2001

SECTION – B

Q. 5. **Attempt any *five* of the following :**

(a) A continuous function $y(t)$ satisfies the differential equation

$$\frac{dy}{dt} = 1 + e^{1-t}, \qquad 0 \le t < 1$$

$$= 2 + 2t - 3t^2, \; 1 \le t < 5$$

If $y(0) = -e$, find $y(2)$. 12

(b) Solve : 12

$$x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log_e x$$

Q. 6. (a) Solve :

$$\frac{dy}{dx}+\frac{y}{x}\log_e y=\frac{y(\log_e y)^2}{x^2}$$

(b) Find the general solution of

$ayp^2+(2x-b)\,p-y=0,\ a>0$ 15

(c) Solve :

$(D^2+1)^2\,y=24x\cos x$

given that $y=Dy=D^2y=0$ and $D^3y=12$ when $x=0$.

15

(d) Using the method of variation of parameters, solve

$$\frac{d^2y}{dx^2}+4y=4\tan 2x$$ 15

2002

SECTION 'B'

Q. 5. Attempt any *five* of the following :

(a) Solve :

$$x\frac{dy}{dx}+3y=x^3y^2$$ 12

(b) Find the values of λ for which all solutions of

$x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}-\lambda y=0$ tend to zero as $x \to \infty$. 12

Q. 6. (a) Find the value of constant λ such that the following differential equation becomes exact.

$$(2xe^y+3y^2)\frac{dy}{dx}+(3x^2+\lambda e^y)=0$$

Further, for this value of λ, solve the equation. 15

(b) Solve : $\frac{dy}{dx}=\frac{x+y+4}{x-y-6}$. 15

(c) Using the method of variation of parameters, find the solution of

$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x\sin x$ with $y(0)=0$ and $\left(\frac{dy}{dx}\right)_{x=0}=0$.

15

(d) Solve : $(D-1)(D^2-2D+2)y=e^x$ where $D\equiv\frac{d}{dx}$. 15

2003

as the intersection of two planes. 15

SECTION 'B'

Q. 5. Attempt any FIVE of the following–

(a) Show that the orthogonal trajectory of a system of confocal ellipses is self orthogonal. 12

(b) Solve : $x\dfrac{dy}{dx} + y\log y = xye^{x}$. 12

Q. 6. (a) Solve $(D^5 - D)y = 4(e^x + \cos x + x^3)$, where $D = \dfrac{d}{dx}$. 15

(b) Solve the differential equation $(px^2 + y^2)(px + y) = (P + 1)^2$, where $p = \dfrac{dy}{dx}$, by reducing it to Clairaut's form using suitable substitutions. 15

(c) Solve

$(1 + x)^2 y'' + (1 + x)y' + y = \sin 2[\log(1 + x)]$. 15

(b) Solve the differential equation.

$$x^2 y'' - 4xy' + 6y = x^4 \sec^2 x$$

by variation of parameters. 15

2004

SECTION 'B'

Q. 5. Attempt any *five* of the following :

(a) Find the solution of the following differential equation

$$\frac{dy}{dx} + y\cos x = \frac{1}{2}\sin 2x.$$ 12

(b) Solve : $y(xy + 2x^2y^2)\,dx + x(xy - x^2y^2)\,dy = 0.$ 12

Q. 6. (a) Solve : $(D^4 - 4D^2 - 5)\,y = e^x(x + \cos x).$ 15

(b) Reduce the equation $(px - y)(py + x) = 2p$, where $p = \frac{dy}{dx}$ to Clairaut's equation and hence solve it. 15

(c) Solve : $(x+2)\frac{d^2y}{dx^2} - (2x+5)\frac{dy}{dx} + 2y = (x+1)e^x.$ 15

(d) Solve the following differential equation :

$$(1-x^2)\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} - (1+x^2)y = x.$$ 15

2005

SECTION 'B'

Q. 5. Attempt any *five* of the following :

(a) Find the orthogonal trajectory of a system of co-axial circles $x^2 + y^2 + 2gx + c = 0$, where g is the parameter. 12

(b) Solve :

$$xy\frac{dy}{dx} = \sqrt{(x^2 - y^2 - x^2y^2 - 1)}$$

Q. 6. (a) Solve the differential equation : 15

$$[(x+1)^4 D^3 + 2(x+1)^3 D^2 - (x+1)^2 D + (x+1)]\, y = \frac{1}{x+1}$$

(b) Solve the differential equation

$$(x^2+y^2)(1+p)^2 - 2(x+y)(1+p)(x+yp) + (x+yp)^2 = 0$$

where $p = \frac{dy}{dx}$, by reducing it to Clairaut's form by using suitable substitution. 15

