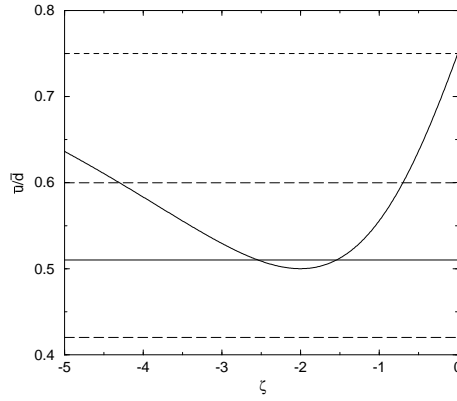


Figure 4.4: The \bar{u}/\bar{d} ratio as a function of ζ . The dashed line shows the \bar{u}/\bar{d} ratio without U(3) symmetry breaking. The solid line is the experimental value. The long dashed lines indicate the experimental error bars.

Taking the value $\zeta = -1.2$ and inserting this into Eq. (4.59), one obtains $a \approx 0.10$ [42]. It is pleasant that a is indeed small, fulfilling the hope that once the features of the non-perturbative phenomenon of spontaneous symmetry breaking are collected in the GBs, the remanent dynamics is perturbative.

Inserting the values $a = 0.10$ and $\zeta = -1.2$ into Eqs. (4.58) - (4.60) gives

$$\Delta\Sigma^p \approx 0.37, \quad \bar{u} - \bar{d} \approx -0.15, \quad \text{and} \quad \bar{u}/\bar{d} \approx 0.53. \quad (4.61)$$

These values agree very well with the experiments.

4.2.4 The χ QM with SU(3) Symmetry Breaking

The fact that the strange quark s is heavier than the non-strange quarks u and d , *i.e.* $m_s > m$, where $m = m_u = m_d$, and the GB non-degeneracy,

would affect the phase space factors for various GB emission processes. Such SU(3) symmetry breaking effects will be introduced in the Lagrangian, simply through the insertion of the following suppression factors: α for the kaons (K), β for the eta (η), and as before ζ for the eta prime (η'), as these strange quark carrying GBs are more massive than the pions (π). The suppression factors α and β are SU(3) symmetry breaking parameters, and the ζ is a U(3) symmetry breaking parameter.

The Lagrangian of interaction is now

$$\mathcal{L}_I = g_8 \bar{\Psi} \Phi \Psi, \quad (4.62)$$

where

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} & \pi^+ & \alpha K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} & \alpha K^0 \\ \alpha K^- & \alpha \bar{K}^0 & -\beta \frac{2\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} \end{pmatrix}.$$

Simple physical considerations would suggest that it should be harder to emit a heavier GB, than a lighter one. Therefore, these suppression factors have been introduced, and the probabilities for processes involving strange quarks will be accordingly modified.

Introducing the suppression factors α and β , one obtains the following quark spin polarizations for the proton [44, 45]

$$\Delta u^p = \frac{4}{3} - \frac{1}{9} (21 + 12\alpha^2 + 4\beta^2 + 8\zeta^2) a, \quad (4.63)$$

$$\Delta d^p = -\frac{1}{3} - \frac{1}{9} (6 - 3\alpha^2 - \beta^2 - 2\zeta^2) a, \quad (4.64)$$

$$\Delta s^p = -\alpha^2 a. \quad (4.65)$$

The total quark spin polarization now turns into

$$\Delta \Sigma^p = 1 - \frac{1}{3} (9 + 6\alpha^2 + \beta^2 + 2\zeta^2) a \quad (4.66)$$

and the \bar{u} - \bar{d} asymmetry formulas become

$$\bar{u} - \bar{d} = \left(\frac{2\zeta + \beta}{3} - 1 \right) \quad (4.67)$$

and

$$\bar{u}/\bar{d} = \frac{21 + 2(2\zeta + \beta) + (2\zeta + \beta)^2}{33 - 2(2\zeta + \beta) + (2\zeta + \beta)^2}. \quad (4.68)$$

SU(3) symmetry breaking in the χ QM has recently been considered by several authors [44–47].

