2020

(e) दीर्घवृत्तज $2x^2+6y^2+3z^2=27$ के स्पर्श समतल का समीकरण निकालिए, जो रेखा $x-y-z=0=x-y+2z-9$ से होकर गुजरता है।

Find the equations of the tangent plane to the ellipsoid $2x^2+6y^2+3z^2=27$ which passes through the line $x-y-z=0=x-y+2z-9$. 10

(c) एक ऐसे बेलन का समीकरण निकालिए, जिसकी जनक-रेखाएँ, रेखा $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ के समांतर हैं तथा जिसका मार्गदर्शक वक्र $x^2+y^2=4,\ z=2$ है।

Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and whose guiding curve is $x^2+y^2=4,\ z=2$. 15

(c) यदि सरल रेखा $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ शंकु $5yz-8zx-3xy=0$ के तीन परस्पर लांबिक जनकों के समुच्चय में से एक है, तब अन्य दो जनकों के समीकरण निकालिए।

If the straight line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ represents one of a set of three mutually perpendicular generators of the cone $5yz-8zx-3xy=0$, then find the equations of the other two generators. 15

(b) अतिपरवलयिक परवलयज $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z$ के लांबिक जनकों के प्रतिच्छेद बिंदु का बिंदुपथ निकालिए।

Find the locus of the point of intersection of the perpendicular generators of the hyperbolic paraboloid $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z$. 15

2019

(e) दर्शाइए कि

$$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1} \quad \text{और} \quad \frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$$

प्रतिच्छेदी रेखाएँ हैं। प्रतिच्छेद बिंदु के निर्देशांकों और उस समतल, जिसमें दोनों रेखाएँ हैं, का समीकरण ज्ञात कीजिए।

Show that the lines

$$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1} \quad \text{and} \quad \frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$$

intersect. Find the coordinates of the point of intersection and the equation of the plane containing them. 10

(i) The plane $x+2y+3z=12$ cuts the axes of coordinates in A, B, C. Find the equations of the circle circumscribing the triangle ABC. 10

(ii) Prove that the plane $z=0$ cuts the enveloping cone of the sphere $x^2+y^2+z^2=11$ which has the vertex at (2, 4, 1) in a rectangular hyperbola. 10

(b) सिद्ध कीजिए कि साधारणतः किसी एक बिंदु से परवलयज $x^2+y^2=2az$ पर तीन अभिलंब बनाए जा सकते हैं, लेकिन अगर बिंदु सतह $27a(x^2+y^2)+8(a-z)^3=0$ पर स्थित है, तो इन तीन अभिलंबों में से दो अभिलंब एक ही हैं।

Prove that, in general, three normals can be drawn from a given point to the paraboloid $x^2+y^2=2az$, but if the point lies on the surface

$$27a(x^2+y^2)+8(a-z)^3=0$$

then two of the three normals coincide. 15

(b) बिंदु P से गुजरने वाली दीर्घवृत्तज

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

की अभिलंब जीवा की लंबाई ज्ञात कीजिए और सिद्ध कीजिए कि अगर यह $4PG_3$ के समान है, जहाँ G_3 वह बिंदु है जहाँ P से गुजरने वाली अभिलंब जीवा xy-तल पर मिलती है, तो P शंकु

$$\frac{x^2}{a^6}(2c^2-a^2)+\frac{y^2}{b^6}(2c^2-b^2)+\frac{z^2}{c^4}=0$$

पर स्थित है।

Find the length of the normal chord through a point P of the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

and prove that if it is equal to $4PG_3$, where G_3 is the point where the normal chord through P meets the xy-plane, then P lies on the cone

$$\frac{x^2}{a^6}(2c^2-a^2)+\frac{y^2}{b^6}(2c^2-b^2)+\frac{z^2}{c^4}=0$$

15

2018

(e) सरल रेखा $\frac{x-1}{2}=\frac{y-1}{3}=\frac{z+1}{-1}$ का समतल $x+y+2z=6$ पर प्रक्षेपण ज्ञात कीजिये।

Find the projection of the straight line $\frac{x-1}{2}=\frac{y-1}{3}=\frac{z+1}{-1}$ on the plane $x+y+2z=6$. 10