(c) Solve the differential equation

$(\sin x - x\cos x)\, y'' - x\sin x\, y' + y\sin x = 0$ given that $y = \sin x$ is a solution of this equation. 15

(d) Solve the differential equation

$$x^2 y'' - 2xy' + 2y = x\log x,\ x > 0$$

by variation of parameters. 15

2006

(b) Solve the differential equation

$$\left(xy^2 + e^{-\frac{1}{x^3}}\right)dx - x^2 y\, dy = 0$$ 12

Q. 6. (a) Solve :

$$(1+y^2)+(x-e^{-\tan^{-1}y})\frac{dy}{dx}=0$$ 15

(b) Solve the equiation

$$x^2p^2+yp(2x+y)+y^2=0$$

using the substitution $y = u$ and $xy = v$ and find its singular solution, where

$$p=\frac{dy}{dx}$$ 15

(c) Solve the differential equation

$$x^2\frac{d^3y}{dx^3}+2x\frac{d^2y}{dx^2}+2\frac{y}{x}=10\left(1+\frac{1}{x^2}\right)$$ 15

(d) Solve the differential equation

$$(D^2-2D+2)y=e^x\tan x,\ D\equiv\frac{d}{dx}$$ by the method of variation of parameters. 15

2007

SECTION B

Q. 5. Attempt any *five* of the following:

(a) Solve the ordinary differential equation

$$\cos 3x\frac{dy}{dx}-3y\sin 3x=\frac{1}{2}\sin 6x+\sin^2 3x,\quad 0<x<\frac{\pi}{2}$$ 12

(b) Find the solution of the equation

$$\frac{dy}{y}+xy^2dx=-\,4x\,dx$$ 12

Q. 1. Attempt any *five* of the following :

(a) Let S be the vector space of all polynomials $p(x)$, with real co-efficients, of degree less than or equal to two considered over the real field $\mathbb{R}$ such that $p(0)$ 0 and $p(1) = 0$. Determine a basis for S and hence its dimension. 12

(b) Let T be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^4$ defined by

$$T(x_1, x_2, x_3) = (2x_1 + x_2 + x_3,\ x_1 + x_2,\ x_1 + x_3,\ 3x_1 + x_2 = 2x_3) \text{ for each } (x_1, x_2, x_3) \in \mathbb{R}^3.$$

Determine a basis for the Null space of T. What is the

dimension of the Range space of T?

Q. 6. (a) Determine the general and singular solutions of the equation

$$y = x\frac{dy}{dx} + a\frac{dy}{dx}\left[1+\left(\frac{dy}{dx}\right)^2\right]^{-\frac{1}{2}}$$, a being a constant. 15

(b) Obtain the general solution of

$$[D^3 - 6D^2 + 12D - 8]\, y = 12\left(e^{2x} + \frac{9}{4}e^{-x}\right),$$

Where $D \equiv \frac{d}{dx}$. 15

(c) Solve the equation

$$2x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 3y = x^3$$ 15

(d) Use the method of variation of parameters to find the general solution of the equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2e^x$$ 15

2008

SECTION—B

Q. 5. Attempt any FIVE of the following :—

(a) Solve the differential equation

$ydx + (x + x^3y^2)\, dy = 0.$ 12

(b) Use the method of variation of parameters to find the general solution of

$x^2y'' - 4xy' + 6y = -x^4 \sin x.$ 12

(b) Using Laplace transform, solve the initial value problem

$y'' - 3y' + 2y = 4t + e^{3t},$

with $y(0) = 1, \quad y'(0) = -1.$ 15

(c) Solve the differential equation

$x^3y'' - 3x^2y' + xy = \sin(lnx) + 1.$ 15

(d) Solve the equation

$y - 2xp + yp^2 = 0$, where $p = \dfrac{dy}{dx}$. 15

2009

12

(b) Find the Wronskian of the set of functions :

$\{3x^3, |3x^3|\}$

on the interval [–1, 1] and determine whether the set is linearly dependent on [–1, 1].

12

(c) A uniform rod AB is movable ...

