2020

खण्ड 'B' SECTION 'B'

5.(a) $z = y f(x) + x g(y)$ से स्वैच्छिक फलनों $f(x)$ व $g(y)$ का विलोपन कर आंशिक अवकल समीकरण बनाइए तथा इसकी प्रकृति (दीर्घवृत्तीय, अतिपरवलीय या परवलीय) $x>0, y>0$ क्षेत्र में इंगित कीजिए।

Form a partial differential equation by eliminating the arbitrary functions $f(x)$ and $g(y)$ from $z = y f(x) + x g(y)$ and specify its nature (elliptic, hyperbolic or parabolic) in the region $x>0, y>0$. 10

5.(d) आंशिक अवकल समीकरण :

$$(D^3 - 2D^2D' - DD'^2 + 2D'^3)z = e^{2x+y} + \sin(x-2y);$$

$$D \equiv \frac{\partial}{\partial x}, \quad D' \equiv \frac{\partial}{\partial y}$$

को हल कीजिए।

Solve the partial differential equation :

$$(D^3 - 2D^2D' - DD'^2 + 2D'^3)z = e^{2x+y} + \sin(x-2y);$$

$$D \equiv \frac{\partial}{\partial x}, \quad D' \equiv \frac{\partial}{\partial y}$$ 10

6.(a) आंशिक अवकल समीकरण :

$$(x-y)y^2\frac{\partial z}{\partial x} + (y-x)x^2\frac{\partial z}{\partial y} = (x^2+y^2)z$$

के वक्र : $xz = a^3, y = 0$ को अपने ऊपर समाहित करने वाले समाकल पृष्ठ को ज्ञात कीजिए।

Find the integral surface of the partial differential equation : G-20 Maths

$$(x-y)y^2\frac{\partial z}{\partial x} + (y-x)x^2\frac{\partial z}{\partial y} = (x^2+y^2)z$$

that contains the curve : $xz = a^3, y = 0$ on it. 15

..., k is a constant.

20

7.(a) आंशिक अवकल समीकरण :

$$z = \tfrac{1}{2}(p^2+q^2) + (p-x)(q-y); \quad p \equiv \frac{\partial z}{\partial x}, q \equiv \frac{\partial z}{\partial y}$$

का हल ज्ञात कीजिये जो कि x-अक्ष से गुजरता हो।

Find the solution of the partial differential equation :

$$z = \tfrac{1}{2}(p^2+q^2) + (p-x)(q-y); \quad p \equiv \frac{\partial z}{\partial x}, q \equiv \frac{\partial z}{\partial y}$$

which passes through the x-axis. 15

2019

5.(a) निम्नलिखित व्यंजक :

$\psi(x^2+y^2+2z^2, y^2-2zx)=0$

के द्वारा दिए गए पृष्ठ कुल का एक आंशिक अवकल समीकरण बनायें ।

Form a partial differential equation of the family of surfaces given by the following expression :

$\psi(x^2+y^2+2z^2, y^2-2zx)=0.$ 10

6.(a) प्रथम कोटि रैखिककल्प आंशिक अवकल समीकरण

$x\frac{\partial u}{\partial x}+(u-x-y)\frac{\partial u}{\partial y}=x+2y$ में $x>0, -\infty<y<\infty$ को $u=1+y$ के साथ $x=1$ पर अभिलाक्षणिक विधि के द्वारा हल करें ।

Solve the first order quasilinear partial differential equation by the method of characteristics :

$x\frac{\partial u}{\partial x}+(u-x-y)\frac{\partial u}{\partial y}=x+2y$ in $x>0, -\infty<y<\infty$ with $u=1+y$ on $x=1$. 15

7.(c) निम्नलिखित द्वितीय कोटि के आंशिक अवकलन समीकरण को विहित रूप में समानीत करें और सामान्य हल ज्ञात करें :

$\frac{\partial^2 u}{\partial x^2}-2x\frac{\partial^2 u}{\partial x\partial y}+x^2\frac{\partial^2 u}{\partial y^2}=\frac{\partial u}{\partial y}+12x$ ।

Reduce the following second order partial differential equation to canonical form and find the general solution :

$\frac{\partial^2 u}{\partial x^2}-2x\frac{\partial^2 u}{\partial x\partial y}+x^2\frac{\partial^2 u}{\partial y^2}=\frac{\partial u}{\partial y}+12x.$ 20

