On the Logical Constraints on Legal Norms*

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Abstract：The problem of the logical constraints of legal norms can be divided into three levels: the "practical problem", the "specific problem" and the "meta-problem". At the level of the "meta-problem", the normative logic skepticism negates the possibility of making logical inferences from valid legal norms, but it points out the possibility of normative logic dealing with the normative statements of the idealized director. By the conception of the idealized director, legal norms must satisfy the two logical constraints of inference-licensing and consistency, and deontic logic can account for the forms of existence of these two constraints. In legal practice, the judiciary creates laws under the constraints of deontic logic through the "refraction mode".

Key Words：legal norms; normative logic ; permission ; consistency ; refraction mode

Introduction

In the methodology of law, the judicial process is considered to follow the logical deduction from general norms to individual norms, that is, the "judicial syllogism";[1]The sources of these constraints on legal norms are often considered to be logical.

There are discussions at three levels, namely the "practical problem", the "specific problem" and the "meta-problem", regarding the logical constraints on legal norms. The "practical problem" discusses how to apply logical rules in the application of legal norms, and the "specific problem" discusses what logical rules are followed by legal norms, For example, what kinds of legal norms cannot coexist logically, and what kinds of legal norms can be obtained logically from other kinds of legal norms.

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[1] Vgl. Karl Larenz. (1991). *Methodenlehre der Rechtswissenschaft*, Berlin: Springer-Verlag, 271-277.

However, these specific problems are faced with many challenges. For example, there are disputes about whether the conflict between norms in the form of "one ought to do something" and "it is prohibited to do something" is logical inconsistent, and whether individual norms can be obtained logically from general norms. [2]These problems also lead to disputes about practical problems: in the case where the relationship between logic and law is unclear, does the judge create laws or apply laws? In my opinion, it is difficult to solve these problems because we have not clarified what the nature of logic of legal norms is and in what sense it constrains legal norms. Therefore, we must return to the level of the "meta-problem" to discuss the logical constraints on legal norms.。

I will first discuss the "meta-problem" about the constraints of legal norms.Then, based on the discussion of the meta-problems, I will discuss of the "specific problem", and finally move on to the "practical problem". In the first part of this article, by the discussion of normative logic skepticism, I will illustrate in what sense normative logic can constitute constraints on legal norms. In the second part, by explaining the semantics of Standard Deontic Logic (SDL), I will justify the logical constraints on legal norms. In the third part, I will explain how these constraints work in judicial practice.

The Possibility of logical constraints on legal norms

There are lots of advantages to assume that legal norms are indeed validly constrained by logical rules: We can not only explain our intuition about legal reasoning, but enable the existing legal arguments, which generally play a good role in legal practice, to be based on a stable and rational foundation. However, the "success" of this concept does not mean that it is actually valid - at most, this can only be defended by pragmatical arguments. Jörgen Jörgensen proposed: “So we have the following puzzle: According to a generally accepted definition of logical inference only sentences which are capable of being true or false can function as premisses or conclusions in an inference; nevertheless it seems evident that a conclusion in the imperative mood may be drawn from two premisses one of which or both of which are in the imperative mood.”[3] This problem is therefore called "Jörgensen's Dilemma".

In the years of debate, Jörgensen's Dilemma has been summarized as a conflict between two horns: ① Imperative sentences do not have truth value (true or false), and since logic only deals with sentences that are true of false, imperative sentences cannot be the premises or conclusions of logical inferences (Horn 1, or the Prohibition Thesis); ② The normative arguments in our daily lives seem obviously valid, so imperative sentences can be the premises or conclusions of logical inferences (Horn 2,

[2] See Kramer, M. H. (2024). *Rights and Right-Holding: A Philosophical Investigation,* Oxford: Oxford University Press, 28-34; Jörgensen, J. (1938). Imperatives and Logic, *Erkenntnis* Vol.7, 290.

[3] Jörgensen, J. (1938). Imperatives and Logic, *Erkenntnis* Vol.7, 290.

or the Permission Thesis). [4]the "Prohibition Thesis" can be reconstructed into such a set of arguments:

① Imperative sentences cannot be true or false;

② The premises and conclusions of logical inferences are, and can only be, sentences that are true or false.

③ Therefore, imperative sentences cannot be the premises or conclusions of logical inferences.

The "Permission Thesis" does not constitute an argument, but is merely an intuition, and thus requires further argumentation from its supporters.[5]

In this process, supporters of the so-called "normative logic skepticism" claim, legal norms can not be dealt by logic. If they are true, then there are no so-called "logical constraints" on legal norms. In the process of reflecting on normative logic skepticism, I will not only re-establish the constraints of normative logic on legal norms, but also explain the nature of such constraints.。

Normative Logic Skepticism

The basic viewpoints of normative logic skepticism are: ① Normative sentences can not be true or false, therefore cannot be premises or conclusions of logical inferences; ② Propositions about the existence of norms (that is, normative statements or normative sentences) may be the premises and conclusions of logical inferences, and the intuitively valid inferences about norms proposed by the Permission Thesis are inferences about normative statements; or ②' The intuitively valid inferences about norms proposed by the Permission Thesis are invalid.

