LPP PYQs – 2000-2020

2000

Qu.

(c) Solve the following assignment problem for the given assignment costs : 20

		Person				
		I	II	III	IV	V
Job	1	11	17	8	16	20
	2	9	7	12	6	15
	3	13	16	15	12	16
	4	21	24	17	28	26
	5	14	10	12	11	13

2001

Qu.

(f) Compute all basic feasible solutions of the linear programming problem [12

$$\text{Max } z = 2x_1 + 3x_2 + 2x_3$$

$$\text{Subject to } 2x_1 + 3x_2 - x_3 = 8$$

$$x_1 - 2x_2 + 6x_3 = -3$$

$$x_1, x_2, x_3 \geq 0,$$

and hence indicate the optimal solution. [12

Qu.

(b) (i) Using duality or otherwise solve the linear programming problem

Minimize $18x_1 + 12x_2$

Subject to $2x_1 - 2x_2 \geq -3$

$3x_1 + 2x_2 \geq 3$

$x_1, x_2 \geq 0$ [15

Qu.

(ii) A manufacturer has distribution centres at Delhi, Kolkata and Chennai. These centres have available 30, 50 and 70 units of his product. His four retail outlets require the following number of units :

A, 30 ; B, 20 ; C, 60 ; D, 40.

The transportation cost per unit in rupees between each centre and outlet is given in the following table :

Distribution Centres	*Retail outlets*			
	A	B	C	D
Delhi	10	7	3	6
Kolkata	1	6	7	3
Chennai	7	4	5	3

Determine the minimum transportation cost. [15

2002

Qu.

(b) (i) Using Simplex method maximize

$Z = 5x_1 + 3x_2$ subject to

$x_1 + x_2 \leq 2,$

$5x_1 + 2x_2 \leq 10,$

$3x_1 + 8x_2 \leq 12,$

$x_1, x_2 \geq 0.$ 15

Qu.

(ii) A company has 3 factories A, B and C which supply units to warehouses X, Y and Z. Every month the capacities of the factories per month are 60, 70 and 80 units at A, B and C respectively. The requirements of X, Y and Z per month are 50, 80 and 80 respectively. The necessary data in terms of unit transportation costs in rupees, factory capacities and warehouse requirements are given below :

	X	Y	Z	
A	8	7	5	60
B	6	8	9	70
C	9	6	5	80
	50	80	80	210

Find the minimum distribution cost. 15

2003

Qu.

(f) For the following system of equations

$$x_1 + x_2 + x_3 = 3$$
$$2x_1 - x_2 + 3x_3 = 4$$

determine :

(i) all basic solutions

(ii) all basic feasible solutions

(iii) a feasible solution which is not a basic feasible solution. 12

Qu.

(b) (i) An animal feed company must produce 200 kg of a mixture consisting of ingredients X_1 and X_2 daily. X_1 costs Rs. 3 per kg and X_2 costs Rs 8 per kg. No more than 80 kg of X_1 can be used, and at least 60 kg of X_2 must be used. Formulate a linear programming model of the problem and use Simplex method to determine the ingredients X_1 and X_2 to be used to minimize cost. 15

(ii) Find the optimal solution for the assignment problem with the following cost matrix :

$$\begin{bmatrix} 6 & 1 & 9 & 11 & 12 \\ 2 & 8 & 17 & 2 & 5 \\ 11 & 8 & 3 & 3 & 3 \\ 4 & 10 & 8 & 6 & 11 \\ 8 & 10 & 11 & 5 & 13 \end{bmatrix}$$

Indicate clearly the rule you apply to arrive at the complete assignment. 15

SECTION 'B'

2004

Qu.

(f) Use Simplex method to solve the linear programming problem:

$$\text{Max.} \quad z = 3x_1 + 2x_2,$$

$$\text{subject to} \quad x_1 + x_2 \leq 4$$

$$x_1 - x_2 \leq 2$$

$$x_1, x_2 \geq 0.$$

12

Qu.

(c) A travelling salesman has to visit 5 cities. He wishes to start from a particular city, visit each city once and them return to his starting point. Cost of going from one city to another is given below :

	A	B	C	D	E
A	∞	4	10	14	2
B	12	∞	6	10	4
C	16	14	∞	8	14
D	24	8	12	∞	10
E	2	6	4	16	∞

You are required to find the least cost route.