(d) रेखाओं

$$a_1x+b_1y+c_1z+d_1=0$$
$$a_2x+b_2y+c_2z+d_2=0$$

और z-अक्ष के बीच की न्यूनतम दूरी ज्ञात कीजिये।

Find the shortest distance between the lines

$$a_1x+b_1y+c_1z+d_1=0$$
$$a_2x+b_2y+c_2z+d_2=0$$

and the z-axis. 12

(c) परवलयज $(x+y+z)(2x+y-z)=6z$ की उन जनक रेखाओं के समीकरणों को ज्ञात कीजिये, जो बिन्दु (1, 1, 1) में से गुज़रती हैं।

Find the equations to the generating lines of the paraboloid $(x+y+z)(2x+y-z)=6z$ which pass through the point (1, 1, 1). 13

(d) xyz-समतल में स्थित, बिन्दुओं (0, 0, 0), (0, 1, −1), (−1, 2, 0) और (1, 2, 3) में से गुज़रते हुये गोले का समीकरण ज्ञात कीजिये।

Find the equation of the sphere in xyz-plane passing through the points (0, 0, 0), (0, 1, −1), (−1, 2, 0) and (1, 2, 3). 12

(c) उस शंकु, जिसका शीर्ष (0, 0, 1) है और जिसका निर्देशक वक्र $2x^2-y^2=4,\ z=0$ है, का समीकरण ज्ञात कीजिये।

Find the equation of the cone with (0, 0, 1) as the vertex and $2x^2-y^2=4,\ z=0$ as the guiding curve. 13

(d) $3x-y+3z=8$ के समांतर और बिन्दु (1, 1, 1) में से गुज़रते हुये समतल का समीकरण ज्ञात कीजिये।

Find the equation of the plane parallel to $3x-y+3z=8$ and passing through the point (1, 1, 1). 12

2017

1.(d) बिन्दु (1, 1, 1) पर शांकवज $3x^2-y^2=2z$ के स्पर्श-तल का समीकरण निकालिए।

Find the equation of the tangent plane at point (1, 1, 1) to the conicoid $3x^2-y^2=2z$. 10

1.(e) विषमतलीय रेखाओं $\frac{x-3}{3}=\frac{8-y}{1}=\frac{z-3}{1}$ व $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ के बीच न्यूनतम-दूरी ज्ञात कीजिए।

Find the shortest distance between the skew lines :

$\frac{x-3}{3}=\frac{8-y}{1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$. 10

2.(b) एक समतल, नियत बिन्दु (a, b, c) में से गुज़रता है तथा अक्षों को क्रमशः बिन्दुओं A, B व C पर काटता है। मूल बिन्दु O तथा A, B व C में से गुज़रने वाले गोले के केन्द्र का बिन्दु-पथ ज्ञात कीजिए।

A plane passes through a fixed point (a, b, c) and cuts the axes at the points A, B, C respectively. Find the locus of the centre of the sphere which passes through the origin O and A, B, C. 15

2.(c) दर्शाइए कि समतल $2x - 2y + z + 12 = 0$, गोले $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$, को स्पर्श करता है। सम्पर्क बिन्दु ज्ञात कीजिए।

Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$. Find the point of contact. 10

3.(d) $ax^2 + by^2 + cz^2 = 1$ के तीन परस्पर लम्बवत् स्पर्शतलों के प्रतिच्छेदन बिन्दु का बिन्दु-पथ ज्ञात कीजिए।

Find the locus of the point of intersection of three mutually perpendicular tangent planes to $ax^2 + by^2 + cz^2 = 1$. 10

4.(a) समीकरण $x^2 + y^2 + z^2 - yz - zx - xy - 3x - 6y - 9z + 21 = 0$ को प्रमाणिक-रूप में व्यक्त कीजिए अतः शांकवज की प्रकृति निर्धारित कीजिए।

Reduce the following equation to the standard form and hence determine the nature of the conicoid : $x^2 + y^2 + z^2 - yz - zx - xy - 3x - 6y - 9z + 21 = 0$. 15

2016

(d) उस गोले (sphere) का समीकरण निकालिये, जो वृत्त $x^2 + y^2 = 4;\ z = 0$ से गुजरता है और जो तल $x + 2y + 2z = 0$ से एक वृत्त, जिसकी त्रिज्या 3 है, में काटा जाता है।

Find the equation of the sphere which passes through the circle $x^2 + y^2 = 4;\ z = 0$ and is cut by the plane $x + 2y + 2z = 0$ in a circle of radius 3. 10

(e) रेखाओं $\frac{x-1}{2} = \frac{y-2}{4} = z - 3$ तथा $y - mx = z = 0$ के बीच लघुतम दूरी (shortest distance) निकालिये। m के किस मान के लिए दोनों रेखाएँ प्रतिच्छेद (intersect) करेंगी?

Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{4} = z - 3$ and $y - mx = z = 0$. For what value of m will the two lines intersect? 10

4. *(a)* एक रेखा, जो रेखाओं $y=a=z,\ x+3z=a=y+z$ को प्रतिच्छेद (intersect) करती है तथा तल $x+y=0$ के समानान्तर है, द्वारा जनित सतह (surface generated) निकालिये।

Find the surface generated by a line which intersects the lines $y=a=z$, $x+3z=a=y+z$ and parallel to the plane $x+y=0$. 10

(b) सिद्ध कीजिये कि शंकु (cone) $3yz-2zx-2xy=0$ के तीन परस्पर लम्बीय जनकों (generators) का एक अनन्त समुच्चय है। यदि $\frac{x}{1}=\frac{y}{1}=\frac{z}{2}$ ऐसे किसी समुच्चय का एक जनक (generator) हो, तो बाकी दो निकालिये।

Show that the cone $3yz-2zx-2xy=0$ has an infinite set of three mutually perpendicular generators. If $\frac{x}{1}=\frac{y}{1}=\frac{z}{2}$ is a generator belonging to one such set, find the other two. 10

(d) शांकवज (conicoid) $ax^2+by^2+cz^2=1$ के तीन पारस्परिक लम्बीय स्पर्शी तलों के प्रतिच्छेदन बिन्दु का बिन्दुपथ निकालिये।

Find the locus of the point of intersection of three mutually perpendicular tangent planes to the conicoid $ax^2+by^2+cz^2=1$. 15

2015

Q. 1(e) **'a' के किस धनात्मक मान के लिए, समतल $ax-2y+z+12=0$, गोलक $x^2+y^2+z^2-2x-4y+2z-3=0$ को स्पर्श करता है। स्पर्श बिन्दु को भी ज्ञात कीजिये।**

For what positive value of a, the plane $ax-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and hence find the point of contact. 10

Q. 2(d) **यदि शंकु $5yz-8zx-3xy=0$ की तीन परस्पर लम्बवत् जनक रेखाओं में से एक जनक रेखा $6x=3y=2z$ हो, तब अन्य दो जनक रेखाओं के समीकरण मालूम कीजिये।**

If $6x=3y=2z$ represents one of the three mutually perpendicular generators of the cone $5yz-8zx-3xy=0$ then obtain the equations of the other two generators. 13

Q. 3(c) (i) उस समतल का समीकरण निकालिए जो बिन्दुओं (2, 3, 1) एवं (4, –5, 3) से गुजरता है व x-अक्ष के समान्तर है।

Obtain the equation of the plane passing through the points (2, 3, 1) and (4, –5, 3) parallel to x-axis. 6

(ii) सत्यापित कीजिये कि रेखाएँ :

$$\frac{x-a+d}{\alpha-\delta}=\frac{y-a}{\alpha}=\frac{z-a-d}{\alpha+\delta} \quad \text{तथा} \quad \frac{x-b+c}{\beta-\gamma}=\frac{y-b}{\beta}=\frac{z-b-c}{\beta+\gamma}$$

समतलीय हैं। यदि हाँ, तो उस समतल का समीकरण ज्ञात कीजिए, जिसमें उपरोक्त दोनों रेखाएँ स्थित हैं।

Verify if the lines :

$$\frac{x-a+d}{\alpha-\delta}=\frac{y-a}{\alpha}=\frac{z-a-d}{\alpha+\delta} \quad \text{and} \quad \frac{x-b+c}{\beta-\gamma}=\frac{y-b}{\beta}=\frac{z-b-c}{\beta+\gamma}$$

are coplanar. If yes, then find the equation of the plane in which they lie. 7

Q. 4(c) यदि परवलयज $x^2 + y^2 = 2z$ पर दो लम्बवत् स्पर्शीय समतल एक सीधी रेखा में, जो समतल $x = 0$ में, पर काटते हैं। उस वक्र को प्राप्त कीजिए जिस पर वह सीधी रेखा स्पर्श करती है।

Two perpendicular tangent planes to the paraboloid $x^2 + y^2 = 2z$ intersect in a straight line in the plane $x = 0$. Obtain the curve to which this straight line touches. 13