It should be particularly noted here that not all supporters of normative logic skepticism agree the following Thesis of the irrationality of legal argumentation: The reasoning in legal argumentation is based on will rather than reason. This is because Thesis① and the Thesis of the irrationality of legal argumentation cannot be derived from each other. Quite a number of the supporters of normative logic skepticism merely hold the view that norms can not be dealt with by formal logic, yet they believe that formal logic should be followed in the process of norm formulation and interpretation. In addition, from the perspective of the history of philosophy, the motivation for some advocates of normative logic skepticism is to separate practical reason from formal logic, and they claim that the argumentation of norms can conform to practical reason without formal logic.

The "realist argumentation" of norms is the most common argument of normative logic skepticism. This argument holds that existence of norms is similar to that of natural entity, natural entity exists in the natural world, and norms exist in the legal

[4] See Coyle, S. (2020). Facing Jörgensen's Dilemma, *Northern Ireland Legal Quarterly* Vol. 55 No.4, 351-353; Leon-Untiveros, M. (2016). Jörgensen's Dilemma: The Quest for Semantic Foundation of Imperatives, *Phainomenon* Vol. 15, 120.

[5] Ross, A. (1968). *Directives and Norms*, New York: Humanity Press, 139-140.

system. The existence of natural entity makes it only possible for logic to "talk about" it, that is, it becomes the truth-maker of a proposition, but it cannot become premises or conclusions of logical inferences; similarly, the existence of norms also makes it so that logic can only "talk about" norms, and cannot make norms premises or conclusions of logical inferences. Norms can only be the truth-maker of those "normative statements" that talk about norms.[6]

There are stronger version and a weaker version in the "realist argumentation". The most famous of the weaker versions comes from Hans Kelsen. He pointed out that norms cannot contain both an imperative or prescriptive and an indicative or descriptive factor. The former is the meaning of an act of thought, while the latter is the meaning of an act of will. Truth and falsity are properties of statements or assertions, while validity and invalidity are not properties of the norm, but of the existence of the norm. A false statement is still a statement, but the invalidity of the existence of a norm means that the norm no longer exists. [7] Kelsen emphasized that although both norms and statements have an inherent connection with language, the similarity between norms and statements is only prima facie. On the other hand, a normative statement that describes whether a norm exists or not is a kind of statement.

The stronger version, on the other hand, posits the existence of a "normative world". Robert Walter holds the view that whether a proposition is true or false does not depend on whether it conforms to the actual state of affairs in a certain factual world it describes, but rather on the fact that this proposition is regarded by people as a reasonable interpretation of the factual world. Therefore, Walter does not confer a special ontological status on the "factual world" and the propositions that describe the "factual world", but rather they have the same status as other statements that are considered reasonable by people. Thus, those statements about norms that are regarded as reasonable can also be considered as propositions possessing truth values, and there is also a corresponding "normative world". Furthermore, Walter supports a normative statement theory similar to that of Kelsen, and classifies the intuitively valid inferences about norms as logical inferences among the normative statements that describe the normative world. [8]Regarding the question of "whether general norms can logically deduce individual norms", Walter gives a negative response: those normative statements that seem to describe individual norms and are derived from the normative statements describing general norms cannot create a new individual norm that exists in the normative world, but merely re-express the general norm.[9]In other words, Walter believes that the proposition about a general norm and the proposition about an individual norm deduced from it have the same meaning and describe the same general norm. Thus, Walter denies the existence of logical

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[6] "Normative statement" refers to a proposition that discusses whether a individual norm exists in the legal system. This concept is often used, but different scholars have given it different names. For example, von Wright calls it "normative propositions", and Kelsen calls it "normative statement".

[7] Kelsen, H. (1973). Law and Logic, in: *Essays in Legal and Moral Philosophy,* selected by Ota Weinberger, trans. By Peter Heath,Dordrecht: D. Reidel Publishing Company, 230.

[8] Walter, R. (1996). Jörgensen's Dilemma and How to Face it, *Ratio Juris* Vol.9 No.2, 169-170.

[9] Walter, R. (1997). Some Thoughts on Peczenik's Replies to "Jörgensen's Dilemma and How to Face it", *Ratio Juris* Vol.10 No.4, 393.

relationships between norms. He treats the intuitively valid logical inferences about norms as the logical relationships between normative statements. Moreover, following Kelsen's approach in his later years, Walter denies that the logical inferences between normative statements can be indirectly applied to the norms themselves.

There always seems to be exceptions for general norms, which also supports one of the arguments of normative logic skepticism, I conclude it as “exception argumentation”. It seems that all general norms have exceptions, and these exceptions are inexhaustible. This makes it necessary, in the inference from a general norm to a special norm, that the general norm as the premise must conjoin an infinite number of exceptions as premises. However, a well-formed formula in classical logic is always a finite symbol string. Therefore, when general norms are in logical inferences, they cannot be formalized into well-formed formulas. Sean Coyle, for example, believes that the deontic logicians simplify the norms in moral and legal arguments into simple universal propositions such as "Love your neighbor" (which he calls "Jörgensen sentences"), but in actual moral and legal arguments, such sentences are hardly ever used.[10]A more radical view holds that even in statements, the inference from a universal proposition to a singular proposition is not necessarily logically valid. A typical example is that some scientific theories expressed as universal propositions have counterexamples, but we do not regard such a proposition as "false" or "erroneous", but merely consider that the scope of this proposition must be narrowed. In this way, even if a universal proposition is true, the singular propositions that can be deduced from it are not necessarily all true. [11]These illustrate the two forms of “exception argumentation”, the first form requires its opponents to retain the classical definition of logical implication and admit that general norms cannot be formalized into well-formed formulas, the second form requires them to directly give up the classical definition of logical implication. And both of these points are difficult for the opponents of normative logic skepticism to accept.