Qu.

(d) A department has 4 technicians and 4 tasks are to be performed. The technicians differ in efficiency and tasks differ in their intrinsic difficulty. The estimate of time (in hours), each technician would take to perform a task is given below. How should the tasks be allotted, one to a technician, so as to minimize the total work hours ? 15

Technician \ Task	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

2005

Qu.

(b) Use simplex method to solve the following : 30

Maximize $z = 5x_1 + 2x_2$

subject to $6x_1 + x_2 \geq 6$

$4x_1 + 3x_2 \geq 12$

$x_1 + 2x_2 \geq 4$

and $x_1, x_2 \geq 0$.

Qu.

(f) Put the following program in standard form :

Minimize $z = 25x_1 + 30x_2$

subject to $4x_1 + 7x_2 \geq 1$

$8x_1 + 5x_2 \geq 3$

$6x_1 + 9x_2 \geq -2$

and hence obtain an initial feasible solution. 12

2006

Qu.

(f) Given the programme

$$\text{Maximize } u = 5x + 2y$$

subject to $x + 3y \leq 12$

$3x - 4y \leq 9$

$7x + 8y \leq 20$

$x, y \geq 0$

Write its dual in the standard form. 12

Qu.

(c) Use the simplex method to solve the problem 15

$$\text{Maximize } u = 2x + 3y$$

subject to $-2x + 3y \leq 2$

$3x + 2y \leq 5$

$x, y \geq 0$ 30

2007

Qu.

(f) Put the following in slack form and describe which of the variables are 0 at each of the vertices of the constraint set and hence determine the vertices algebraically :

Maximize $u = 4x + 3y$

Subject to $x + y \leq 4$

$-x + y \leq 2$

$x, y \geq 0$

12

Qu.

(c) Solve the following by Simplex method:

Maximize $u = x + y$

subject to

$-x + y \leq 1$

$x - 2y \leq 4$

$x, y \geq 0$

30

2008

Qu.

(f) Find the dual of the following linear programming problem :

$$\text{Max. } Z = 2x_1 - x_2 + x_3$$

such that

$$x_1 + x_2 - 3x_3 \leq 8$$
$$4x_1 - x_2 + x_3 = 2$$
$$2x_1 + 3x_2 - x_3 \geq 5$$
$$x_1, x_2, x_3 \geq 0.$$

12

Qu.

Factories \ Destinations	D_1	D_2	D_3	D_4	D_5	D_6	Availability
F_1	2	1	3	3	2	5	50
F_2	3	2	2	4	3	4	40
F_3	3	5	4	2	4	1	60
F_4	4	2	2	1	2	2	30
Demand	30	50	20	40	30	10	

finding the initial solution by Matrix Minima Method. 30

2009

Qu.

(f) A paint factory produces both interior and exterior paint from two raw materials M_1 and M_2. The basic data is as follows :

	Tons of raw material per ton of		Maximum daily availability
	Exterior paint	Interior paint	
Raw Material M_1	6	4	24
Raw Material M_2	1	2	6
Profit per ton (Rs. 1,000)	5	4	

Qu.

A market survey indicates that the daily demand for interior paint cannot exceed that of exterior paint by more than 1 ton. The maximum daily demand of interior paint is 2 tons. The factory wants to determine the optimum product mix of interior and exterior paint that maximizes daily profits. Formulate the LP problems for this situation. 12

Qu.

(b) Maximize : $Z = 3x_1 + 5x_2 + 4x_3$
subject to :

$$2x_1 + 3x_2 \leq 8,$$
$$3x_1 + 2x_2 + 4x_3 \leq 15,$$
$$2x_2 + 5x_3 \leq 10,$$
$$x_i \geq 0.$$

30

2010

Qu.

(f) Construct the dual of the primal problem :

Maximize $z = 2x_1 + x_2 + x_3$, subject to the constraints $x_1 + x_2 + x_3 \geq 6$, $3x_1 - 2x_2 + 3x_3 = 3$, $-4x_1 + 3x_2 - 6x_3 = 1$, and $x_1, x_2, x_3 \geq 0$. 12

Qu.

