Unit 1 Practice Problems

1. If a rocket system delivers 133 kN of thrust with a propellant flow rate of 31.90 kg/s, what is the specific impulse?

\[ T = 133 \text{ kN} = 1.33 \times 10^5 \text{ N} \]

\[ \dot{m}_f = 31.90 \text{ kg/s} \]

\[ g_0 = 9.807 \text{ m/s}^2 \]

\[ I_{sp} = \frac{T}{\dot{m}_f g_0} \]

\[ \Rightarrow I_{sp} \approx 425.133 \text{ s} \]

2. The Merlin 1D Vacuum+ rocket system used on the Falcon 9 upper stage delivers 934 kN of thrust in vacuum with a vacuum Isp of 348 s.

a) If the dry (empty/structural) mass of the upper stage is 3900 kg, and the engine operates for 397 s, determine the initial combined mass of the upper stage (before the engine starts).

b) Suppose that this stage must push a payload to a minimum \( \Delta V \) of 6 km/s. Determine the maximum payload mass.

\[ T = 934 \text{ kN} = 9.34 \times 10^5 \text{ N} \]

\[ I_{sp} = 348 \text{ s} \]

\[ m_0 = 3,900 \text{ kg} \]

\[ \Delta V = 6 \text{ km/s} \]

\[ g_0 = 9.807 \text{ m/s}^2 \]

\[ m_p = \dot{m}_f \Delta t = 1.0865 \times 10^5 \text{ kg} \]

\[ m_0^* + m_s = 112,548 \text{ kg} \]

\[ m^* = m_p + m_s \]

\[ \frac{m^*}{m_0} = 0.03465 \]

\[ \gamma = \frac{m_0}{\dot{m}_f g_0} \]

\[ \frac{1}{\gamma} = 0.16641 \]

\[ \dot{m}_p = \frac{\dot{m}_f}{\gamma} \]

\[ m_p = m_0^* \lambda = 18,729 \text{ kg} \]

3. The Merlin 1D+ rocket system used on the Falcon 9 delivers 480 kN of thrust in vacuum with a vacuum Isp of 304.8 s. What is the propellant flow rate?

- The definition of specific impulse, \( I_{sp} \), connects the thrust, \( T \), the mass flow rate, \( \dot{m}_f \), and sea level gravity, \( g_0 \). Rearranging this equation provides the desired result directly.

Answer: 160.6 kg/s
4. A spacecraft has a monopropellant rocket system that delivers an $I_{sp}$ of 275 s. An orbital trim maneuver is planned that will require 175 m/s $\Delta V$. All propellant is stored internal to the spacecraft, and no change to the structural mass will occur due to the burn. **How much propellant will be consumed** if the spacecraft must have a mass of 1400 kg after the maneuver?
   - The ideal rocket equation (1.5) connects the $\Delta V$, the $I_{sp}$, the initial mass, $m_0$, and the final mass, $m_f$. The units on $\Delta V$ and $g_0$ need to be compatible. $m_0$ is found from this equation. The propellant consumed is found by subtracting the final mass from the initial mass.
   Answer: 93.9 kg

5. The Centaur upper stage has the following characteristics: stage dry mass = 2935 kg, propellant mass = 20,950 kg, $I_{sp} = 444$ s.
   a) **What is the maximum velocity increase** that the upper stage will impart to a spacecraft with a mass of 3000 kg?
   b) **What is the total vehicle burnout mass** before spacecraft separation?
   - The final mass is computed as the Centaur dry mass plus the spacecraft mass. The initial mass is the final mass plus the propellant mass.
   - A) The velocity increase is found using the rocket equation (1.5).
     Answer: 6.58 km/s
   - B) This value is the same as the final mass used in part (a)
     Answer: 5935 kg

6. Suppose that a spacecraft with an orbital transfer package is deployed into Low-Earth Orbit (LEO) by a Falcon 9. The package consists of a two-stage system that will deliver the payload (spacecraft) to the final Geosynchronous Orbit with the following characteristics:

   $\epsilon_1 = 0.07; \quad \epsilon_2 = 0.09; \quad I_{sp,1} = 310$ s; \quad $I_{sp,2} = 285$ s;
   $\Delta V_1 = 2.45$ km/s; \quad $\Delta V_2 = 1.48$ km/s;

   The launch vehicle can deliver a maximum mass of 22,800 kg to LEO. **Determine the maximum mass** that the spacecraft (payload) can have after it separates from the second transfer stage.
   - The initial mass is the maximum launched by the vehicle. The mass ratio for each stage is computed using the rocket equation (1.5) and the given values $\Delta V_i$ and $I_{sp,i}$. The units used for $\Delta V$ and $g_0$ must be consistent. The burnout mass for stage 1 is the initial mass divided by the mass ratio. The difference is these two masses is the first stage propellant mass. The definition of the structural ratio (1.8) can then be used to find the stage 1 structural mass. The second stage initial mass is then the stage 1 payload mass, which is the final mass minus the structure from stage 1. The process is then repeated using the mass ratio for stage 2 to get the system payload mass.
   Answer: 5063.0 kg