Reflection on Normative Logic Skepticism

Normative logic skepticism is, for who attempts to "solve" Jörgensen's Dilemma, must be responded to directly. Relatively, the "exception argumentation" is easier to refute. The "exception argumentation" often comes from the supporters of some informal logic. Whether logic is the ontological structure of the world or the epistemological structure of argumentation is a long-standing issue in the philosophy of logic, and contemporary philosophy of logic supports more the latter view.[12]However, unlike our static assumptions about the ontological structure of the world, the propositions used in the process of argumentation are often defeasible. Thus, if logic is regarded as the epistemological structure of a certain type of

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[10] Coyle, S. (2020). Facing Jörgensen's Dilemma, *Northern Ireland Legal Quarterly* Vol.55, No.4, 351-353.

[11] Mikael M. Karlsson, (1995). Defeating the Inference from General to Particular Norms, *Ratio Juris* Vol.8 No.3, 273-276.

[12] Weinberger, O. (1999). *Against the Ontologization of Logic: A Critical Comment on Robert Walter ’s Tackling Jørgensen’s Dilemma*, Ratio Juris vol. 12 No. 1, 97.

argumentation, one will give up the principle of bivalence in classical logic and the concept of validity derived from it, which is characterized by truth preservation, and instead use the principle of the validity in argumentation. But it can be seen that there is a significant difference between the validity of an argumentation and the validity of logic. The validity in argumentation places more emphasis on the procedural issues of the argument rather than the truth preservation of the reasoning, because the dynamic process of argumentation needs to pay attention not only to the truth of the premises, but also to the reasoning from the premises to the conclusion. However, when we mention logical validity, we are more eager to pursue a static and constant standard, which serves as the limit that the dynamic process of argumentation intends to approach.

The realist argumentation is persuasive. Its persuasive force aspects lie in the following three points: ① As an directive issued by legislators, law is a performative action. Legal norms are not representative statements. They aim to provide reasons rather than represent the existing reasons; ②Under the background of positivism, legal statements about what legal norms are depend on facts, and it is entirely possible that there are seemingly illogical norms in law; ③ According to ① and ②, if we follow Kelsen and regard legal norms as taking the quasi-mental attitude of legislators towards specific states of affairs as a necessary condition, then in the sentences describing the existence of legal norms, the sentences about those specific states of affairs of the norms will be in an intensional context. For example, in the sentence "The legislator supports taxing all the rich", "taxing all the rich" is in an intensional context. If Hans is rich, although "taxing all the rich" implies "taxing Hans", the sentence "The legislator supports taxing all the rich" does not imply "The legislator supports taxing Hans", because the legislator may not even know who Hans is, let alone hold a certain mental state towards him.

Therefore, Those dealt with by normative logic should have the following characteristics: ① Representativeness, that is, sentences that reflect the facts in the world; ② Extensionality, that is, it is able to transform the sentences about specific states of affairs in the sentences describing the quasi-mental attitude of legislators towards specific states of affairs into an extensional context, so that logical inferences can be made about it.

In the following text, we will propose the concept of an "idealized director" and point out that those dealt with by normative logic is not the will of real legislators, but the will of the "idealized director", so as to meet the above requirements. The "idealized director" imitates the non-normative idealization method in meta-ethics. There is no need to consider whether the quasi-mental state of the idealized director towards states of affairs itself meets the requirements of moral normativity, but only needs to meet the requirement of non-conflict. In other words, each legislator has an "idealized director" relative to him. This director retains all his desire states, but only has complete empirical knowledge. The idealized director only needs to possess sufficient non-moral information, enabling them to recognize the potentially conflicting states of affairs in the world without contradiction and maintain psychological coherence.

This coherence has two requirements: consistency and inference-licensing. The former means that once the premises of a logical inference are accepted, the conclusion cannot be denied. The latter means that when accepting the premises, one not only cannot deny the conclusion, but also must accept the conclusion.[13]

The mental states of the idealized directors, which possess these two characteristics, can be explained with the theory of high-order attitudes proposed by Simmon Blackburn. For the idealized directors, not only do they have desires for specific states of affairs, but due to their complete empirical knowledge, they also have high-order desires that their own desires conform to logical requirements. For example, the idealized director may desire that all subjects pay taxes, or may desire that all subjects do not pay taxes. However, he opposes desiring that all subjects both pay and do not pay taxes. He also opposes desiring that all subjects pay taxes while lacking the desire for subjects to pay taxes. Therefore, he will not desire that all subjects both pay and do not pay taxes, nor will he desire that they pay taxes while not desiring that those under his governance pay taxes. In this way, the idealized director opposes inconsistent desires within himself and also opposes the inconsistency of his own desires. The former corresponds to the principle of inference-licensing, and the latter corresponds to the principle of consistency.

The trouble is that there are two types of "inconsistency" in legal norms, namely, "ought to do vs. permitted to do (something is an obligation and at the same time not an obligation)" and "ought to do vs. forbidden to do (something is an obligation and at the same time prohibited)". Can both of these two types of inconsistency be supported?