If $\zeta = 0$ (*i.e.* no η') and $\alpha = \beta = 1$ (no suppression), then the formulas in this subsection will reduce to the ones in Subsections 4.2.1 and 4.2.2.

We end this subsection with a discussion about the total quark spin polarizations of the decuplet baryons. The total quark spin polarization for a decuplet baryon B is given by [48]

$$\Delta\Sigma^B = 3 - a(9 + 6\alpha^2 + \beta^2 + 2\zeta^2) + a(3 - 2\alpha^2 - \beta^2)x, \quad (4.69)$$

where x is the number of s quarks in the baryon.

When $a = 0$, *i.e.* GB emission is not possible, then Eq. (4.69) reduces to the NQM result $\Delta\Sigma^B = 3$, as it should. We observe that the NQM result is independent of the number of s quarks in the baryon.

If we have SU(3) symmetry, *i.e.* $\alpha = \beta = 1$, then Eq. (4.69) will reduce to $\Delta\Sigma^B = 3 - 2a(8 + \zeta^2)$, which is equal to about 1.11, since $a = 0.10$ and $\zeta = -1.2$. This result is also independent of the number of s quarks.

Remember that the experimental value of the total quark spin polarization for the proton is about one third of the corresponding NQM value. If we believe that the situation is the same for the decuplet baryons, then the χ QM result of 1.11 is very good. However, there are no experimental data for the total quark spin polarizations of the decuplet baryons.

When we include SU(3) symmetry breaking, the total quark spin polarization becomes larger and it increases linearly with the number of s quarks. For the numerical results, see Table III in Paper 2.

A formula similar to Eq. (4.69) does not exist for the octet baryons.

The quark spin polarizations for the decuplet baryons are listed in Paper 2.

4.2.5 The Probability Parameter a

The parameter a can be calculated, using the chiral field theory approach, to be [41]

$$\begin{aligned} a = & \frac{g_8^2}{32\pi^2} \int_0^1 \theta(\Lambda_{\chi\text{SB}}^2 - \tau(z))z \left\{ \ln \frac{\Lambda_{\chi\text{SB}}^2 + m_\pi^2}{\tau(z) + m_\pi^2} \right. \\ & \left. + m_\pi^2 \left[\frac{1}{\Lambda_{\chi\text{SB}}^2 + m_\pi^2} - \frac{1}{\tau(z) + m_\pi^2} \right] \right\} dz, \end{aligned} \quad (4.70)$$

where $g_8 \equiv \frac{2m}{f_\pi}$, $\tau(z) \equiv m^2 \frac{z^2}{1-z}$, and θ is the Heaviside function.

Using the data $\Lambda_{\chi\text{SB}} = 1169$ MeV, $m = 350$ MeV, $f_\pi = 93$ MeV, and $m_\pi = 140$ MeV, one obtains $a \approx 0.15$. If one instead uses the effective quark mass of

the non-strange quarks $m = 232 \text{ MeV}$ obtained from the χQM [49], then the result is $a \approx 0.083$.

Using the \bar{u} - \bar{d} asymmetry formulas for the χQM with $\text{SU}(3)$ symmetry breaking, Eqs. (4.67) and (4.68), one can construct the following system of equations

$$\begin{cases} \bar{u} - \bar{d} = \left(\frac{2\zeta + \beta}{3} - 1 \right) a \approx -0.15 \\ \bar{u}/\bar{d} = \frac{21 + 2(2\zeta + \beta) + (2\zeta + \beta)^2}{33 - 2(2\zeta + \beta) + (2\zeta + \beta)^2} \approx 0.51 \end{cases}, \quad (4.71)$$

where the right hand sides are the experimental results. Solving the system of equations (4.71), one finds $2\zeta + \beta \approx \begin{cases} -4.1 \\ -2.1 \end{cases}$ and $a \approx \begin{cases} 0.064 \\ 0.088 \end{cases}$.

All these values of a are consistent with $a = 0.1$.