Q. 6. (a) Find the differential equation of the family of circles n the xy-plane passing through (–1, 1) and (1, 1). 20

(b) Find the inverse Laplace transform of

$$F(s) = \ln\left(\frac{s+1}{s+5}\right)$$ 20

(c) Solve :

$$\frac{dy}{dx} = \frac{y^2(x-y)}{3xy^2 - x^2y - 4y^3}, \; y(0) = 1$$

2010

5. Attempt any *five* of the following :

(a) Consider the differential equation

$$y' = \alpha x, \quad x > 0$$

where α is a constant. Show that—

(i) if $\phi(x)$ is any solution and $\psi(x) = \phi(x)e^{-\alpha x}$, then $\psi(x)$ is a constant;

(ii) if $\alpha < 0$, then every solution tends to zero as $x \to \infty$. 12

(b) Show that the differential equation

$$(3y^2 - x) + 2y(y^2 - 3x)y' = 0$$

admits an integrating factor which is a function of $(x + y^2)$. Hence solve the equation. 12

6. *(a)* Verify that

$$\frac{1}{2}(Mx + Ny)\,d(\log_e(xy)) + \frac{1}{2}(Mx - Ny)\,d(\log_e(\tfrac{x}{y})) = M\,dx + N\,dy$$

Hence show that—

(i) if the differential equation $M\,dx + N\,dy = 0$ is homogeneous, then $(Mx + Ny)$ is an integrating factor unless $Mx + Ny \equiv 0$;

(ii) if the differential equation $M\,dx + N\,dy = 0$ is not exact but is of the form

$$f_1(x\,y)\,y\,dx + f_2(x\,y)\,x\,dy = 0$$

then $(Mx - Ny)^{-1}$ is an integrating factor unless $Mx - Ny \equiv 0$. 20

8. *(a)* Use the method of undetermined coefficients to find the particular solution of

$$y'' + y = \sin x + (1 + x^2)e^x$$

and hence find its general solution. 20

2011

SECTION—B

5. (a) Obtain the solution of the ordinary differential equation $\frac{dy}{dx}=(4x+y+1)^2$, if $y(0)=1$. 10

(b) Determine the orthogonal trajectory of a family of curves represented by the polar equation

$$r = a(1 - \cos\theta),$$

(r, θ) being the plane polar coordinates of any point. 10

6. (a) Obtain Clairaut's form of the differential equation

$$\left(x\frac{dy}{dx}-y\right)\left(y\frac{dy}{dx}+x\right)=a^2\frac{dy}{dx}.$$

Also find its general solution. 15

(b) Obtain the general solution of the second order ordinary differential equation

$$y'' - 2y' + 2y = x + e^x \cos x,$$

where dashes denote derivatives w.r. to x. 15

(c) Using the method of variation of parameters, solve the second order differential equation

$$\frac{d^2y}{dx^2}+4y=\tan 2x.$$ 15

(d) Use Laplace transform method to solve the following initial value problem :

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t, x(0) = 2 \text{ and } \left.\frac{dx}{dt}\right|_{t=0} = -1.$$

15

2012

5. *(a)* Solve

$$\frac{dy}{dx} = \frac{2xy\,e^{(x/y)^2}}{y^2(1 + e^{(x/y)^2}) + 2x^2 e^{(x/y)^2}}$$ 12

(b) Find the orthogonal trajectories of the family of curves $x^2 + y^2 = ax$. 12

(c) Using Laplace transforms, solve the initial value problem

$$y'' + 2y' + y = e^{-t},\ y(0) = -1,\ y'(0) = 1$$ 12

6. *(a)* Show that the differential equation

$$(2xy\log y)\,dx + (x^2 + y^2\sqrt{y^2+1})\,dy = 0$$

is not exact. Find an integrating factor and hence, the solution of the equation. 20

(b) Find the general solution of the equation $y''' - y'' = 12x^2 + 6x$. 20

(c) Solve the ordinary differential equation

$$x(x-1)y'' - (2x-1)y' + 2y = x^2(2x-3)$$ 20

2013

SECTION 'B'

5. Answer all the questions :

5.(a) y is a function of x, such that the differential coefficient $\frac{dy}{dx}$ is equal to $\cos(x+y) + \sin(x+y)$. Find out a relation between x and y, which is free from any derivative/differential. 10

5.(b) Obtain the equation of the orthogonal trajectory of the family of curves represented by $r^n = a \sin n\theta$, (r, θ) being the plane polar coordinates. 10

5.(c) A body is performing S.H.M. in a straight line OPQ. Its velocity ...