Mark Schroeder has defined the attitude of "Inconsistency-Transmitting". An attitude is inconsistency-transmitting if two instances of this attitude are inconsistent with each other when they have incompatible contents. Intentions and beliefs are both attitudes of "inconsistency-transmitting". If the attitude of the legislator's requirement is of this kind, then it is easy to explain that "ought to do something" and "be prohibited from doing something" in the normative system are inconsistent, because it means that the authority has a demanding attitude towards a set of inconsistent contents (p and ¬p) at the same time.[14]This can explain the possibility of logical inconsistency between "ought to do something" and "be prohibited from doing something", but it cannot explain the logical inconsistency between "ought to do something" and "be permitted not to do something (not ought to do something)".

According to Kelsen, norms, as the meaning of acts of will, express attitudes towards certain states of affairs. However, in addition to having attitudes towards states of affairs expressed through norms, legislators also have attitudes towards norms expressed through the creation of norms. In my opinion, if we want to impose certain constraints on the extreme irrationality of legislators, we must rule out the possibility of the simultaneous existence of opposing attitudes towards norms. That is to say:

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[13] Schroeder, M. (2024). *Noncognitivism in Ethics*, New York: Routledge.115-116.
[14] Schroeder, M. (2024). *Noncognitivism in Ethics*, New York: Routledge.139-140.

Legislators cannot hold both a supportive attitude and an unsupportive attitude towards a certain norm.

Suppose someone asserts the proposition p. The proposition that someone believes in this natural state of affairs can be written as b(p). The negation of the proposition p is ¬p, and the proposition that someone believes in the negation of the proposition p in this natural state of affairs can be written as b(¬p), while the proposition that someone does not believe in the proposition p in this natural state of affairs can be written as ¬b(p). An idealized person with sufficient knowledge of the real world cannot simultaneously believe the proposition p and the proposition ¬p.

In this example, we can impose the following requirement on the idealized individual: One cannot believe both p and ¬p (or disbelieve p). We term "believing" as an attitude towards a proposition. As stated above, the act of a legislator creating a norm is not merely a simple natural state of affairs; rather, through his act of will, it expresses an attitude towards the norm (which we refer to as support and non-support). The attitude of a legislator in supporting the existence of norm n is denoted as s(n*), where n* represents the normative statement: Norm n exists. By drawing an analogy with the attitude b of "believing", we can conclude that for an idealized director, s(n*) and s(¬n*) are inconsistent. This is because the function of a norm is to provide reasons to guide people's actions. If norm n both exists and does not exist, then it is impossible to achieve the guiding purpose of the norm.

I assume that participants in practical argumentation, just like those in theoretical argumentation, must be sincere, that is, they cannot hold contradictory attitudes towards the statements they express. This requirement endows the statements expressed by each participant in the argumentation with two different aspects. The first is the aspect of the natural state of affairs, that is, whether the participants in the argumentation express these statements in a certain way (such as speaking with their mouths or writing with a pen). The second is the aspect of the idealized argumentation field, that is, if the participants in the argumentation are idealized, whether they can assert the proposition expressed by this statement without conflict. We assume that an arguer expresses the assertion of the proposition p and also expresses the non-assertion of the proposition p. At this time, this arguer will be accused of insincerity. But he can claim that it is just a mistake and that he wants to retract the assertion of the proposition p. Then, although at the level of the natural state of affairs he has expressed the assertion of the proposition p and the assertion of the proposition ¬p (according to the previous text, not asserting the proposition p is equivalent to asserting the proposition ¬p), at the level of the argumentation field, the proposition p no longer exists. Similarly, a sincere legislator must also follow similar rules: he cannot both support the existence of the norm n and support the non-existence of the norm n. According to Kelsen's assertion, all norms established by authorized legislators are valid norms, and we can put them all at the level of the natural state of affairs; after the process of argumentation, by eliminating those norms that are both supported and opposed by the legislator among the valid norms, the remaining norms are the norms that enter the level of the idealized argumentation field.

Participants in theoretical argumentation have diverse ways of expressing their attitudes towards propositions. Among them, there are two most important types: One is the "elliptical expression", that is, expressing the assertion of a proposition by directly stating it. The other is the "complete expression", that is, reporting the assertion of a proposition by stating the state of affairs of one's assertion of this proposition. However, the way in which legislators express their attitudes towards norms generally only be carried out through elliptical expressions. That is, they change the norms themselves by using normative modal words such as "(not) allowed" and "(not) ought to", rather than expressing their attitudes towards the norms. A person can express a certain negative attitude towards the proposition p by stating "I do not assert p", but a legislator generally will not express a similar negative attitude towards the existence of the norm n that he does not want to create in a similar way.

This is because the complete expression of the legislator's attitude towards the norm is not to state his own attitude, but rather the act of "creation" itself and the act of "non-creation" which is the opposite of the act of creation. The act of "creation" can be carried out in various ways, but the act of "non-creation" can only be carried out through silence or abolishing.

However, the ideal director cannot express a negative attitude towards a certain norm through "silence". The reason is that an idealized normative system established by the legislator is presupposed to be complete. That is to say, in every real or hypothetical situation, every choice is either permitted, prohibited, or required.[15]But the legislator cannot cover the behaviors of an infinite number of subjects. This requires the legislator to ensure the integrity of his own normative system by setting some rules, such as "What is not prohibited by law is free", "No crime without a legal provision", "What is not authorized by law is prohibited", etc. These rules give specific meanings to the legislator's silence on individual norms, rather than simply expressing opposition to the norms. The legislator also cannot express a negative attitude towards a norm through "abolish" because he can only abolish the existent norms, and not on the absent norms.

