`eforensics` Analysis of Three Pennsylvania Counties in the 2024 Presidential Election*

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Data are 2024 president election precinct counts from three counties in Pennsylvania. The counties are Allegheny, Erie and Philadelphia. Statewide Trump received 3,543,308 votes and Harris received 3,423,042 votes, but in the three counties the candidates' votes total respectively 496,505 and 1,063,766.

For `eforensics`-plots and subsequent `eforensics` model estimation the leader is the candidate with the most votes statewide. The `eforensics`-plots for precinct turnout and leader vote choice proportion data reveal strong multimodality in vote choice proportions in the original data (Figure 1(a)): both Trump's support and voter turnout are higher in Erie county than in the other two counties. The distribution irregularities persist even when county fixed effects are removed (Figure 1(b)). The data are clumpy (efficiency .9671).[1]

To estimate the `eforensics` model in a way that provides information about the difference between election-day votes and other kinds of votes I create a variable that measures the proportion of the votes cast for each precinct that are election-day votes. Using V_i to denote the number of votes cast for either Trump or Harris at precinct i and E_i to denote the number of election-day votes,[2] I define

$$\text{ED proportion}_i = E_i/V_i\,.$$

The ED proportion_i variable has a minimum of .462, a maximum of .906 and a median of .696. I include the ED proportion_i variable as a covariate in x_i^ι and x_i^υ in the frauds magnitude proportions

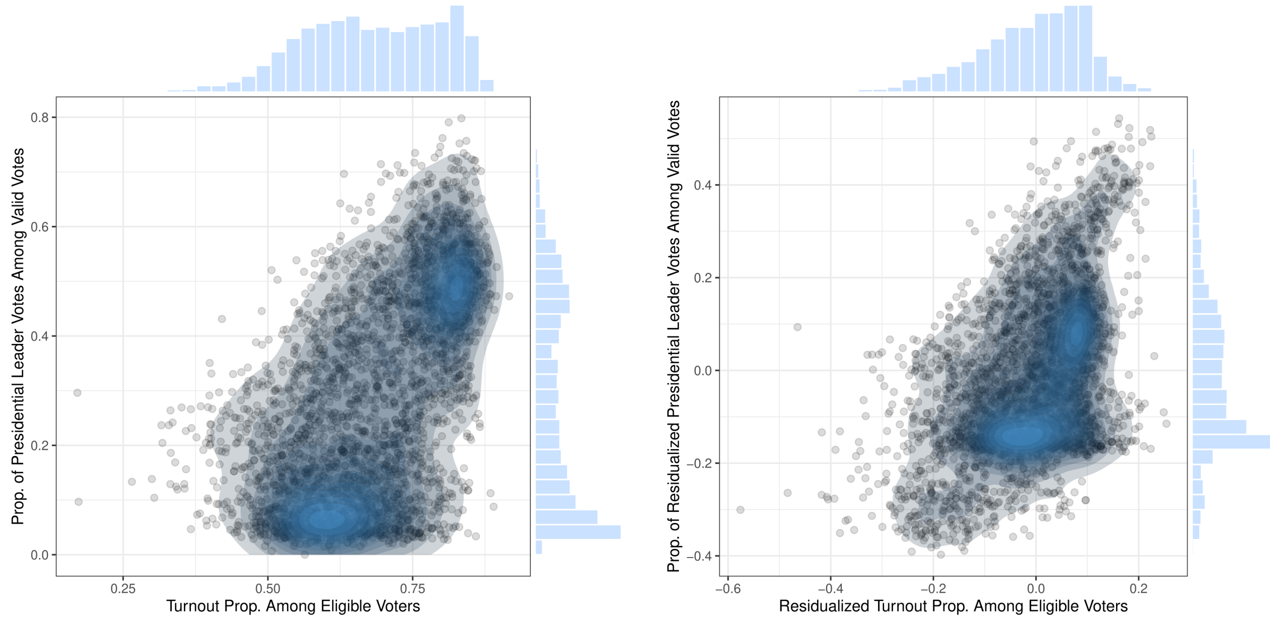
$$\iota_i^l = \frac{k}{1+\exp[-(\rho_l^\top x_i^\iota + \kappa_i^{\iota l})]}\,, l \in \{M,S\} \tag{1a}$$

$$\upsilon_i^l = k + \frac{1-k}{1+\exp[-(\delta_l^\top x_i^\upsilon + \kappa_i^{\upsilon l})]}\,, l \in \{M,S\} \tag{1b}$$

[1]To compute entropy measures I use a 102 × 102 grid. See Mebane (2023, 19) for the definition of the efficiency measure.