6.(a) Solve the differential equation
$(5x^3 + 12x^2 + 6y^2)dx + 6xydy = 0.$ 10

6.(b) Using the method of variation of parameters, solve the differential equation
$\frac{d^2y}{dx^2} + a^2y = \sec ax.$ 10

6.(c) Find the general solution of the equation
$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \ln x \sin(\ln x).$ 15

6.(d) By using Laplace transform method, solve the differential equation
$\left(D^2 + n^2\right)x = a \sin(nt + \alpha),\ D^2 \equiv \frac{d^2}{dt^2}$ subject to the initial conditions
$x = 0$ and $\frac{dx}{dt} = 0$, at $t = 0$, in which a, n and α are constants. 15

2014

Q5. ***सभी*** प्रश्नों के उत्तर दीजिए :
Answer ***all*** the questions : *10×5=50*

(a) उचित ठहराइए कि

$$[y + x\, f(x^2 + y^2)]\, dx + [y\, f(x^2 + y^2) - x]\, dy = 0$$

की भांति अवकल समीकरण जहाँ कि $f(x^2 + y^2)$, $(x^2 + y^2)$ का एक स्वेच्छ फलन है, एक यथातथ अवकल समीकरण नहीं है तथा इसका $\frac{1}{x^2 + y^2}$ एक समाकलन गुणक है । अतएव इस अवकल समीकरण को $f(x^2 + y^2) = (x^2 + y^2)^2$ के लिए हल कीजिए ।

Justify that a differential equation of the form :

$$[y + x\, f(x^2 + y^2)]\, dx + [y\, f(x^2 + y^2) - x]\, dy = 0,$$

where $f(x^2 + y^2)$ is an arbitrary function of $(x^2 + y^2)$, is not an exact differential equation and $\frac{1}{x^2 + y^2}$ is an integrating factor for it. Hence solve this differential equation for $f(x^2 + y^2) = (x^2 + y^2)^2$. *10*

Q6. (a) प्राचलों के विचरण की विधि के द्वारा हल कीजिए :

$$\frac{dy}{dx} - 5y = \sin x$$

Solve by the method of variation of parameters : *10*

$$\frac{dy}{dx} - 5y = \sin x$$

(b) अवकल समीकरण हल कीजिए :

$$x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + 8y = 65 \cos (\log_e x)$$

Solve the differential equation : *20*

$$x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + 8y = 65 \cos (\log_e x)$$

Q7. (a) निम्नलिखित अवकल समीकरण हल कीजिए :

$$x \frac{d^2y}{dx^2} - 2(x+1) \frac{dy}{dx} + (x+2)\, y = (x-2)e^{2x}$$

जबकि e^x इसके संगत समघात अवकल समीकरण का एक हल है ।

Solve the following differential equation :

$$x \frac{d^2y}{dx^2} - 2(x+1) \frac{dy}{dx} + (x+2)\, y = (x-2)e^{2x},$$

when e^x is a solution to its corresponding homogeneous differential equation. *15*

Q8. (a) अवकल समीकरण $M(x, y)\, dx + N(x, y)\, dy = 0$ के लिए पर्याप्त शर्त ज्ञात कीजिए ताकि उसका समाकलन गुणक, $(x + y)$ का फलन हो । उस दशा में समाकलन गुणक क्या होगा ? अतएव अवकल समीकरण $(x^2 + xy)\, dx + (y^2 + xy)\, dy = 0$ का समाकलन गुणक ज्ञात कीजिए तथा हल कीजिए ।

Find the sufficient condition for the differential equation $M(x, y)\, dx + N(x, y)\, dy = 0$ to have an integrating factor as a function of $(x + y)$. What will be the integrating factor in that case ? Hence find the integrating factor for the differential equation

$$(x^2 + xy)\, dx + (y^2 + xy)\, dy = 0,$$

and solve it. *15*

(c) प्रारंभिक मान समस्या

$$\frac{d^2y}{dt^2} + y = 8\,e^{-2t} \sin t, \quad y(0) = 0, \; y'(0) = 0$$

को लाप्लास-रूपांतर के प्रयोग से हल कीजिए ।

Solve the initial value problem

$$\frac{d^2y}{dt^2} + y = 8\,e^{-2t} \sin t, \quad y(0) = 0, \; y'(0) = 0$$

by using Laplace-transform. 20

2015

Q. 5(a) निम्नलिखित अवकल समीकरण को हल कीजिये :

$$x \cos x \frac{dy}{dx} + y(x \sin x + \cos x) = 1.$$

Solve the differential equation :

$$x \cos x \frac{dy}{dx} + y(x \sin x + \cos x) = 1.$$ 10

Q. 5(b) निम्नलिखित अवकल समीकरण का हल निकालिये :

$$(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0.$$

Solve the differential equation :

$$(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0.$$ 10

Q. 6(a) यदि $(x + y)^a$, निम्न अवकल समीकरण $(4x^2 + 2xy + 6y)dx + (2x^2 + 9y + 3x)dy = 0$ का समाकलन गुणांक है तो 'a' का मान मालूम कीजिये। तत्पश्चात् अवकल समीकरण का हल निकालिए।