If we agree that the legislator's attitude towards a norm cannot be inconsistent, then we also agree that normative modal words such as "permitted" and "ought to" can directly express the attitude towards the norm. Correspondingly, when the legislator makes inconsistent statements regarding the normative words, then this legislator is either making a mistake or being insincere. For a legislator who makes a mistake, he holds false beliefs and thus fails to realize that his attitude towards the norm is inconsistent. When he realizes this, he will either seek to correct the norm or transform into an insincere legislator. As for an insincere legislator, since his attitude towards behavior is false, the norms he formulates cannot participate in the rational discourse about norms.

In conclusion, we have basically explained the "meta-problem" of the logical

[15] Gibbard, A. (1993). *Wise Choices, Apt Feelings*, Oxford: Clarendon Press, 88.

constraints on legal norms. In summary, although normative logic skepticism refutes the possibility of logical inferences at the level of valid norms, normative logic can instead deal with the will of the idealized director. There are two main principles about the will of the idealized director: consistency and inference-licensing, which will constitute the source of the logical constraints on legal norms.

Justification of logical constraints on legal norms

After explaining the "meta-problem", this section will move on to the level of "specific problems" and elaborate on how the semantics of Standard Deontic Logic (SDL) accounts for the inference-licensing and consistency of the will of the ideal director, thereby transforming these two characteristics into logical constraints.

Standard Deontic Logic

In the middle of the 20th century, with von Wright establishing the first formal deontic logic system and the new-established possible world semantics expanding into generalized modal logic, logicians discovered a new train of thought for assigning values to norms: while retaining the assignment of truth values to propositions, deontic modalities are interpreted as relations between possible worlds. This train of thought seems to solve Jørgensen's dilemma at one stroke, and because it is based on the field of modal logic, the reliability and completeness of the logical system are also well resolved.

The standard deontic logic system uses a language Ld expanded from the propositional logic language Lp. Its initial symbols are divided into three categories:

The first category includes countably infinite propositional symbols: p0, p1, p2, ⋯, pm, ⋯, where m is a natural number;

The second category includes three connectives: O, ¬, →.

The third category includes two punctuation marks: (,).

A formula of Ld is only a string of symbols constructed according to the following rules:

(1) A single propositional symbol;

(2) If α is a formula, then ¬α and Oα are formulas;

(3) If α and β are formulas, then (α→β) is a formula.

The following logical connectives are defined as supplements:

(1) (αΛβ) =df¬(α→¬β);

(2) (αvβ) =df¬α→β;

(3) (α↔β) =df (α→β) Λ (β→α);

(4) Pα=df¬O¬α;

(5) Fα=dfO¬α.。

The basic semantics of this logical system can be formalized as follows:

Given a non-empty set W, R is a binary relation on W. Given a set F(Ld), the elements in it are arbitrary well-formed formulas. The valuation V is a function from W×F(Ld) to the value set {0, 1}. For any well-formed formulas α, β □ Ld and any element w □ W in W, the following holds:

(1) V(¬α, w) = 1 if and only if V(α, w) = 0;

(2) V(α→β, w) = 1 if and only if V(α, w) = 0 or V(β, w) = 1;

(3) V(Oα, w) = 1 if and only if for any w' □ W, if wRw', then V(α, w') = 1;

(4) V(Pα, w) = 1 if and only if there exists a w' □ W such that wRw' and V(α, w') = 1;

(5) V(Fα, w) = 1 if and only if there does not exist a w' □ W such that wRw' and V(α, w') = 1.

In addition to the tautologies and inference rules of propositional logic being valid, the standard deontic logic also adds a new axiom O(α→β)→Oα→Oβ and an inference rule α□Oα.[16]

In addition, we can reflect some unique philosophical properties of norms by endowing the structure <W, R> with certain characteristics, and these structural characteristics will also be reflected in the syntactic system in the form of adding axioms. This system can be conveniently further expanded into a first-order predicate deontic logic.

The core of this semantics lies in the understanding of possible worlds. In the possible world semantics of deontic logic, we can imagine countless possible worlds similar to our real world, and each norm will select some possible worlds from these possible worlds, and in these possible worlds, the states of affairs involved in the norms conform to the situations described by the norms. We call such possible worlds "deontic ideal worlds". Furthermore, we define that "the state of affairs A ought to be realized" means that in all deontic ideal worlds, the proposition "the state of affairs A is realized" is true; "the state of affairs A is prohibited from being realized" means that in all deontic ideal worlds, the proposition "the state of affairs A is realized" is false; "the state of affairs A is permitted to be realized" means that there exist deontic ideal worlds in which the proposition "the state of affairs A is realized" is true. Thus, we correspond the truth or falsehood of norms with deontic modal words to the truth or falsehood of propositions without deontic modal words in deontic ideal worlds. The deontic modal words imposed on states of affairs correspond to the specific relations existing between possible worlds.

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[16] Hilpinen, R., McNamara, P. (2013). Deontic Logic: A History Survey and Introduction, in Gabby, D., Horty, J., Parent X.(eds.) *Handbook of Deontic Logic and Normative Systems*. College Publications, 36-47.

