

The issue of urban subway station construction spacing and operating profits

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I. Summary

The construction of subway stations is a necessity for economic and social development. During the construction process, builders often carefully consider how to determine the spacing between stations to maximize operating costs. With this goal in mind, our team abstracted the problem, formulated hypotheses, and developed a mathematical model. Using elementary function methods, we described the relationship between operating profit and the distance between stations. We then used real-world data to validate the model.

Keywords: urban planning, subway construction, mathematical modeling, functional thinking

II. Introduction

With the rapid development of urbanization, a large number of people are concentrating in large cities. Coupled with China's vast population base, this has led to the formation of numerous megacities. As a crucial means of transportation for residents of large cities, the subway's development in each city has attracted local attention. However, losses are a common phenomenon in subway operations. According to incomplete statistics, as of July 7, 2023, 32 subway companies in Beijing, Shanghai, Shenzhen, Guangzhou, Wuhan, Changsha, and Chengdu had released their 2022 annual reports and audit reports. The results show that: First, after accounting for local government subsidies, only one subway company in Shenyang among the 32 cities had a negative net profit. Second, after deducting local government subsidies, only Wuhan, Shenzhen, Jinan, Shanghai, and Changzhou had positive subway profits; the remaining cities all suffered losses. Therefore, in subway planning and construction, how to maximize profits has become a crucial consideration. The authors aim to mathematically describe the relationship between subway profits and construction distance.

3. Main text

3.1 Problem Restatement

The problem requires designing a new subway system for a city with only one line. The system aims to maximize profits by building subway stations while taking into account the cost. The following assumptions are made in the design of the line:

1. The population distribution in the city is even.
2. The radiation range of the subway station is a circular area with a radius of 2km.
3. The total length of the subway is 25km.
4. Within the radiation range of the subway station, the daily passenger flow is fixed.
5. Metro station stops need to be evenly distributed throughout the city.
6. The radiation ranges of the two subway stations need to overlap.
7. Subway maintenance costs and operating costs also need to be considered.

3.2 Problem Analysis

To meet the city's needs and maximize operational profitability, we need to carefully analyze and consider the following key factors:

1. Population distribution and station location: Since the population of a city is evenly distributed, the location of subway stations is crucial. We need to ensure that there are enough people within the radiation range of each subway station to attract sufficient passenger flow.
2. Overlapping radiation range: In order to meet the overlapping requirements, we need to pay attention to the fact that the radiation ranges of adjacent subway stations may have a certain overlap, and the passenger flow of the overlapping parts should not be counted repeatedly.
3. Even distribution of stations: Stations need to be evenly distributed across the city, which means we need to carefully plan the location of stations to ensure that every area is served by a subway station and the distance between stations should be consistent.
4. Fixed passenger flow and operating profit: Within the radiation area of a subway station, the daily passenger flow is fixed. This means that we need to adjust the location and number of stations based on this fixed passenger flow to maximize operating profit.
5. System design and cost considerations: When designing a subway system, in addition to meeting the above requirements, construction and operating costs must also be considered. Otherwise, the system can be built indefinitely, resulting in a very large radiation area.

Based on this problem, we have a modeling idea as shown in Figure 1.

3.3 Model Assumptions

1. The number of passengers taking the subway per day per unit area is constant, without considering the changes in daily passenger flow.
2. The amount each person spends on the subway every day is equal and constant.
3. The population is evenly distributed within the city.

3.4 Explanation of symbols

Table 1 Symbols

symbol	illustrate	unit
r	Average radius of passengers attracted by a subway station	km
L	Total length of subway	km
k	Average ticket price	Ten thousand yuan
s	Number of subway passengers per day per unit area	People/km ²
m	Daily operating costs for each site	Ten thousand yuan
n	Daily maintenance cost per kilometer of railway	Ten thousand yuan
x	Site distance	km
p	Total daily profit	Ten thousand yuan
S	Total coverage area	km ²

f The area of a rectangle with a length of $4r$ and a width of $2r$ is the difference between the area of a semicircle with a radius of $2r$ and the area of a circle with a radius of $1r$ (as shown in the shaded area in Figure 2). km²

The significance of Figure 2f

3.5 Model Building

Based on the actual research of this problem, we consider establishing an elementary mathematical function model.

Considering that the radiation ranges of two subway stations need to overlap and each quantity must be meaningful, the above quantities are restricted:

$$k, \sigma, m, n, S > 0$$

$$0 < x < 4$$

3.6 Model Solution

Based on this problem, we should first consider the overlapping coverage area in the model, and then determine the total coverage area, and then we can determine the function expression of total revenue and total profit related to x .