Find the constant a so that $(x + y)^a$ is the Integrating factor of $(4x^2 + 2xy + 6y)dx + (2x^2 + 9y + 3x)dy = 0$ and hence solve the differential equation. 12

Q. 7(a) (i) निम्नलिखित का लाप्लास विलोम रूपांतर प्राप्त कीजिये :

$$\left\{\ell n\left(1+\frac{1}{s^2}\right)+\frac{s}{s^2+25}e^{-\pi s}\right\}.$$

(ii) लाप्लास रूपांतर का प्रयोग करके, निम्नलिखित

$$y'' + y = t,\ y(0) = 1,\ y'(0) = -2$$

का हल निकालिए।

(i) Obtain Laplace Inverse transform of

$$\left\{\ell n\left(1+\frac{1}{s^2}\right)+\frac{s}{s^2+25}e^{-\pi s}\right\}.$$

(ii) Using Laplace transform, solve

$$y'' + y = t,\ y(0) = 1,\ y'(0) = -2.$$ 6+6=12

Q. 7(d) अवकल समीकरण

$$x = py - p^2$$

का हल निकालिए, जहाँ $p=\frac{dy}{dx}$.

Solve the differential equation

$$x = py - p^2 \text{ where } p=\frac{dy}{dx}.$$ 13

Q. 8(d) निम्न अवकल समीकरण को हल करें :

$$x^4\frac{d^4y}{dx^4}+6x^3\frac{d^3y}{dx^3}+4x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^2+2\cos(\log_e x).$$

Solve :

$$x^4\frac{d^4y}{dx^4}+6x^3\frac{d^3y}{dx^3}+4x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^2+2\cos(\log_e x).$$ 13

2016

5. (a) $\frac{d^2y}{dx^2}+y=e^{x/2}\sin\frac{x\sqrt{3}}{2}$ का विशेष समाकल (particular integral) निकालिये।

Find a particular integral of $\frac{d^2y}{dx^2}+y=e^{x/2}\sin\frac{x\sqrt{3}}{2}$. 10

(c) हल कीजिये :

Solve : 10

$$\frac{dy}{dx} = \frac{1}{1+x^2}(e^{\tan^{-1}x} - y)$$

(d) दर्शाइये कि परवलय-कुल $y^2 = 4cx + 4c^2$ स्वलांबिक (self-orthogonal) है।

Show that the family of parabolas $y^2 = 4cx + 4c^2$ is self-orthogonal. 10

6. (a) हल कीजिये :

Solve : 10

$$\{y(1 - x\tan x) + x^2 \cos x\}\, dx - x\, dy = 0$$

(b) अवकल समीकरण (differential equation)

$$(D^2 + 2D + 1)\, y = e^{-x} \log(x), \qquad \left[D \equiv \frac{d}{dx}\right]$$

को प्राचल-विचरण (variation of parameters) विधि से हल कीजिये।

Using the method of variation of parameters, solve the differential equation

$$(D^2 + 2D + 1)\, y = e^{-x} \log(x), \qquad \left[D \equiv \frac{d}{dx}\right]$$ 15

(c) समीकरण $x^2 \frac{d^3y}{dx^3} - 4x\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 4$ का व्यापक हल (general solution) निकालिये।

Find the general solution of the equation $x^2 \frac{d^3y}{dx^3} - 4x\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 4$. 15

(d) लाप्लास रूपांतरण (Laplace transformation) की मदद से निम्न का हल निकालिये :

Using Laplace transformation, solve the following : 10

$$y'' - 2y' - 8y = 0, \quad y(0) = 3, \quad y'(0) = 6$$

2017

खण्ड 'B' SECTION 'B'

5.(a) x-y समतल में सभी वृत्तों को निरूपित करने वाला अवकल समीकरण ज्ञात कीजिए।

Find the differential equation representing all the circles in the x-y plane. 10

6.(a) (i) निम्नलिखित युगपत रेखीय अवकल समीकरणों को हल कीजिए :

$(D+1)y = z + e^x$ व $(D+1)z = y + e^x$, जहाँ y व z स्वतंत्र चर x के फलन हैं तथा $D \equiv \frac{d}{dx}$ ।