4.3 The Coleman–Glashow Sum-Rule

Let us now turn the discussion to the Coleman–Glashow sum-rule, which imposes a constraint how well the octet baryon magnetic moments can be accounted for.

The Coleman–Glashow sum-rule for the octet baryon magnetic moments is as before

$$\mu(p) - \mu(n) + \mu(\Sigma^-) - \mu(\Sigma^+) + \mu(\Xi^0) - \mu(\Xi^-) = 0. \quad (4.72)$$

This sum-rule is fulfilled in the NQM, which can easily be seen by inserting Eqs. (2.13) - (2.18) into Eq. (4.72). In previous studies of the χQM , which consider the octet baryon magnetic moments [8, 43], it also turns out that this sum-rule is always fulfilled. This is not good since the sum-rule is experimentally found to be broken with about ten standard deviations. Putting the experimental results of the magnetic moments [4] into the left hand side of Eq. (4.72), one finds $(0.49 \pm 0.05) \mu_N$.

In Paper 1 we show that the Coleman–Glashow sum-rule is always fulfilled in the χQM to lowest order, irrespective of $\text{SU}(3)$ symmetry breaking. This is because all model calculations have been carried out by using the usual $\text{SU}(6)$ wave functions for the octet baryons.

In the next section and Paper 1 we will discuss a possible way of overcoming this problem.

4.4 The Chiral Quark Model with Configuration Mixing

In this section we will consider yet another extension of the χ QM. This extension though is of a completely different character, since until now, we have kept the usual $SU(6)$ wave functions when we have calculated the quark spin polarizations of the octet baryons. Now we will introduce the concept of wave function configuration mixing. Configuration mixing is realized by introducing symmetry breaking in the wave functions. The reason why we are led to introduce configuration mixing is that this is a possible way of breaking the Coleman–Glashow sum-rule. There are other models which also violate the Coleman–Glashow sum-rule, some of them can be found in Refs. [22–25]. However, these models do not include \bar{u} – \bar{d} asymmetry and are therefore defective in this aspect.

One way of realizing configuration mixing in wave functions is to write the baryon states as

$$|B\rangle = \cos\theta_B|B_1\rangle + \sin\theta_B|B_2\rangle, \quad (4.73)$$

where $|B_1\rangle$ is the usual $SU(6)$ wave function and $|B_2\rangle$ is the configuration mixing term. The parameter θ_B is a measure of the amount of mixing.

In Paper 1 we study two types of configuration mixing in the wave functions of the octet baryons. These are quark-gluon and quark-diquark configuration mixings and they will be discussed below.

Especially with the quark-diquark configuration mixing, we are able to obtain values of the octet baryon magnetic moments, which are in very good agreement with the experimental results. As a consequence of this, the χ QM with quark-diquark configuration mixing also violates the Coleman–Glashow sum-rule to the right degree.

4.4.1 Quark-Gluon Configuration Mixing

The wave function for an octet baryon with quark-gluon configuration mixing is given by

$$|B^\uparrow\rangle = \cos\theta_B^g|B_1^\uparrow\rangle + \sin\theta_B^g|(B_8G)^\uparrow\rangle \quad (4.74)$$

where $|(B_8G)^\uparrow\rangle$ is the gluonic octet baryon color-singlet wave function and θ_B^g is the quark-gluon mixing angle.

The wave function $|(B_8G)^\uparrow\rangle$ is a coupling of an octet baryon color-octet wave function $|B_8\rangle$ and a spin-one color-octet gluon wave function $|G\rangle$. This type of configuration mixing has been investigated by Lipkin [50] in order to improve the ratio of proton and neutron magnetic moments $\frac{\mu(p)}{\mu(n)}$ and also by Noda *et al.* [51].

The quark spin polarizations with quark-gluon configuration mixing are listed in Appendix B1 at the end of Paper 1.

4.4.2 Diquarks

Before we turn to the subject of quark-diquark configuration mixing, we will briefly discuss diquarks.