2018

5.(a) दीर्घवृत्तज : $x^2+4y^2+4z^2=4$ के उन सभी स्पर्श-तलों के संकाय का आंशिक अवकल समीकरण ज्ञात कीजिए, जो xy समतल के लम्बवत नहीं हैं ।

Find the partial differential equation of the family of all tangent planes to the ellipsoid : $x^2+4y^2+4z^2=4$, which are not perpendicular to the xy plane. 10

6.(a) आंशिक अवकल समीकरण :

$(y^3x - 2x^4)p + (2y^4 - x^3y)q = 9z(x^3 - y^3)$, का,

जहाँ $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$ है, का व्यापक हल ज्ञात कीजिए, तथा इसके, वक्र: $x = t, y = t^2, z = 1$ में से गुजरने वाले समाकल पृष्ठ को भी ज्ञात कीजिए ।

Find the general solution of the partial differential equation :

$(y^3x - 2x^4)p + (2y^4 - x^3y)q = 9z(x^3 - y^3)$,

where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$, and find its integral surface that passes through the curve : $x = t, y = t^2, z = 1$. 15

7.(a) आंशिक अवकल समीकरण :

$(2D^2 - 5DD' + 2D'^2)z = 5\sin(2x + y) + 24(y - x) + e^{3x+4y}$ को हल कीजिए

जहाँ $D \equiv \frac{\partial}{\partial x}$, $D' \equiv \frac{\partial}{\partial y}$.

Solve the partial differential equation :

$(2D^2 - 5DD' + 2D'^2)z = 5\sin(2x + y) + 24(y - x) + e^{3x+4y}$

where $D \equiv \frac{\partial}{\partial x}$, $D' \equiv \frac{\partial}{\partial y}$. 15

2017

Q5. (a) $(D^2 - 2DD' + D'^2)\, z = e^{x + 2y} + x^3 + \sin 2x$ को हल कीजिए, जहाँ

$$D \equiv \frac{\partial}{\partial x},\ D' \equiv \frac{\partial}{\partial y},\ D^2 \equiv \frac{\partial^2}{\partial x^2},\ D'^2 \equiv \frac{\partial^2}{\partial y^2}$$ हैं ।

Solve $(D^2 - 2DD' + D'^2)\, z = e^{x + 2y} + x^3 + \sin 2x$, where

$$D \equiv \frac{\partial}{\partial x},\ D' \equiv \frac{\partial}{\partial y},\ D^2 \equiv \frac{\partial^2}{\partial x^2},\ D'^2 \equiv \frac{\partial^2}{\partial y^2}.$$ 10

Q6. (a) आंशिक अवकल समीकरण

$$2\,(pq + yp + qx) + x^2 + y^2 = 0$$

का पूर्ण समाकल ज्ञात कीजिए ।

Find a complete integral of the partial differential equation

$$2\,(pq + yp + qx) + x^2 + y^2 = 0.$$ 15

7. (a) समीकरण

$$y^2 \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \, \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x} \frac{\partial z}{\partial x} + \frac{x^2}{y} \frac{\partial z}{\partial y}$$

को विहित रूप में समानीत कीजिए और अतएव इसका हल ज्ञात कीजिए ।

Reduce the equation

$$y^2 \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \, \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x} \frac{\partial z}{\partial x} + \frac{x^2}{y} \frac{\partial z}{\partial y}$$

to canonical form and hence solve it. *15*

2016

Q5. सभी प्रश्नों के उत्तर दीजिए :

Answer all the questions : *10×5=50*

(a) $x^2 + y^2 + z^2 = cz$ द्वारा दिए गए गोलों के कुल के लम्बकोणीय पृष्ठों का व्यापक समीकरण ज्ञात कीजिए ।

Find the general equation of surfaces orthogonal to the family of spheres given by $x^2 + y^2 + z^2 = cz$. *10*

(e) आंशिक अवकल समीकरण

$$(y + zx)\,p - (x + yz)\,q = x^2 - y^2$$

का व्यापक समाकल ज्ञात कीजिए ।

Find the general integral of the partial differential equation

$$(y + zx)\,p - (x + yz)\,q = x^2 - y^2.$$ *10*

Q7. (a) आंशिक अवकल समीकरण

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x\, \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$$

को हल कीजिए ।

Solve the partial differential equation

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x\, \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$$ *15*