(c) Determine an optimal transportation programme so that the transportation cost of 340 tons of a certain type of material from three factories F_1, F_2, F_3 to five warehouses W_1, W_2, W_3, W_4, W_5 is minimized. The five warehouses must receive 40 tons, 50 tons, 70 tons, 90 tons and 90 tons respectively. The availability of the material at F_1, F_2, F_3 is 100 tons, 120 tons, 120 tons respectively. The transportation costs per ton from factories to warehouses are given in the table below :

	W_1	W_2	W_3	W_4	W_5
F_1	4	1	2	6	9
F_2	6	4	3	5	7
F_3	5	2	6	4	8

Use Vogel's approximation method to obtain the initial basic feasible solution. 30

2011

Qu.

(d) Solve by Simplex method, the following LP Problem :

Maximize, $Z = 5x_1 + 3x_2$

Constraints, $3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0$ 12

2012

Qu.

integrable. Justify. 20

(c) By the method of Vogel, determine an initial basic feasible solution for the following transportation problem :

Products P_1, P_2, P_3 and P_4 have to be sent to destinations D_1, D_2 and D_3. The cost of sending product P_i to destinations D_j is C_{ij}, where the matrix

$$[C_{ij}] = \begin{bmatrix} 10 & 0 & 15 & 5 \\ 7 & 3 & 6 & 15 \\ 0 & 11 & 9 & 13 \end{bmatrix}.$$

The total requirements of destinations D_1, D_2 and D_3 are given by 45, 45, 95 respectively and the availability of the products P_1, P_2, P_3 and P_4 are respectively 25, 35, 55 and 70. 20

Qu.

(c) Write down the dual of the following LP problem and hence solve it by graphical method :

Minimize, $Z = 6x_1 + 4x_2$

Constraints, $2x_1 + x_2 \geq 1$

$3x_1 + 4x_2 \geq 1.5$

$x_1, x_2 \geq 0$ 20

Qu.

(d) For each hour per day that Ashok studies mathematics, it yields him 10 marks and for each hour that he studies physics, it yields him 5 marks. He can study at most 14 hours a day and he must get at least 40 marks in each. Determine graphically how many hours a day he should study mathematics and physics each, in order to maximize his marks ? 12

2013

Qu.

(e) Maximize $z = 2x_1 + 3x_2 - 5x_3$
subject to $x_1 + x_2 + x_3 = 7$
and $2x_1 - 5x_2 + x_3 \geq 10,\ x_i \geq 0.$ *10*

Qu.

(a) Solve the minimum time assignment problem : 15

Jobs \ Machines	M_1	M_2	M_3	M_4
J_1	3	12	5	14
J_2	7	9	8	12
J_3	5	11	10	12
J_4	6	14	4	11

(b) Using Cauchy's residue theorem, evaluate the integral

$$I = \int_0^{\pi} \sin^4 \theta \, d\theta$$ 15

(c) Minimize $z = 5x_1 - 4x_2 + 6x_3 - 8x_4$

subject to the constraints

$$x_1 + 2x_2 - 2x_3 + 4x_4 \leq 40$$

$$2x_1 - x_2 + x_3 + 2x_4 \leq 8$$

$$4x_1 - 2x_2 + x_3 - x_4 \leq 10$$

$$x_i \geq 0$$ 20

2014

Qu.

Solve graphically :

Maximize $Z = 6x_1 + 5x_2$

subject to

$$2x_1 + x_2 \leq 16$$
$$x_1 + x_2 \leq 11$$
$$x_1 + 2x_2 \geq 6$$
$$5x_1 + 6x_2 \leq 90$$
$$x_1, x_2 \geq 0$$

10

Qu.

Find the initial basic feasible solution to the following transportation problem by Vogel's approximation method. Also, find its optimal solution and the minimum transportation cost :

		Destinations				
		D_1	D_2	D_3	D_4	*Supply*
Origins	O_1	6	4	1	5	14
	O_2	8	9	2	7	16
	O_3	4	3	6	2	5
Demand		6	10	15	4	

20

Qu.

Find all optimal solutions of the following linear programming problem by the simplex method :

Maximize $Z = 30x_1 + 24x_2$

subject to

$$5x_1 + 4x_2 \leq 200$$
$$x_1 \leq 32$$
$$x_2 \leq 40$$
$$x_1, x_2 \geq 0$$

20

2015

Qu.

