# Tautological Risk Model: Formalizing Exposure, Mitigation, and Residual Risk

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## Posit

*Where threat aligns with vulnerability there is exposure that implies contingent risk to the extent that exposure is unmitigated and of which residual risk exists regardless of exposure mitigation.*

## Formula

Given Exposure ($e$), Threat ($t$), Vulnerability ($v$), Mitigation ($m$), Contingent Risk ($r_c$), and Residual Risk ($r_r$) and $r_r \equiv$ True,

$[e \equiv (t \land v)] \implies [(e \land \neg m) \rightarrow (r_c \lor r_r)]$

for $f(e, m, r_c, r_r) = (e \land \neg m) \rightarrow (r_c \lor r_r) \equiv \neg(e \land \neg m) \lor (r_c \lor r_r)$

## Explanation

The formula states that if exposure is equivalent to the conjunction of threat and vulnerability, then unmitigated exposure implies the existence of either contingent risk or residual risk, and residual risk is always present regardless of mitigation.

## The Big Picture: Why This Formula Exists

This formula is built to overcome a problem: In real life, just because an Exposure exists, it doesn't always mean Risk happens (because of Mitigation). This formula is designed to be a universally true logical rule (a Tautology) that accounts for the success of Mitigation and guarantees the existence of a minimal level of Residual Risk.

Following is the plain language of this formula explaining how this deeply structured, formal logic statement maps directly to real-world risk management.

### Step 1: Defining the Premise (The Setup)

Given $e$, $t$, $v$, $m$, $r_c$, $r_r$,

$[e \equiv (t \land v)]$

This part simply sets the stage and establishes the definition of Exposure in the model:

- Exposure ($e$) is logically equivalent to (or defined as) a Threat ($t$) AND a Vulnerability ($v$) existing at the same time.

### Step 2: The Logic Flow (The Argument)

$\implies [(e \land \neg m) \rightarrow (r_c \lor r_r)]$

This is the central argument, known as the main conditional. The external implication ($\Longrightarrow$) asserts that, given the definition of Exposure, the truth of the conditional premise logically necessitates the validity of the rule:

- The rule is: "IF (Exposure exists AND Mitigation is NOT successful), THEN Total Risk ($r_c \vee r_r$) must follow." (This means, if the exposure is unmitigated, either Contingent Risk OR the always-present Residual Risk will be True).

### Step 3: The Mathematical Proof (The Justification)

for $f(e, m, r_c, r_r) = (e \wedge \neg m) \rightarrow (r_c \vee r_r) \equiv \neg(e \wedge \neg m) \vee (r_c \vee r_r)$

This section is the mathematical proof that the rule in Step 2 works flawlessly. It uses the Law of Material Implication to prove the truth conditions:

- It means: The statement "Unmitigated Exposure implies Total Risk" is logically guaranteed to be true because it is identical in every possible scenario to the statement "EITHER Unmitigated Exposure does NOT happen OR Total Risk happens."

## Summary of the Model's Tautology

The entire complex formula is structured as a Tautology, meaning the final logical result is always True in all scenarios given $r_r \equiv$ True (see Table 1). This proves two key principles of the model:

1. **Risk is Inescapable**: Because the conclusion of the implication ($r_c \vee r_r$) is always True (due to $r_r$ being True), the rule itself can never be broken or False. This mathematically guarantees the principle that Residual Risk always exists.
2. **Unmitigated Exposures Are Logically Forced to Result in Risk**: The model is designed so that the only logical possibility is that an Unmitigated Exposure results in a Risk event, otherwise the formula would be invalidated.

**Contextual Truth Table for the Core Implication Rule (Given $r_r \equiv$ True)** $C = (e \wedge \neg m) \rightarrow (r_c \vee r_r)$

| **Exposure ($e$)** | **Mitigation ($m$)** | **Contingent Risk ($r_c$)** | **Residual Risk ($r_r$)** | **Premise ($e \wedge \neg m$)** | **Conclusion ($r_c \vee r_r$)** | **Result $C$** | **Scenario Interpretation** |
|---|---|---|---|---|---|---|---|
| T | T | T | T | F | T | T | Mitigation successful; Risk ($r_r$) still exists. |
| T | T | F | T | F | T | T | Mitigation successful; Risk ($r_r$) still exists. |
| T | F | T | T | T | T | T | Unmitigated: $r_c$ is T and $r_r$ is T. |
| T | F | F | T | T | T | T | Unmitigated Critical Case: $r_c$ is F, but $r_r$ being |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | | T prevents the implication from failing. |
| F | T | T | T | F | T | T | No exposure; Risk exists independently. |
| F | T | F | T | F | T | T | No exposure; Risk ($r_r$) still exists. |
| F | F | T | T | F | T | T | No exposure; Risk exists independently. |
| F | F | F | T | F | T | T | No exposure; Risk ($r_r$) still exists. |