Q8. (a) लम्बाई 10 cm तथा अचर अनुप्रस्थ-परिच्छेद का क्षेत्रफल 1 cm^2 की चाँदी की एक छड़ में तापमान $u(x, t)$ ज्ञात कीजिए । मान लीजिए घनत्व $\rho = 10{\cdot}6$ g/cm^3, ऊष्मा चालकता $K = 1{\cdot}04$ cal / (cm sec °C) तथा विशिष्ट ऊष्मा $\sigma = 0{\cdot}056$ cal/g °C. छड़ पूर्णत: पार्श्विक वियुक्त (perfectly isolated laterally) है, सिरों को 0°C पर रखा गया है तथा प्रारम्भिक तापमान $f(x) = \sin(0{\cdot}1\,\pi x)$ °C है । ध्यान रखिए कि $u(x, t)$ तापीय समीकरण $u_t = c^2 u_{xx}$ का अनुगमन करता है, जहाँ $c^2 = K / (\rho\,\sigma)$ है ।

Find the temperature $u(x, t)$ in a bar of silver of length 10 cm and constant cross-section of area 1 cm^2. Let density $\rho = 10{\cdot}6$ g/cm^3, thermal conductivity $K = 1{\cdot}04$ cal / (cm sec °C) and specific heat $\sigma = 0{\cdot}056$ cal/g °C. The bar is perfectly isolated laterally, with ends kept at 0°C and initial temperature $f(x) = \sin(0{\cdot}1\,\pi x)$ °C. Note that $u(x, t)$ follows the heat equation $u_t = c^2 u_{xx}$, where $c^2 = K / (\rho\,\sigma)$. *20*

2015

5. *(a)* आंशिक अवकल समीकरण

$$(y^2 + z^2 - x^2)p - 2xyq + 2xz = 0$$

जहाँ $p = \frac{\partial z}{\partial x}$ तथा $q = \frac{\partial z}{\partial y}$, को हल कीजिए।

Solve the partial differential equation

$$(y^2 + z^2 - x^2)p - 2xyq + 2xz = 0$$

where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. 10

(b) $(D^2 + DD' - 2D'^2)u = e^{x+y}$ को हल कीजिए, जहाँ $D = \frac{\partial}{\partial x}$ तथा $D' = \frac{\partial}{\partial y}$.

Solve $(D^2 + DD' - 2D'^2)u = e^{x+y}$, where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$. 10

6. *(a)* $p\cos(x+y) + q\sin(x+y) = z$, जहाँ $p = \frac{\partial z}{\partial x}$ तथा $q = \frac{\partial z}{\partial y}$, को व्यापक हल के लिए हल कीजिए।

Solve for the general solution $p\cos(x+y) + q\sin(x+y) = z$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. 15

Find the solution of the initial-boundary value problem

$$u_t - u_{xx} + u = 0, \quad 0 < x < l, \ t > 0$$
$$u(0, t) = u(l, t) = 0, \quad t \geq 0$$
$$u(x, 0) = x(l - x), \quad 0 < x < l$$

15

8. *(a)* द्वितीय-कोटि आंशिक अवकल समीकरण

$$x^2 \frac{\partial^2 u}{\partial x^2} - 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

को विहित रूप में समानीत कीजिए तथा इसका व्यापक हल ज्ञात कीजिए।

Reduce the second-order partial differential equation

$$x^2 \frac{\partial^2 u}{\partial x^2} - 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

into canonical form. Hence, find its general solution. 15

2014

5. *(a)* आंशिक अवकल समीकरण $(2D^2 - 5DD' + 2D'^2)\, z = 24(y - x)$ को हल कीजिए।

Solve the partial differential equation $(2D^2 - 5DD' + 2D'^2)\, z = 24(y - x)$. 10

6. *(a)* समीकरण $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ को विहित रूप में समानीत कीजिए।

Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

7. *(a)* एक कंपमान डोरी (लम्बाई = π, स्थिर सिरे, $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$) का विक्षेप ज्ञात कीजिए, यदि प्रारंभिक वेग शून्य हो और प्रारंभिक विक्षेप $f(x) = k(\sin x - \sin 2x)$ हो।

Find the deflection of a vibrating string (length = π, ends fixed, $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$) corresponding to zero initial velocity and initial deflection

$$f(x) = k(\sin x - \sin 2x)$$ 15

8. *(a)* हल कीजिए $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1$, $t > 0$, दिया है कि