What kind of world is a "deontic ideal" one? In legal norms, which kind of world is deontic ideal should be regarded as determined by the legislator.[17]That is to say, through issuing instructions, the legislator determines which kind of world belongs to the deontic ideal world, or in other words, provides a specific model of the deontic logic of law. However, as mentioned above, real-life legislators lack sufficient non-moral information, making the deontic ideal world they set up possibly logically imperfect. Therefore, the "deontic ideal world" should be established by the idealized director relative to each legislator, and form a constraint on real-life legislators.

Meta-normative justification of logical constraints

It is generally believed that as a meaningful sentence, a norm has an important difference from a statement, that is, it can provide reasons for actions (or practices). In the field of practical reasoning, taking the beliefs and desires of the agent as premises respectively, the goals of the agent can be obtained. Among them, there may be two kinds of inconsistencies in the desires of the agent, that is, the agent may desire two incompatible states of affairs or have a vague attitude towards the same state of affairs (or the propositions describing such an attitude are inconsistent with each other). The former will lead to the logical unrealizability of the agent's goal, and the latter will lead to the inability to determine whether the agent has a certain desire. They respectively correspond to the two properties of the idealized director, namely consistency and inference-licensing.

Therefore, for a legislator as an agent, there are at least two "meta-norms": Firstly, the legislator must desire a world in which all the propositions described by him are consistent. Only such a world is realizable, that is, the "principle of realizability"; Secondly, the desires of the legislator must be clear. He cannot desire and not desire the same state of affairs. In other words, the statements describing the legislator's desires must be consistent. Such a statement is exactly the interpretation of the axioms and inference rules involving deontic modalities in SDL: For the deontic inference rule $\alpha \vdash O\alpha$, it requires that all deontic ideal worlds must be logically consistent worlds, and the logical theorems in the real world must also be valid in the deontic ideal world; For the K axiom $O(\alpha \rightarrow \beta) \rightarrow O\alpha \rightarrow O\beta$, it requires the clarity of the legislator's desires, and there should be no conflict attitudes in desires.

The openness of the axioms of modal logic makes the above axioms or inference rules not absolutely necessary, and these are not the only possible axioms. However, these two axioms or inference rules ensure that such modal logic is normal. Non-normal logical systems that do not use these axioms or inference rules will allow the emergence of worlds with confused or inconsistent attitudes of the legislator, which is not what we desire. According to our further requirements for the legislator, we can also expand our deontic logic system and add more axioms as "meta-norms".

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[17] Stelmach, J. Brozek, B. (2006). *Methods of Legal Reasoning*. Dordrecht: Springer, 31.

Semantic justification of logical constraints

1.semantic justification of consistency

Firstly, as previously stated, a model determines the truth values of formulas over a set of possible worlds. For a certain possible world w □ W and a formula α, we can denote V(α, w) = T as <W, R, V> □w α. If for any w □ W, <W, R, V> □w α, then α is said to be valid in the model <W, R, V>, and it is denoted as <W, R, V> □ α.

For any propositions α and β, for any model M, if M □ α, then M □ β, and if M □ β, then M □ α. In this case, α and β are called a set of contradictory propositions.

That is to say, if proposition α and proposition β are contradictory, then in any situation, proposition α and proposition β have different truth values. It can be proven that if α represents any formula, then the proposition "Oα" and the proposition "¬Oα" form a logical contradiction. Similarly, "Pα" and "¬Pα", "Fα" and "¬Fα" also form contradictory expressions. From an intuitive semantic perspective, they respectively mean: □ In all deontic ideal worlds established by the will of the legislator, it is impossible that α is true in all deontic ideal worlds while α is false in some deontic ideal worlds; □ In all deontic ideal worlds established by the will of the legislator, it is impossible that α is true in some deontic ideal worlds while α is false in all deontic ideal worlds.

However, obviously, in such possible world semantics, it is easy to prove that "Oα" and "Fα" ("ought to do" vs "ought not to do") do not form a contradiction. Intuitively, consider such a situation: A legal provision states that "Concluding a contract is permitted, and not concluding a contract is also permitted." One lawyer claims that "Concluding a contract is obligatory", and the other claims that "Concluding a contract is prohibited." Obviously, the claims of these two lawyers are both wrong. That is to say, in this case, these two propositions have the same truth value: both can be false. But whether they can both be true is still controversial. If they cannot both be true, then "There should be no conflict between 'ought to do' and 'ought not to do'" is the logical constraint that legal norms should follow. Here, we need to further examine the nature of the R relation. This requires us to return to the philosophical discussion: What is the relationship between the world where the legislator is and the ideal world he establishes?

It is easy to prove that if the conflict between "ought to do" and "ought not to do" constitutes a logical inconsistency, then there exists w'□W such that wRw'. In the semantics of possible worlds, we call this property "serial". There is controversy regarding the intuitive interpretation of this property. However, in the philosophy of law, this interpretation can be regarded as "for a possible world where there is a legislator with the power to formulate norms, there is always an ideal possible world envisioned by the legislator". According to the semantics of possible worlds, if such an ideal possible world does not exist, then the legislator cannot formulate norms. That is to say, if the legislator can formulate norms, then there must be such an ideal possible world. Given that the world is consistent, seriality implies that the content of

the norms formulated by the legislator must be logically realizable.[18] It can be further proved that if we accept seriality, then we will accept the following axiom*: Oα→Pα.