2014

0

(e) परीक्षण कीजिए कि क्या समतल $x + y + z = 0$ शंकु $yz + zx + xy = 0$ को समकोणीय (लंब) रेखाओं में काटता है।

Examine whether the plane $x + y + z = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines. *10*

Q4. (a) (i) गोलक $x^2 + y^2 + z^2 - 4x + 2y = 4$ के बिन्दुओं के निर्देशांक ज्ञात कीजिए जिसके स्पर्शी समतल, समतल $2x - y + 2z = 1$ के समांतर हैं ।

Find the co-ordinates of the points on the sphere $x^2 + y^2 + z^2 - 4x + 2y = 4$, the tangent planes at which are parallel to the plane $2x - y + 2z = 1$. *10*

(ii) सिद्ध कीजिए कि समीकरण $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ एक शंकु निरूपित करता है, यदि $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = d$ हो तो ।

Prove that the equation $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$, represents a cone if $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = d$. *10*

(b) दर्शाइए कि उद्गम (मूल-बिन्दु) से खींची हुई रेखाएँ, जो केन्द्रीय शांकवज $ax^2 + by^2 + cz^2 = 1$ के समतल $lx + my + nz = p$ के साथ प्रतिच्छेदन बिन्दुओं पर लम्बों के समान्तर हैं, शंकु $p^2\left(\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c}\right) = \left(\frac{lx}{a} + \frac{my}{b} + \frac{nz}{c}\right)^2$ का जनन करती हैं ।

Show that the lines drawn from the origin parallel to the normals to the central conicoid $ax^2 + by^2 + cz^2 = 1$, at its points of intersection with the plane $lx + my + nz = p$ generate the cone

$$p^2\left(\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c}\right) = \left(\frac{lx}{a} + \frac{my}{b} + \frac{nz}{c}\right)^2.$$ *15*

(c) अतिपरवलयज के समतल $z = 0$ के द्वारा मुख्य दीर्घवृत्तीय खण्ड $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ z = 0$ के कोई बिन्दु $(a\cos\theta,\ b\sin\theta,\ 0)$ में से दो जनक रेखाओं के समीकरण ज्ञात कीजिए ।

Find the equations of the two generating lines through any point $(a\cos\theta, b\sin\theta, 0)$, of the principal elliptic section $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ z = 0$, of the hyperboloid by the plane $z = 0$. *15*

2013

1.(d) Find the equation of the plane which passes through the points (0, 1, 1) and (2, 0, –1), and is parallel to the line joining the points (–1, 1, –2), (3, –2, 4). Find also the distance between the line and the plane. 10

1.(e) A sphere S has points (0, 1, 0), (3, –5, 2) at opposite ends of a diameter. Find the equation of the sphere having the intersection of the sphere S with the plane $5x - 2y + 4z + 7 = 0$ as a great circle. 10

4.(a) Show that three mutually perpendicular tangent lines can be drawn to the sphere $x^2 + y^2 + z^2 = r^2$ from any point on the sphere $2(x^2 + y^2 + z^2) = 3r^2$. 15

4.(b) A cone has for its guiding curve the circle $x^2 + y^2 + 2ax + 2by = 0$, $z = 0$ and passes through a fixed point $(0, 0, c)$. If the section of the cone by the plane $y = 0$ is a rectangular hyperbola, prove that the vertex lies on the fixed circle

$$x^2 + y^2 + z^2 + 2ax + 2by = 0$$
$$2ax + 2by + cz = 0.$$ 15

4.(c) A variable generator meets two generators of the system through the extremities B and B' of the minor axis of the principal elliptic section of the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} - z^2c^2 = 1$ in P and P'. Prove that $BP \cdot B'P' = a^2 + c^2$. 20

SECTION 'B'

2012

(b) A variable plane is parallel to the plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$$

and meets the axes in A, B, C respectively. Prove that the circle ABC lies on the cone

$$yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$$ 20

(c) Show that the locus of a point from which the three mutually perpendicular tangent lines can be drawn to the paraboloid $x^2 + y^2 + 2z = 0$ is

$$x^2 + y^2 + 4z = 1$$ 20

(e) Prove that two of the straight lines represented by the equation

$$x^3 + bx^2y + cxy^2 + y^3 = 0$$

will be at right angles, if $b + c = -2$. 12

2011

(f) Show that the equation of the sphere which touches the sphere

$$4(x^2 + y^2 + z^2) + 10x - 25y - 2z = 0$$

at the point (1, 2, –2) and passes through the point (–1, 0, 0) is

$$x^2 + y^2 + z^2 + 2x - 6y + 1 = 0.$$ 10

4. (a) Three points P, Q, R are taken on the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ so that the lines joining P, Q, R to the origin are mutually perpendicular. Prove that the plane PQR touches a fixed sphere. 20