After calculation and analysis, we can get

The total coverage area can be obtained

Total income is

The total profit can be obtained

In order to simplify the calculation and avoid unnecessary calculations, we use GeoGebra software to draw the image. At the same time, based on this problem, we consider assigning values to the six quantities r, L, k, σ, m , and n according to the problem setting and the existing subway conditions in real life.

Table 2 Symbol assignment

symbol	Assignment
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r	2
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L	25
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k	0.0002
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s	5000
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m	1.5
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n	1.2
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We can plot the graph of p .

Figure 3 Image of p

Therefore, under this data, combined with software analysis, we can find that

Table 3 Study on the expression of p

x	(0,2.5010)	2.5010	(2.5010,4)
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p	Monotonically increasing	Maximum	Monotonically decreasing
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Combined with the image, it is not difficult to find that the zero point of function p is at $x=0.4639$, and the maximum value of p is 59.1081 at $x=2.5010$.

In other words, based on this data, we can roughly estimate by drawing a graph that when the station distance $x = 2.5010$ km, the total profit is the largest, at 591,081 yuan. When the station distance $x > 0.4639$ km, the subway will not lose money.

3.7 Model Checking

After studying the expression of p , we concluded that: "When the station distance $x=2.5010$ km, the total profit is the largest, which is 5.91081 million yuan. When the station distance $x>0.4639$ km, the subway will not lose money." By selecting Changsha Metro Line 1 (2019) data close to the model data, we found that it basically conforms to real life.

To further validate the model, we selected data from Shenzhen Metro Line 1, where $L = 40.88$ km, $\sigma = 3851$ people/km², and $k = 0.0004$ million yuan. All other parameters remained unchanged. We found that the maximum profit was achieved when $x = 2.1927$ km. When $x = 2.1927$ km, building 20 metro stations would result in the maximum profit, $p = 17.94663$ million yuan. However, Shenzhen Metro Line 1 actually has 30 stations (28 underground and 2 elevated). Does this mean the model is incorrect? The authors believe that urban metro construction should not only prioritize profitability but also convenience for residents. Calculations show that the average station spacing for Shenzhen Metro Line 1 is 1.3793 km, and its graph intersects with p at (1.3793, 171.1453), which is approximately 4.6% lower than the theoretical maximum profit. In other words, the profit obtained by substituting the station spacing into the model in real life is not much different from the maximum profit calculated by the model. In real life, this design is likely to be based on considerations of urban functions and convenience. Therefore, the author believes that this model is basically in line with real life.

3.8 Model Evaluation

Model advantages:

- 1. Comprehensive consideration of multiple factors: The model comprehensively considers multiple factors such as population distribution, station location, subway line length, overlapping radiation range, etc., to ensure the comprehensiveness and rationality of the design.*
- 2. Maximizing operating profit: The goal of this model is to maximize daily operating profit, which is a practical and important consideration and has certain guiding significance for improving the economic benefits of subway systems in real life.*
- 3. Flexible scalability: Considering the future development and expansion of the city, the model has certain flexibility and scalability, which facilitates the adjustment or expansion of sites and lines in the future.*
- 4. Design of overlapping radiation range: The overlapping radiation range conforms to the actual situation in real life.*

Model Disadvantages:

1. *Large data requirements:* In order to implement this model, a large amount of data support is required, such as population distribution data, geographic information data, etc.
2. *High computational complexity:* Since multiple factors need to be considered, the computational complexity of this model is high.
3. *Insufficient cost consideration:* Although future scalability and system design costs are mentioned, the model lacks specific cost analysis. In real life, cost is a very important consideration.
4. *Fixed customer flow assumption:* This model assumes that daily customer flow is fixed, which may not be entirely accurate in reality. Actual customer flow may be affected by many factors, such as holidays and weather.
5. *Line Assumption:* This model assumes a single line, which is acceptable for some cities in the early stages of subway construction. However, for super first-tier and first-tier cities, there are obviously more than one subway line, which is not realistic. Furthermore, the line length should be scalable rather than fixed.

Overall, the model has certain advantages, such as comprehensiveness, scalability, and practicality. However, in real life, it is necessary to combine the actual situation, pay attention to its shortcomings, and make corresponding improvements and perfections.

IV. Conclusion

For a subway line with a length of 25km, the total profit is maximized when the station distance is 2.5010km.

V. Summary

This paper analyzes actual data in real life, and through public information and mathematical modeling analysis and research, proposes a mathematical model of site distance and profit, which basically conforms to real life, indicating that our model has certain reliability and reference value.

At the same time, relying on this mathematical model, we can provide guidance on subway planning for cities that have not yet started building subway lines, based on actual conditions.

6. References

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