In the ordinary non-relativistic quark model, baryons consist of three point-like quarks with spin-parity $\frac{1}{2}^+$, *i.e.* the light quarks u, d, s with charges $2/3, -1/3, -1/3$, and so on. This model is successful in explaining gross properties of baryons, *e.g.* it gives good mass spectra for the octet and decuplet baryons. However, it is difficult to obtain good values of the magnetic moments for these baryons in this model (in principle impossible).

The diquark model is a modification of the usual quark model by considering that two quarks are glued together to form a diquark [3].

In the SU(6) model, two quarks can form 21 symmetric states and 15 anti-symmetric states:

$$\mathbf{6} \otimes \mathbf{6} = \mathbf{21} \oplus \mathbf{15}. \quad (4.75)$$

The SU(3) content of the representation $\mathbf{21}$ can be expressed as

$$\mathbf{21} = \{\mathbf{6}\} \times 3 + \{\mathbf{3}^*\} \times 1, \quad (4.76)$$

where the brackets indicate irreducible representations of SU(3) and the factors 3 and 1 are spin degrees of freedom of the diquark. There are SU(3)-sextet axial-vector diquarks and SU(3)-triplet scalar diquarks.

We will only consider the scalar diquarks. The symbol $(q_1 q_2)_d$ will denote a scalar diquark built up of the quarks q_1 and q_2 .

4.4.3 Quark-Diquark Configuration Mixing

The wave function for an octet baryon with quark-diquark configuration mixing is given by

$$|B^\uparrow\rangle = \cos \theta_B^d |B_1^\uparrow\rangle + \sin \theta_B^d |B_d^\uparrow\rangle, \quad (4.77)$$

where $|B_d^\uparrow\rangle$ is the quark-diquark wave function and θ_B^d is the quark-diquark mixing angle.

This type of quark-diquark configuration mixing has been studied earlier by Noda *et al.* [51]. They found that it can give a good fit to existing data on the strange sea polarization in the proton.

In Paper 1 we illustrate how a simple mechanism in the form of a toy model can bring about quark-diquark configuration mixing in the wave functions for the octet baryons.

The quark spin polarizations with quark-diquark configuration mixing are listed in Appendix B2 at the end of Paper 1.

4.5 Weak Form Factors in the χ QM

The transition matrix element $\mathcal{M}_{B \rightarrow B' l^- \bar{\nu}_l}$ for the decay $B \rightarrow B' + l^- + \bar{\nu}_l$ ($q \rightarrow q' + l^- + \bar{\nu}_l$), is given by

$$\mathcal{M}_{B \rightarrow B' l^- \bar{\nu}_l} = \frac{G}{\sqrt{2}} V_{qq'} \langle B'(p') | J_{\text{weak}}^\mu | B(p) \rangle L_\mu, \quad (4.78)$$

where G is the Fermi coupling constant, $V_{qq'}$ is the qq' -element of the Cabibbo–Kobayashi–Maskawa mixing matrix [52, 53], and conservation of momentum gives that $p = p' + p_{l^-} - p_{\bar{\nu}_l}$. The leptonic current is

$$L_\mu = \bar{u}_{l^-}(p_{l^-}) \gamma_\mu (1 - \gamma^5) v_{\bar{\nu}_l}(p_{\bar{\nu}_l}), \quad (4.79)$$

where p_{l^-} ($p_{\bar{\nu}_l}$) and u_{l^-} ($v_{\bar{\nu}_l}$) are the momentum and Dirac spinor of the lepton (antineutrino), respectively. The hadronic weak current is

$$J_{\text{weak}}^\mu = J_V^\mu - J_A^\mu, \quad (4.80)$$

where J_V^μ is the vector current and J_A^μ is the axial-vector current. The matrix element of the vector current in momentum space of the transition $B \rightarrow B' + l^- + \bar{\nu}_l$ is given by