(i) Solve the following simultaneous linear differential equations :

$(D+1)y = z + e^x$ and $(D+1)z = y + e^x$ where y and z are functions of independent variable x and $D \equiv \frac{d}{dx}$. 8

6.(b) (i) अवकल समीकरण : $xy\,p^2 - (x^2 + y^2 - 1)\,p + xy = 0$, जहाँ पर $p = \frac{dy}{dx}$ है, पर विचार कीजिए। $u = x^2$ तथा $v = y^2$ द्वारा प्रतिस्थापन कर क्लेराउट्स रूप (Clairaut's form) में u, v तथा $p' = \frac{dv}{du}$ में व्यक्त कीजिए। अतएव या अन्यथा समीकरण को हल कीजिए।

(i) Consider the differential equation $xy\,p^2 - (x^2 + y^2 - 1)\,p + xy = 0$ where $p = \frac{dy}{dx}$. Substituting $u = x^2$ and $v = y^2$ reduce the equation to Clairaut's form in terms of u, v and $p' = \frac{dv}{du}$. Hence, or otherwise solve the equation. 10

6.(b) (ii) निम्नलिखित प्रारम्भिक-मान अवकल समीकरणों को हल कीजिए :

$20y'' + 4y' + y = 0,\ y(0) = 3{\cdot}2$ व $y'(0) = 0$ ।

(ii) Solve the following initial value differential equations :

$20y'' + 4y' + y = 0,\ y(0) = 3{\cdot}2$ and $y'(0) = 0$. 7

7.(b) (i) निम्नलिखित अवकल समीकरण को हल कीजिए :

$$x\frac{d^2y}{dx^2}-\frac{dy}{dx}-4x^3y=8x^3\sin(x^2)\text{ ।}$$

(i) Solve the differential equation :

$$x\frac{d^2y}{dx^2}-\frac{dy}{dx}-4x^3y=8x^3\sin(x^2).$$ 9

7.(b) (ii) निम्नलिखित अवकल समीकरण को प्राचल-विचरण विधि के द्वारा हल कीजिए :

$$\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=44-76x-48x^2\text{ ।}$$

(ii) Solve the following differential equation using method of variation of parameters :

$$\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=44-76x-48x^2.$$ 8

8.(b) निम्नलिखित प्रारम्भिक-मान समस्या को लैपलास रूपान्तरण के द्वारा हल कीजिए :

$$\frac{d^2y}{dx^2}+9y=r(x),\quad y(0)=0,\quad y'(0)=4$$

जहाँ पर $r(x)=\begin{cases}8\sin x & \text{यदि } 0<x<\pi\\ 0 & \text{यदि } x\geqslant\pi\end{cases}$

Solve the following initial value problem using Laplace transform :

$$\frac{d^2y}{dx^2}+9y=r(x),\quad y(0)=0,\quad y'(0)=4$$

where $r(x)=\begin{cases}8\sin x & \text{if } 0<x<\pi\\ 0 & \text{if } x\geqslant\pi\end{cases}$ 17

2018

5. *(a)* हल कीजिये/Solve : 10

$$y''-y=x^2e^{2x}$$

(c) हल कीजिये/Solve : 10

$$y''' - 6y'' + 12y' - 8y = 12e^{2x} + 27e^{-x}$$

(d) (i) $f(t) = \frac{1}{\sqrt{t}}$ का लाप्लास रूपान्तर ज्ञात कीजिये।

Find the Laplace transform of $f(t) = \frac{1}{\sqrt{t}}$.

(ii) $\frac{5s^2 + 3s - 16}{(s-1)(s-2)(s+3)}$ का विलोम लाप्लास रूपान्तर ज्ञात कीजिये।

Find the inverse Laplace transform of $\frac{5s^2 + 3s - 16}{(s-1)(s-2)(s+3)}$. 10

6. (a) हल कीजिये/Solve : 13

$$\left(\frac{dy}{dx}\right)^2 y + 2\frac{dy}{dx}x - y = 0$$

(c) हल कीजिये/Solve : 13

$$y'' + 16y = 32\sec 2x$$

(c) प्रारंभिक मान समस्या

$$y'' - 5y' + 4y = e^{2t}$$
$$y(0) = \frac{19}{12},\ y'(0) = \frac{8}{3}$$

को हल कीजिये।

Solve the initial value problem

$$y'' - 5y' + 4y = e^{2t}$$
$$y(0) = \frac{19}{12},\ y'(0) = \frac{8}{3}$$

13

(d) α और β को, जिसके लिये $x^{\alpha}y^{\beta}$ समीकरण $(4y^2 + 3xy)dx - (3xy + 2x^2)dy = 0$ का एक समाकलन गुणक है, ज्ञात कीजिये और समीकरण हल कीजिये।