(b) Show that the cone $yz + zx + xy = 0$ cuts the sphere $x^2 + y^2 + z^2 = a^2$ in two equal circles, and find their area. 20

(c) Show that the generators through any one of the ends of an equiconjugate diameter of the principal elliptic section of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ are inclined to each other at an angle of $60°$ if $a^2 + b^2 = 6c^2$. Find also the condition for the generators to be perpendicular to each other. 20

2010

(e) Show that the plane $x+y-2z=3$ cuts the sphere $x^2+y^2+z^2-x+y=2$ in a circle of radius 1 and find the equation of the sphere which has this circle as a great circle. 12

(c) Show that the plane $3x + 4y + 7z + \frac{5}{2} = 0$ touches the paraboloid $3x^2 + 4y^2 = 10z$ and find the point of contact. 20

(c) Show that every sphere through the circle

$$x^2 + y^2 - 2ax + r^2 = 0, \quad z = 0$$

cuts orthogonally every sphere through the circle

$$x^2 + z^2 = r^2, \quad y = 0$$ 20

(c) Find the vertices of the skew quadrilateral formed by the four generators of the hyperboloid

$$\frac{x^2}{4} + y^2 - z^2 = 49$$

passing through (10, 5, 1) and (14, 2, – 2). 20

2009

(e) A line is drawn through a variable point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \ z = 0$ to meet two fixed lines $y = mx, \ z = c$ and $y = -mx, \ z = -c$. Find the locus of the line. 12

(f) Find the equation of the sphere having its centre on the plane $4x-5y-z=3$, and passing through the circle

$$x^2+y^2+z^2-12x-3y+4z+8=0$$
$$3x+4y-5z+3=0$$ 12

(c) Prove that the normals from the point (α, β, γ) to the paraboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$ lie on the cone

$$\frac{\alpha}{x-\alpha}+\frac{\beta}{y-\beta}+\frac{a^2-b^2}{z-\gamma}=0$$ 20

2008

(e) The plane $x-2y+3z=0$ is rotated through a right angle about its line of intersection with the plane $2x+3y-4z-5=0$; find the equation of the plane in its new position. 12

$+6y^2=140$.

(c) A sphere S has points (0, 1, 0), (3, −5, 2) at opposite ends of a diameter. Find the equation of the sphere having the intersection of the sphere S with the plane $5x-2y+4z+7=0$ as a great circle. 20

(c) If $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ represent one of a set of three mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$, find the equations of the other two. 20

(b) Obtain the volume bounded by the elliptic paraboloids given by the equations

$z - x^2 + 9y^2$ and $z = 18 - x^2 - 9y^2$. 20

(c) Show that the enveloping cylinders of the ellipsoid $ax^2 + by + cz^2 = 1$ with generators perpendicular to z axis meet the plane $z = 0$ in parabolas. 20

2007

revolution.

(e) Find the locus of the point which moves so that its distance from the plane $x + y - z = 1$ is twice its distance from the line $x = -y = z$. 12

Q. 4. (a) Show that the spheres $x^2 + y^2 + z^2 - x + z - 2 = 0$ and $3x^2 + 3y^2 + 3z^2 - 8x - 10y + 8z + 14 = 0$ cut orthogonally. Find the centre and radius of their common circle. 15

(b) A line with direction ratios 2, 7, –5 is drawn to intersect the lines

$$\frac{x}{3} = \frac{y-1}{2} = \frac{z-2}{4} \text{ and } \frac{x-11}{3} = \frac{y-5}{-1} = \frac{z}{1}.$$

Find the coordinates of the points of intersection and the length intercepted on it. 15

(c) Show that the plane $2x - y + 2z = 0$ cuts the cone $xy + yz + zx = 0$ in perpendicular lines. 15

(d) Show that the feet of the normals from the point P (α, β, γ), $\beta = 0$ on the paraboloid $x^2 + y^2 = 4z$ lie on the sphere

$$2\beta (x^2 + y^2 + z^2) - (\alpha^2 + \beta^2)\ y - 2\beta (2 + \gamma)\ z = 0$$ 15

SECTION 'B'