$$\begin{aligned} \langle B'(p') | J_V^\mu | B(p) \rangle &= \bar{u}'(p') \left(f_1(q^2) \gamma^\mu - i \frac{f_2(q^2)}{M_B + M_{B'}} \sigma^{\mu\nu} q_\nu \right. \\ &\quad \left. + \frac{f_3(q^2)}{M_B + M_{B'}} q^\mu \right) u(p) \end{aligned} \quad (4.81)$$

and the matrix element of the axial-vector current by

$$\begin{aligned} \langle B'(p') | J_A^\mu | B(p) \rangle &= \bar{u}'(p') \left(g_1(q^2) \gamma^\mu \gamma^5 - i \frac{g_2(q^2)}{M_B + M_{B'}} \sigma^{\mu\nu} q_\nu \gamma^5 \right. \\ &\quad \left. + \frac{g_3(q^2)}{M_B + M_{B'}} q^\mu \gamma^5 \right) u(p), \end{aligned} \quad (4.82)$$

where M_B ($M_{B'}$), p (p'), $u(p)$ ($u'(p')$), and $|B(p)\rangle$ ($|B'(p')\rangle$) are the mass, momentum, Dirac spinor, and external baryon state of the initial (final) baryon B (B'), respectively, and $q = p - p'$ is the momentum transfer [9]. The functions $f_i(q^2)$, $i = 1, 2, 3$, are the vector current form factors and the functions $g_i(q^2)$, $i = 1, 2, 3$, are the axial-vector current form factors. The form factors

are Lorentz scalars and they contain all the information about the hadron dynamics. f_1 is the *vector* form factor, f_2 is the *induced tensor* form factor (or *weak magnetism* form factor or *anomalous magnetic moment* form factor), f_3 is the *induced scalar* form factor, g_1 is the *axial-vector* form factor, g_2 is the *induced pseudotensor* form factor (or *weak electric* form factor), and g_3 is the *induced pseudoscalar* form factor.

Under G -parity, the f_2 transforms with the same sign as the f_1 , whereas the f_3 has the opposite sign, and the g_3 transforms with the same sign as the g_1 , whereas the g_2 has the opposite sign. The currents with form factors f_3 and g_2 are therefore called *second-class currents*, and the others are *first-class currents* [54].

One also introduces the notations

$$\rho_f \equiv \frac{f_2}{f_1}, \quad \rho_g \equiv \frac{g_2}{g_1}, \quad g_A \equiv \frac{g_1}{f_1}, \quad g_{PT} \equiv \frac{g_2}{f_1}, \quad (4.83)$$

where g_A is called the weak axial-vector form factor and g_{PT} is called the weak pseudotensor form factor.

When measuring the weak form factors, it is normally assumed that the second-class current form factors f_3 and g_2 are negligible. However, for strangeness-changing transitions, *i.e.* $\Delta S = 1$ transitions, these currents may be non-negligible. Hsueh *et al.* [55] have measured the second-class current form factor g_2 for the $\Sigma^- \rightarrow n + e^- + \bar{\nu}_e$ transition and found it quite large. All existing experimental data have quite large errors, so while waiting for the experiments to become better, several theoretical works in this field have been made [56–62]. Most of them have been done in the framework of the MIT bag model.

In Paper 3 we derive the following estimates of the weak vector and axial-vector form factors for the semileptonic octet baryon decays $B \rightarrow B' + l^- + \bar{\nu}_l$ in the χ QM up to first order in the $SU(3)$ symmetry breaking mass differences

$$f_1 = f_1^{\text{QM}} \quad (4.84)$$

$$f_2 = \left(\frac{\Sigma}{\sigma} G_A - 1 \right) f_1^{\text{QM}} \quad (4.85)$$

$$f_3 = \frac{\Sigma}{\sigma} (E G_A - \epsilon) f_1^{\text{QM}} \quad (4.86)$$

and

$$g_1 = g_a g_1^{\text{QM}} \quad (4.87)$$

$$g_2 = \left(\frac{\Sigma}{\sigma} \epsilon - \frac{1}{2} \left(1 + \frac{\Sigma^2}{\sigma^2} \right) E \right) g_a g_1^{\text{QM}} \quad (4.88)$$