Find α and β such that $x^{\alpha}y^{\beta}$ is an integrating factor of $(4y^2 + 3xy)dx - (3xy + 2x^2)dy = 0$ and solve the equation. 12

(d) $f(y)$, जिसके लिये समीकरण $(2xe^y + 3y^2)dy + (3x^2 + f(y))dx = 0$ यथातथ्य है, ज्ञात कीजिये और हल निकालिये।

Find $f(y)$ such that $(2xe^y + 3y^2)dy + (3x^2 + f(y))dx = 0$ is exact and hence solve. 12

2019

5. (a) अवकल समीकरण

$$(2y\sin x + 3y^4 \sin x\cos x)\,dx - (4y^3\cos^2 x + \cos x)\,dy = 0$$

को हल कीजिए।

Solve the differential equation

$$(2y\sin x + 3y^4 \sin x\cos x)\,dx - (4y^3\cos^2 x + \cos x)\,dy = 0$$ 10

(b) अवकल समीकरण

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 3x^2e^{2x}\sin 2x$$

का पूर्ण हल ज्ञात कीजिए।

Determine the complete solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 3x^2e^{2x}\sin 2x$$ 10

(c) (i) अवकल समीकरण

$$\frac{d^2y}{dx^2} + (3\sin x - \cot x)\frac{dy}{dx} + 2y\sin^2 x = e^{-\cos x}\sin^2 x$$

को हल कीजिए।

(ii) $t^{-1/2}$ तथा $t^{1/2}$ का लाप्लास रूपांतर ज्ञात कीजिए। सिद्ध कीजिए कि $t^{n+\frac{1}{2}}$ का लाप्लास रूपांतर

$$\frac{\Gamma\left(n+1+\frac{1}{2}\right)}{s^{n+1+\frac{1}{2}}}$$

होता है, जहाँ $n \in \mathbf{N}$.

(i) Solve the differential equation

$$\frac{d^2y}{dx^2} + (3\sin x - \cot x)\frac{dy}{dx} + 2y\sin^2 x = e^{-\cos x}\sin^2 x$$ 10

(ii) Find the Laplace transforms of $t^{-1/2}$ and $t^{1/2}$. Prove that the Laplace transform of $t^{n+\frac{1}{2}}$, where $n \in \mathbf{N}$, is

$$\frac{\Gamma\left(n+1+\frac{1}{2}\right)}{s^{n+1+\frac{1}{2}}}$$ 10

7. (a) समीकरण $x^2y'' - 2xy' + 2y = x^3 \sin x$ के संगत समांगी अवकल समीकरण का रेखीय स्वतंत्र हल निकालिए और तब दिए गए समीकरण का प्राचल-विचरण विधि द्वारा सामान्य हल निकालिए।

Find the linearly independent solutions of the corresponding homogeneous differential equation of the equation $x^2y'' - 2xy' + 2y = x^3 \sin x$ and then find the general solution of the given equation by the method of variation of parameters. 15

8. *(a)* अवकल समीकरण

$$\left(\frac{dy}{dx}\right)^2\left(\frac{y}{x}\right)^2\cot^2\alpha-2\left(\frac{dy}{dx}\right)\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)^2\operatorname{cosec}^2\alpha=1$$

का विचित्र हल प्राप्त कीजिए। दिए हुए अवकल समीकरण का पूर्ण पूर्वग भी ज्ञात कीजिए। पूर्ण पूर्वग तथा विचित्र हल की ज्यामितीय व्याख्या कीजिए।

Obtain the singular solution of the differential equation

$$\left(\frac{dy}{dx}\right)^2\left(\frac{y}{x}\right)^2\cot^2\alpha-2\left(\frac{dy}{dx}\right)\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)^2\operatorname{cosec}^2\alpha=1$$

Also find the complete primitive of the given differential equation. Give the geometrical interpretations of the complete primitive and singular solution. 15

2020

5. *(a)* निम्न अवकल समीकरण को हल कीजिए :

$$x\cos\left(\frac{y}{x}\right)(y\,dx+x\,dy)=y\sin\left(\frac{y}{x}\right)(x\,dy-y\,dx)$$