Solve the following assignment problem to *maximize* the sales : 10

Salesmen (विक्रेता)	Territories (क्षेत्र) I	II	III	IV	V
A	3	4	5	6	7
B	4	15	13	7	6
C	6	13	12	5	11
D	7	12	15	8	5
E	8	13	10	6	9

Qu.

Consider the following linear programming problem :

$$\text{Maximize } Z = x_1 + 2x_2 - 3x_3 + 4x_4$$

subject to

$$x_1 + x_2 + 2x_3 + 3x_4 = 12$$
$$x_2 + 2x_3 + x_4 = 8$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Qu.

Solve the following linear programming problem by the simplex method. Write its dual. Also, write the optimal solution of the dual from the optimal table of the given problem : 20

$$\text{Maximize } Z = 2x_1 - 4x_2 + 5x_3$$

subject to

$$x_1 + 4x_2 - 2x_3 \leq 2$$
$$-x_1 + 2x_2 + 3x_3 \leq 1$$
$$x_1, x_2, x_3 \geq 0$$

2016

Qu.

(e) व्यवरोधों

$x + 2y \geq 1,\ 2x + y \leq 1,\ x \geq 0$ तथा $y \geq 0$

के साथ $5x + 2y$ का अधिकतम मान आलेखीय विधि द्वारा ज्ञात कीजिए ।

Find the maximum value of

$5x + 2y$

with constraints

$x + 2y \geq 1,\ 2x + y \leq 1,\ x \geq 0$ and $y \geq 0$

by graphical method. *10*

Qu.

(c) अधिकतमीकरण कीजिए

$$z = 2x_1 + 3x_2 + 6x_3$$

बशर्ते कि

$$2x_1 + x_2 + x_3 \leq 5$$

$$3x_2 + 2x_3 \leq 6$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

क्या इष्टतम हल अद्वितीय है ? अपने उत्तर का औचित्य दीजिए ।

Maximize

$$z = 2x_1 + 3x_2 + 6x_3$$

subject to

$$2x_1 + x_2 + x_3 \leq 5$$

$$3x_2 + 2x_3 \leq 6$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

Is the optimal solution unique ? Justify your answer. *20*

2017

Qu.

(e) ग्राफी विधि के इस्तेमाल के द्वारा

$$2x + y$$

का उच्चतम मान, बशर्ते

$$4x + 3y \leq 12$$
$$4x + y \leq 8$$
$$4x - y \leq 8$$
$$x, y \geq 0$$

ज्ञात कीजिए ।

Using graphical method, find the maximum value of

$$2x + y$$

subject to

$$4x + 3y \leq 12$$
$$4x + y \leq 8$$
$$4x - y \leq 8$$
$$x, y \geq 0.$$

10

Qu.

(c) एकधा विधि के द्वारा निम्नलिखित रैखिक प्रोग्रामन समस्या को हल कीजिए :

अधिकतमीकरण कीजिए

$$z = 3x_1 + 5x_2 + 4x_3$$

बशर्ते कि

$$2x_1 + 3x_2 \leq 8$$
$$2x_2 + 5x_3 \leq 10$$
$$3x_1 + 2x_2 + 4x_3 \leq 15$$
$$x_1, x_2, x_3 \geq 0.$$

Solve the following linear programming problem by simplex method :

Maximize

$$z = 3x_1 + 5x_2 + 4x_3$$

subject to

$$2x_1 + 3x_2 \leq 8$$
$$2x_2 + 5x_3 \leq 10$$
$$3x_1 + 2x_2 + 4x_3 \leq 15$$
$$x_1, x_2, x_3 \geq 0.$$

20

(b) निम्नलिखित परिवहन समस्या के लिए, वोगेल की सन्निकटन विधि के द्वारा, आरंभिक आधारिक सुसंगत हल ज्ञात कीजिए तथा लागत ज्ञात कीजिए ।

गन्तव्य

उद्गम		D_1	D_2	D_3	D_4	D_5	पूर्ति
	O_1	4	7	0	3	6	14
	O_2	1	2	– 3	3	8	9
	O_3	3	– 1	4	0	5	17
	माँग	8	3	8	13	8	

Find the initial basic feasible solution of the following transportation problem using Vogel's approximation method and find the cost. *15*