In conclusion, if we accept axiom*, then the relationship between "ought to do" and "ought not to do" becomes a logical inconsistency. Although these two can both be false, they cannot both be true, forming a "contrary relation". I agree with this acceptance because, as mentioned above, if the conflict between a pair of "ought to do" and "ought not to do" is not regarded as a logical contradiction, then in the model-theoretic semantics, the normative system corresponding to this conflict will be incomprehensible.[19]

2.semantic justification of inference-licensing

The technical issues involved in the reasoning from general norms to individual norms are much more complicated. Two common ways of simulation are "O∀x（Tx →Rx）" and "∀x（Tx →ORx）",[20]The intuitive semantics of the former is that for any object in the domain, the following state of affairs is obligatory: if it has the property T, then it has the property R; the intuitive semantics of the latter is that for any object in the domain, if it has the property T, then the following state of affairs is obligatory: it has the property R.

Common works on legal methodology simply reduce the major premise in this reasoning, by combining deontic logic and first-order predicate logic, to "∀x（Tx →ORx）". [21] Intuitively in terms of semantics, it means that for any element in the domain, if it has the property T, then in all deontic ideal worlds, it has property R. And the problem precisely lies in the issue of "domain". The property T is a property in the real world rather than in the deontic ideal world. If this predicate is a relation on the set of individuals in the real world, then if there are individuals in the deontic ideal world that do not exist in the real world, there will be gaps. In terms of the theory of the "exception argument" in the previous text, it means that the legislator cannot consider all future situations and provide exceptions when legislating in the real world. Even if we adopt the attitude of normative logic skepticism, such gaps will still cause difficulties: from the existence of general norms, the existence of individual norms cannot be rationally deduced. Technically speaking, if we want to ensure the validity of "∀x（Tx →ORx）, Ta→ORa" and avoid such gaps, then we must use a kind of "constant domain semantics", keeping the individuals in the real world and all deontic ideal worlds the same, so that the domain in the reasoning remains consistent throughout. However, this kind of semantics has great limitations. From a philosophical perspective, the volitional behavior of the legislator cannot focus on the

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[18] The logical realizability here is only the minimum requirement, and it does not require "being realizable" physically, but merely means that this world can be conceived.。Vgl. Joeden. (2010). Logik im Recht, Berlin: Springer-Verlag.

[19] Compared with M. H. (2024). *Rights and Right-Holding: A Philosophical Investigation,* Oxford: Oxford University Press, 28-34.

[20] Stelmach, J. Brozek, B. (2006). *Methods of Legal Reasoning.* Dordrecht: Springer, 35.

[21] Alexy, R. (1991). *Theorie der juristischen Argumentation : die Theorie des rationalen Diskurses als Theorie der juristischen Begründung*. Frankfurt am Main: Suhrkamp Verlag, 274.

increase and decrease of elements in the real world to adjust the exception premises in the norms, making the elements in the possible world envisioned by the legislator always consistent with those in the real world.

Another serious problem with this reduction is that it will trigger a special case of the "Chisholm Paradox". It is easy to prove that ∀x（¬Tx→（Tx→ORx））, and further that ∀x¬Tx→∀x（Tx→ORx）. That is to say, if all objects in the domain do not have the property T, then there is the "norm" ∀x（Tx→ORx）. This is obviously absurd: if no one in the world kills, then there exists the norm "If anyone kills, then he should be sentenced to death".[22]

According to the previous research results of modal logic, It is better to use "O∀x（Tx →Rx）". The domain in this sentence only includes the deontic ideal world and does not include the real world. From the perspective of the philosophy of logic, the classical transformation is a transformation of "modality de re", while this transformation is a transformation of "modality de dicto". It can be proved that a proposition of modality de re implies a proposition of modality de dicto if and only if the set of elements in the deontic ideal world in the real world is a subset of the set of elements in the real world; a proposition of modality de dicto implies a proposition of modality de re if and only if the set of elements in the real world is a subset of the set of elements in the deontic ideal world. For the theory of legal norms, it is more reasonable to assume that the set of elements in the real world is a subset of the set of elements in the deontic ideal world.

In this way, the reasoning from general norms to individual norms can be expressed as "O∀x (Tx → Rx), OTa→ORa". The validity of this formula can be easily proved. However, its drawback is also quite obvious: the property T is not always desired by the legislator, especially in criminal law norms. Taking "Those who commit murder should be sentenced to death" as an example, it seems to imply that the minor premise must be "Murder should be committed". Here, it is necessary to stratify the deontic ideal world semantically and introduce the order among deontic ideal worlds. [23] In essence, the legislator first conceives an optimal deontic ideal world. Once this ideal world cannot be realized, it will "fall" into a sub-optimal deontic ideal world. In such conditional norms, the deontic modal word "O" expressing "ought" points to the sub-optimal deontic ideal world, rather than the optimal deontic ideal world.

Thus, the argument from general norms to individual norms, that is, the logical constraint of "reasoning permission", is preliminarily established.

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[22] This argument is derived from an extension of the "Chisholm Paradox" in deontic propositional logic. See Hilpinen, R., McNamara, P. (2013). Deontic Logic: A History Survey and Introduction, in Gabby, D., Horty, J., Parent X.(eds.) *Handbook of Deontic Logic and Normative Systems*. College Publications, 85.

[23] See Carmo, J. Jones, A. (2002). Deontic Logic and Contrary-to-duties, in *Handbook of Philosophical Logic*,Second Edition,Vol.8,Gabbay and F. Guenthner(eds), Kluwer Academic Publishers, 265-344.