Solve the following differential equation :

$$x\cos\left(\frac{y}{x}\right)(y\,dx+x\,dy)=y\sin\left(\frac{y}{x}\right)(x\,dy-y\,dx)$$ 10

(b) वृत्त-कुल, जो बिंदु (0, 2) एवं (0, −2) से गुजरता है, का लंबकोणीय संछेदी ज्ञात कीजिए।

Find the orthogonal trajectories of the family of circles passing through the points (0, 2) and (0, −2). 10

6. *(a)* प्राचल विचरण विधि का प्रयोग करके, निम्न अवकल समीकरण का हल निकालिए, यदि $y=e^{-x}$, पूरक फलन (CF) का एक हल है :

$$y''+(1-\cot x)y'-y\cot x=\sin^2 x$$

Using the method of variation of parameters, solve the differential equation $y''+(1-\cot x)y'-y\cot x=\sin^2 x$, if $y=e^{-x}$ is one solution of CF. 20

20

(b) लाप्लास रूपांतरण का प्रयोग करके प्रारंभिक मान समस्या $ty''+2ty'+2y=2$; $y(0)=1$ तथा $y'(0)$ स्वेच्छ है, को हल कीजिए। क्या इस प्रश्न का हल अद्वितीय है?

Using Laplace transform, solve the initial value problem $ty''+2ty'+2y=2$; $y(0)=1$ and $y'(0)$ is arbitrary. Does this problem have a unique solution? 10

8. (a) (i) निम्न अवकल समीकरण हल कीजिए :

$$(x+1)^2 y'' - 4(x+1)y' + 6y = 6(x+1)^2 + \sin\log(x+1)$$

(ii) अवकल समीकरण $9p^2(2-y)^2 = 4(3-y)$ के व्यापक व विचित्र हल निकालिए, जहाँ $p = \frac{dy}{dx}$.

(i) Solve the following differential equation :

$$(x+1)^2 y'' - 4(x+1)y' + 6y = 6(x+1)^2 + \sin\log(x+1)$$ 10

(ii) Find the general and singular solutions of the differential equation $9p^2(2-y)^2 = 4(3-y)$, where $p = \frac{dy}{dx}$. 10

2021

5.(a) अवकल समीकरण :

$$\frac{d^2y}{dx^2} + 2y = x^2 e^{3x} + e^x \cos 2x$$

को हल कीजिए।

Solve the differential equation :

$$\frac{d^2y}{dx^2} + 2y = x^2 e^{3x} + e^x \cos 2x$$ 10

5.(b) लाप्लास रूपान्तर विधि का उपयोग करते हुए प्रारम्भिक मान समस्या :

$$\frac{d^2y}{dx^2} + 4y = e^{-2x} \sin 2x;\ y(0) = y'(0) = 0$$

को हल कीजिए।

Solve the initial value problem :

$$\frac{d^2y}{dx^2} + 4y = e^{-2x} \sin 2x;\ y(0) = y'(0) = 0$$

using Laplace transform method. 10

6.(b) सभी अन्तर्ग्रस्त (शामिल) चरणों को दर्शाते हुए समीकरण :

$$\frac{d^2y}{dx^2} + (\tan x - 3\cos x)\frac{dy}{dx} + 2y\cos^2 x = \cos^4 x$$

को पूर्ण रूप से हल कीजिए।

Solve the equation :

$$\frac{d^2y}{dx^2} + (\tan x - 3\cos x)\frac{dy}{dx} + 2y\cos^2 x = \cos^4 x$$

completely by demonstrating all the steps involved. 15

7.(b) अवकल समीकरण :

$$y^2 \log y = xy\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2$$

के सभी सम्भव हल ज्ञात कीजिए।

Find all possible solutions of the differential equation : 15

$$y^2 \log y = xy\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2.$$

... डोरी से एक स्थिर बिन्दु से टँगा है तथा $\sqrt{2gh}$ वेग से क्षैतिज

... की अवितान्य ...

Find the orthogonal trajectories of the family of confocal conics

$$\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1;$$ $a > b > 0$ are constants and λ is a parameter.

Show that the given family of curves is self orthogonal. 10

8.(a)(ii) अवकल समीकरण : $x^2\frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = 0$ का व्यापक हल ज्ञात कीजिए। अतः अवकल समीकरण : $x^2\frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = x^3$ को प्राचल विचरण विधि द्वारा हल कीजिए।

Find the general solution of the differential equation :

$$x^2\frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = 0.$$

Hence, solve the differential equation : $x^2\frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = x^3$ by the method of variation of parameters. 10