(i) $u(x, 0) = 0$, $0 \le x \le 1$

(ii) $\frac{\partial u}{\partial t}(x, 0) = x^2$, $0 \le x \le 1$

(iii) $u(0, t) = u(1, t) = 0$, सभी t के लिए

Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1$, $t > 0$, given that

(i) $u(x, 0) = 0$, $0 \le x \le 1$

(ii) $\frac{\partial u}{\partial t}(x, 0) = x^2$, $0 \le x \le 1$

(iii) $u(0, t) = u(1, t) = 0$, for all t 15

2013

(a) Form a partial differential equation by eliminating the arbitrary functions f and g from $z = y f(x) + x g(y)$. 10

(b) Reduce the equation

$$y \frac{\partial^2 z}{\partial x^2} + (x+y) \frac{\partial^2 z}{\partial x \partial y} + x \frac{\partial^2 z}{\partial y^2} = 0$$

to its canonical form when $x \neq y$. 10

(a) Solve

$$(D^2 + DD' - 6D'^2) z = x^2 \sin(x+y)$$

where D and D′ denote $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$. 15

(b) Find the surface which intersects the surfaces of the system

$$z(x+y) = C(3z+1), \text{ (C being a constant)}$$

orthogonally and which passes through the circle $x^2 + y^2 = 1$, $z = 1$. 15

(c) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in equilibrium position. If it is set vibrating by giving each point a velocity $\lambda . x (l - x)$, find the displacement of the string at any distance x from one end at any time t. 20

2012

Section 'B'

5. (a) Solve the partial differential equation

$$(D - 2D')(D - D')^2 z = e^{x+y}.$$ 12

6. (a) Solve the partial differential equation $px + qy = 3z$. 20

(b) A string of length l is fixed at its ends. The string from the mid-point is pulled up to a height k and then released from rest. Find the deflection $y(x, t)$ of the vibrating string. 20

(b) The edge $r = a$ of a circular plate is kept at temperature $f(\theta)$. The plate is insulated so that there is no loss of heat from either surface. Find the temperature distribution in steady state. 20

2011

SECTION—B

5. (a) Solve the PDE

$$(D^2 - D'^2 + D + 3D' - 2)\, z = e^{(x-y)} - x^2 y$$

12

(b) Solve the PDE

$$(x + 2z)\frac{\partial z}{\partial x} + (4zx - y)\frac{\partial z}{\partial y} = 2x^2 + y$$

12

6\. (a) Find the surface satisfying $\frac{\partial^2 z}{\partial x^2} = 6x + 2$ and touching $z = x^3 + y^3$ along its section by the plane $x + y + 1 = 0$. 20

(b) Solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 \le x \le a, \ 0 \le y \le b$$

satisfying the boundary conditions

$u(0, y) = 0$, $u(x, 0) = 0$, $u(x, b) = 0$

$\frac{\partial u}{\partial x}(a, y) = T \sin^3 \frac{\pi y}{a}$. 20

(c) Obtain temperature distribution $y(x, t)$ in a uniform bar of unit length whose one end is kept at 10°C and the other end is insulated. Also it is given that $y(x, 0) = 1 - x$, $0 < x < 1$. 20

2010

Section 'B'

5. Attempt any *five* of the following :

(a) Solve the *PDE*

$$\left(D^2 - D'\right)(D-2D')Z = e^{2x+y} + xy.$$ 12

(b) Find the surface satisfying the *PDE* $\left(D^2 - 2DD' + D'^2\right)Z = 0$ and the conditions that $bZ = y^2$ when $x = 0$ and $aZ = x^2$ when $y = 0$. 12

6. (a) Solve the following partial differential equation

$$zp + yq = x$$

$x_0(s) = s,\ y_0(s) = 1,\ z_0(s) = 2s$

by the method of characteristics. 20

(b) Reduce the following 2nd order partial differential equation into canonical form and find its general solution

$x\,u_{xx} + 2x^2 u_{xy} - u_x = 0.$ 20

(c) Solve the following heat equation

$u_t - u_{xx} = 0,\quad 0 < x < 2,\quad t > 0$

$u(0,\ t) = u(2,\ t) = 0,\quad t > 0$

$u(x,\ 0) = x(2-x),\quad 0 \leqslant x \leqslant 2.$ 20