Working mechanism of logical constraints on legal norms

In this section, we will examine how the inference-licensing constraints and consistency constraints operate to restrict the application of legal norms in practice. Intuitively, there exist logical relationships among valid legal norms, and such logical constraints can be directly applied to legal norms. However, the skepticism regarding deontic logic renders this intuition indefensible. If, as stated above, deontic logic is applicable to the normative statements about the deontic ideal world created by the idealized director, then does there exist a reflexive application? That is, will the logical reasoning of normative statements reflect on the valid norms, and will the logical relationships between normative statements be reflected as the logical relationships between the original norms?

This seems to be the only possible way, because only when all legal norms are either obtained from the legislator or through deductive reasoning from the norms issued by the legislator can it be said that the discovery of legal norms is not based on the will of the judiciary, but on rational reasoning. However, this line of thought presupposes a premise, that is, reason can only be realized based on deductive reasoning. For example, when a judge derives an individual norm from the general norm of the legislator, under the premise that the reflexive model holds, it is a form of deductive reasoning and is of course rational. But if the reflexive model does not hold, then this kind of inference is no longer deductive reasoning and is not considered rational. However, in all arguments, there are a large number of non-deductive reasoning, and since they are supported by appropriate reasons, they are not irrational.

The relationship between normative statements should have a refractive influence on legal norms. The appliers of legal norms first acquire a set of beliefs regarding the existence of norms based on these norms, which is also known as a set of normative statements. After the logical relationships between normative statements are determined, we may find that there are certain problems within this set of norms. Corresponding to the constraints of inference-licensing and consistency, it may turn out that the needed norms are absent, or there are norms that conflict with each other. In other words, guided by deontic logic, the judiciary makes a constrained "continuation" of legal norms.

Gaps in legal norms

First, we examine the situation where there is a gap in the norms we need. This is not the common situation of a legal gap beyond the legislator's plan in the methodology of law. Instead, it is a gap caused by the fact that we are unable to directly and logically obtain the individual norms from the original norms. Such a gap needs to be fulfilled by the constraints of inference-licensing.

A typical example in Jørgensen's Dilemma: In the judicial syllogism O□x（Tx →Rx）, OTa→ORa, since it no longer expresses the logical relationship between norms, the legislator cannot derive individual norms from general norms. The most typical obstacle here is the existence of a large number of exceptions relative to general norms. In classical legal methodology, the negation of the conjunction of the premises of these exceptional situations is generally taken as part of the premise of the general norm. However, this is the result of handling by the methodology formed over time in legal practice, not a logical rule, and it cannot rule out the situation where there are exceptions with unclear hierarchies in legal practice. In this case, the deontic ideal world created by the legislator may be in chaos (that is, the propositions describing it are logically inconsistent), which requires us to conduct dynamic argumentation to solve it.

Subsequently, according to the judicial power granted by the legislator, we will refractionally add the derived norms described by the normative statements about the existence of derived norms obtained from the normative statements about the existence of norms (for example, deriving the individual norms described by the normative statements about the existence of individual norms from the normative statements about the existence of general norms) to the set of applicable norms, thus obtaining a new set of norms.

Conflicts in legal norms

Next, we will discuss the situation of normative conflicts. In such cases, we need to carry out subsequent creation based on the consistency constraints.

For example, O□x（Tx →Rx） and ¬O□x（Tx →Rx） are obviously in conflict. Intuitively, the former expresses that "it is obligatory that any individual with the property T has the property R"; the latter expresses that "it is not obligatory that any individual with the property T has the property R". Based on the interpretation of normative statements and the semantics of possible worlds that we have established, these two sentences describe the states of affairs in the deontic ideal world established by the legislator. The accurate semantics of the former is that "in all the deontic ideal worlds established by the legislator, any individual with the property T has the property R", and the accurate semantics of the latter is that "it is not the case that in all the deontic ideal worlds established by the legislator, any individual with the property T has the property R". The logical contradiction in the normative statements is obvious. However, this does not mean that the world created by the norms is logically contradictory or inconsistent. This "possible world" actually has similar logical characteristics to our real world, that is, it is not a logical object.。

Subsequently, according to legal rules, the methods of legal dogmatics, and even value judgments, we will discard some of the inconsistent propositions. Furthermore, by assuming that the propositions in the newly obtained set of normative statements are all true, the appliers of legal norms can obtain a new set of norms. And this new set of norms no longer reflects to the set of the original norms, but is refracted into a

new set of norms.

The necessary condition for the establishment of the refraction theory lies in a subtle property of norms: norms are not completely independent of the mind. For the objective world that can be extremely regarded as independent of the mind, if there is a possible conflict between the objective world and logic, we cannot hope or require the objective world to conform to logic; but for norms and their legislators, this can be achieved. In this sense, the ultimate realization of norms is generated by the interaction between the judiciary and the legislator. Therefore, norms are a cooperative undertaking.

Conclusion

This article arrives at three conclusions: Firstly, in terms of meta problems, deontic logic does not deal with valid norms, but rather with the norms created by the idealized director. Inference-licensing and consistency are two principles of the will of the idealized director. Regarding specific problems, the semantics of possible worlds in deontic logic demonstrates the two logical constraints of inference-licensing and consistency on legal norms. In practical problems, the judiciary completes the "continuation" of legal norms through the "refraction model" guided by deontic logic, thus accomplishing the cooperative undertaking of legal norms.