Destinations

Origins		D_1	D_2	D_3	D_4	D_5	Supply
	O_1	4	7	0	3	6	14
	O_2	1	2	– 3	3	8	9
	O_3	3	– 1	4	0	5	17
	Demand	8	3	8	13	8	

2018

2.(b) निम्नलिखित रैखिक प्रोग्रामन समस्या को Big M विधि से हल कीजिए तथा दर्शाइए कि समस्या के परिमित इष्टतम हल हैं । साथ ही उद्देश्य फलन का मान भी ज्ञात कीजिए :

न्यूनतमीकरण कीजिए $z = 3x_1 + 5x_2$

बशर्ते कि $x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

$5x_1 + 6x_2 \leq 60,$

$x_1, x_2 \geq 0.$

Solve the following linear programming problem by Big M-method and show that the problem has finite optimal solutions. Also find the value of the objective function :

Minimize $z = 3x_1 + 5x_2$

subject to $x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

$5x_1 + 6x_2 \leq 60,$

$x_1, x_2 \geq 0.$ 20

3.(c) अधोलिखित समीकरणों के रैखिकतः स्वतंत्र समुच्चय में कितने आधारी हल हैं ? उन सभी को ज्ञात कीजिए ।

$$2x_1 - x_2 + 3x_3 + x_4 = 6$$
$$4x_1 - 2x_2 - x_3 + 2x_4 = 10.$$

How many basic solutions are there in the following linearly independent set of equations ? Find all of them.

$$2x_1 - x_2 + 3x_3 + x_4 = 6$$
$$4x_1 - 2x_2 - x_3 + 2x_4 = 10.$$

15

4.(c) एक फैक्ट्री में पाँच प्रचालक O_1, O_2, O_3, O_4, O_5 तथा पाँच मशीनें M_1, M_2, M_3, M_4, M_5 हैं । परिचालन लागत, जब कि O_i प्रचालक M_j $(i, j = 1, 2, ..., 5)$ मशीन को परिचालन करता है, दी गई हैं । लेकिन एक प्रतिबन्ध है कि O_3 को तीसरी मशीन M_3 का परिचालन करने तथा O_2 को पाँचवीं मशीन M_5 का परिचालन करने की इजाज़त नहीं दी जा सकती है । लागत आव्यूह नीचे दी है । इष्टतम नियतन तथा इष्टतम नियतन की लागत ज्ञात कीजिए ।

मशीन
Machine

प्रचालक Operator	M_1	M_2	M_3	M_4	M_5
O_1	24	29	18	32	19
O_2	17	26	34	22	21
O_3	27	16	28	17	25
O_4	22	18	28	30	24
O_5	28	16	31	24	27

In a factory there are five operators O_1, O_2, O_3, O_4, O_5 and five machines M_1, M_2, M_3, M_4, M_5. The operating costs are given when the O_i operator operates the M_j machine $(i, j = 1, 2, ..., 5)$. But there is a restriction that O_3 cannot be allowed to operate the third machine M_3 and O_2 cannot be allowed to operate the fifth machine M_5. The cost matrix is given above. Find the optimal assignment and the optimal assignment cost also. 15

2019

1.(e) ग्राफी विधि के इस्तेमाल के द्वारा रैखिक प्रोग्रामन समस्या को हल कीजिए ।

अधिकतमीकरण कीजिए $Z = 3x_1 + 2x_2$

बशर्ते कि

$$x_1 - x_2 \geq 1,$$
$$x_1 + x_3 \geq 3$$

और $x_1, x_2, x_3 \geq 0$

Use graphical method to solve the linear programming problem.

Maximize $Z = 3x_1 + 2x_2$

subject to

$$x_1 - x_2 \geq 1,$$
$$x_1 + x_3 \geq 3$$

and $x_1, x_2, x_3 \geq 0$ 10

3.(b) एकधा विधि का इस्तेमाल करते हुए रैखिक प्रोग्रामन समस्या को हल कीजिये :

न्यूनतमीकरण कीजिए $Z = x_1 + 2x_2 - 3x_3 - 2x_4$

बशर्ते कि

$$x_1 + 2x_2 - 3x_3 + x_4 = 4$$
$$x_1 + 2x_2 + x_3 + 2x_4 = 4$$

और $x_1, x_2, x_3, x_4 \geq 0$

Solve the linear programming problem using Simplex method.