$$g_3 = \left(\frac{1}{2} \left(1 - \frac{\Sigma^2}{\sigma^2} \right) + \frac{\Sigma^2}{\sigma^2} g_3^q \right) g_a g_1^{\text{QM}}, \quad (4.89)$$

where $\Sigma \equiv M_B + M_{B'}$, $\Delta \equiv M_B - M_{B'}$, $E \equiv \frac{\Delta}{\Sigma}$, $\sigma \equiv m_q + m_{q'}$, $\delta \equiv m_q - m_{q'}$, $\epsilon \equiv \frac{\delta}{\sigma}$, and $G_A \equiv \frac{g_1^{\text{QM}}}{f_1^{\text{QM}}}$. Note that the first-class current form factors f_1 , f_2 , g_1 , and g_3 only contain terms with even powers of E and ϵ , while the second-class current form factors f_3 and g_2 only contain terms with odd powers of E and ϵ . This follows from the Ademollo–Gatto theorem [59, 63]. Observe also that the expressions for f_i and g_i , where $i = 1, 2, 3$, in Eqs. (4.84) - (4.89) are evaluated at $q^2 = \Delta^2$.

The parameter g_3^q is found to be

$$g_3^q = \frac{\sigma^2}{\Delta^2 - m_\Phi^2} g_a, \quad (4.90)$$

where m_Φ is the mass of the pseudoscalar GB field.

We here quote the simple result

$$g_A = g_a G_A, \quad (4.91)$$

where g_a is the quark axial-vector current coupling constant and G_A is the SU(6) weak axial-vector form factor. It is argued that g_a should be equal to one in constituent quark models [40], and since the χ QM is such a model, we put $g_a = 1$. In Section 3.3, we have derived the SU(6) weak axial-vector form factors G_A for all semileptonic octet baryon decays.

All the weak form factors are discussed and investigated in Paper 3. The results obtained for the g_A 's are encouraging and represent an improvement of the NQM results. The ρ_f ratios obtained are within the experimental errors and also close in magnitude to the conserved vector current (CVC) results. Unfortunately, the theoretical estimates for the only measured ρ_g and g_{PT} , $\rho_g^{\Sigma^- n}$ and $g_{PT}^{\Sigma^- n}$, are not in agreement with the experimental values by Hsueh *et al.* [55] However, none of the other existing models give values in agreement with the experimental ones either.

Chapter 5

Conclusions

In this thesis we have studied various aspects of baryons in the framework of the chiral quark model (χ QM).

As can be seen from these studies, the χ QM represents a clear improvement over the NQM for the description of baryon properties, such as quark spin polarizations, magnetic moments, and weak form factors. Even so, the χ QM cannot reproduce the experimentally observed breaking of the Coleman–Glashow sum-rule for the octet baryon magnetic moments. Using quark-diquark configuration mixing in the $SU(6)$ wave functions for the octet baryons, we have seen that the Coleman–Glashow sum-rule can be broken to the right degree. As a bonus we also obtain very accurate octet baryon magnetic moments.

Using parameters obtained in the χ QM in Paper 1, we are also able to obtain values of the magnetic moments for the decuplet baryons Δ^{++} and Ω^- , which are in general agreement with the present experimental data.

In addition, using quark spin polarizations from the χ QM, we can calculate weak vector and axial-vector form factors for the semileptonic octet baryon decays. The results are in general agreement with the experiments.

Further research is required to understand how configuration mixing comes about and how gluons and diquarks could change the interaction Lagrangian, since so far such interactions have been neglected. Other future developments could be to introduce the dynamics of the Goldstone bosons and to incorporate relativistic effects in the calculations. Investigation of baryon scattering should also be possible in the spirit of the χ QM.

More experimental results for the other decuplet baryons are needed in order to be able to compare with our predictions, and also, if possible, some data from experiments on the spin polarizations of the decuplet baryons.

Better experimental results for the second-class weak form factors would also be of importance to be able to distinguish between various models.

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Paper 1:

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Paper 2:

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Paper 3:
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Quark Model*
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3