Minimize $Z = x_1 + 2x_2 - 3x_3 - 2x_4$

subject to

$$x_1 + 2x_2 - 3x_3 + x_4 = 4$$
$$x_1 + 2x_2 + x_3 + 2x_4 = 4$$

and $x_1, x_2, x_3, x_4 \geq 0$ 15

4.(d) निम्नलिखित एल. पी. पी. पर विचार करें,

अधिकतमीकरण कीजिए $Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$

बशर्ते कि

$$x_1 + x_2 + x_3 = 4$$
$$x_1 + 4x_2 + x_4 = 8$$

और $x_1, x_2, x_3, x_4 \geq 0$

प्रति समस्या का उपयोग करते हुए, सत्यापित करें कि बुनियादी समाधान (x_1, x_2) इष्टतम नहीं है ।

Consider the following LPP,

Maximize $Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$

subject to

$$x_1 + x_2 + x_3 = 4$$
$$x_1 + 4x_2 + x_4 = 8$$

and $x_1, x_2, x_3, x_4 \geq 0$

Use the dual problem to verify that the basic solution (x_1, x_2) is not optimal. 10

2020

UPSC maintenance section has purchased sufficient number of curtain cloth pieces to meet the curtain requirement of its building. The length of each piece is 17 feet. The requirement according to curtain length is as follows :

Curtain length (in feet)	*Number required*
5	700
9	400
7	300

The width of all curtains is same as that of available pieces. Form a linear programming problem in standard form that decides the number of pieces cut in different ways so that the total trim loss is minimum. Also give a basic feasible solution to it. 10

3.(b) एकधा विधि के द्वारा निम्नलिखित रैखिक प्रोग्रामन समस्या को हल कीजिए :

न्यूनतमीकरण कीजिए $z = -6x_1 - 2x_2 - 5x_3$

बशर्ते कि
$$2x_1 - 3x_2 + x_3 \leq 14$$
$$-4x_1 + 4x_2 + 10x_3 \leq 46$$
$$2x_1 + 2x_2 - 4x_3 \leq 37$$
$$x_1 \geq 2,\ x_2 \geq 1,\ x_3 \geq 3$$

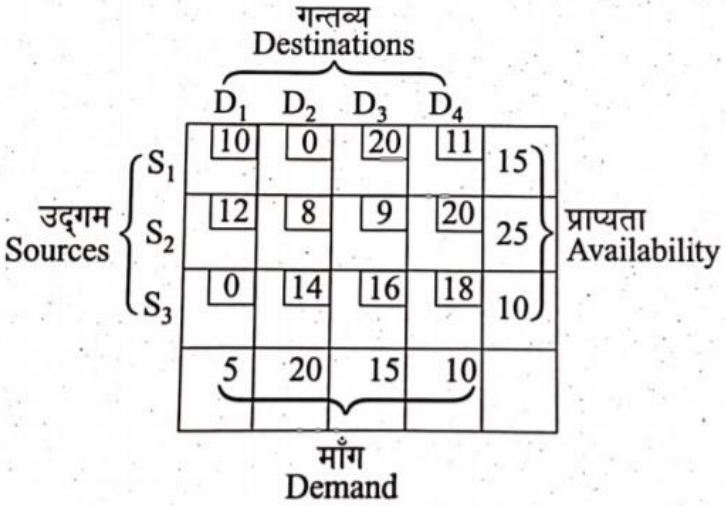
Solve the linear programming problem using simplex method :

Minimize $z = -6x_1 - 2x_2 - 5x_3$

subject to
$$2x_1 - 3x_2 + x_3 \leq 14$$
$$-4x_1 + 4x_2 + 10x_3 \leq 46$$
$$2x_1 + 2x_2 - 4x_3 \leq 37$$
$$x_1 \geq 2,\ x_2 \geq 1,\ x_3 \geq 3$$

15

4.(c) वोगेल की सन्निकटन विधि से निम्नलिखित परिवहन समस्या का आरंभिक आधारिक सुसंगत हल ज्ञात कीजिए। इस हल का उपयोग कर समस्या का इष्टतम हल एवं परिवहन लागत ज्ञात कीजिए।



Find the initial basic feasible solution of the following transportation problem by Vogel's approximation method and use it to find the optimal solution and the transportation cost of the problem. 20