[2]Variables in the three spreadsheets of data I received that I treat as counting the election-day votes are `EDTotalVotes` for Allegheny and Erie and `EDVotes` for Philadelphia.

Figure 1: `eforensics`-plots: Pennsylvania 2024 President Three Counties, Second Round



(a) original data (b) county-residualized data

Note: scatterplots, 2D empirical densities and marginal histograms for turnout and leader vote proportions. $n = 3179$ precincts. For `eforensics` estimates see Tables 1 and 2. Entropy: residualized observed (b), 5.95; Normal simulation, 7.19; efficiency, .9671.

(see Mebane (2023, 5–8) for further details about the formal `eforensics` model definition). If the frauds magnitudes coefficients in ρ_M, ρ_S, δ_M or δ_S are positive then estimated `eforensics`-fraudulent votes for precincts that have active `eforensics`-frauds tend to be larger, and if the coefficients are negative then the estimated `eforensics`-fraudulent votes tend to be smaller.

The `eforensics` estimates reported in Table 1 are for a model specification that includes county fixed effects for turnout and vote choice. Diagnostics signal MCMC posterior multimodality for the mixture probabilities, e.g., $D(\pi_2) = 0$ is significant and $M(\pi_2) = .103$ is large: probably there are lost votes, i.e., turnout rates that differ for would-be supporters of of different candidates. All `eforensics`-frauds are incremental frauds: of $n = 3179$ precincts 57 have incremental frauds. The total of `eforensics`-fraudulent votes, $F_w = 8643.0\ [1618.3, 11848.6]$, is a posterior mean proportion

Table 1: Pennsylvania 2024 President Three Counties `eforensics` Estimates, County Fixed Effects

Type	Parameter	Covariate	Mean	lo[a]	up[b]
mixture probabilities	π_1	No Fraud	.932	.887	.998
	π_2	Incremental Fraud	.0680	.00208	.112
	π_3	Extreme Fraud	.000321	2.71e-08	.000934
incremental frauds	ρ_{M0}	(Intercept)	.143	$-.00122$	.325
	ρ_{M1}	ED proportion	.176	.0302	.304
	ρ_{S0}	(Intercept)	$-.384$	$-.704$	.146
	ρ_{S1}	ED proportion	.154	.0297	.359
extreme frauds	δ_{M0}	(Intercept)	.0657	$-.350$	.499
	δ_{M1}	ED proportion	.134	$-.187$	.385
	δ_{S0}	(Intercept)	$-.131$	$-.634$	.281
	δ_{S1}	ED proportion	.265	$-.319$	.865

MCMC posterior multimodality diagnostics:
dip test p-values $D(\pi_1) = 0$; $D(\pi_2) = 0$; $D(\pi_3) = .997$.[c]
means difference $M(\pi_1) = .103$; $M(\pi_2) = .103$; $M(\pi_3) = 3.36\text{e-}05$.[d]

units `eforensics`-fraudulent: (57 incremental, 0 extreme, 3122 not fraudulent)
manufactured votes $F_t = 3775.1\ [596.1, 5253.5]$[e]
total `eforensics`-fraudulent votes $F_w = 8643.0\ [1618.3, 11848.6]$[e]

Note: selected `eforensics` model parameter estimates (posterior means and credible intervals). County fixed effects for turnout and vote choice are not shown. $n = 3179$ precincts. Electors, votes cast and votes for the leader: $\sum_{i=1}^{n} N_i = 2253256$; $\sum_{i=1}^{n} V_i = 1560271$; $\sum_{i=1}^{n} W_i = 496505$. [a] 95% HPD lower bound. [b] 95% HPD upper bound. [c] dip test for unimodality null hypothesis over all MCMC chains. [d] difference between largest and smallest chain-specific posterior means. [e] posterior mean [99.5% credible interval].

of .072 of the statewide gap of 120266 votes between Trump and Harris: not enough to change the election outcome, but also not negligibly small. The incremental frauds magnitudes are nonnegative: intercepts are nonnegative, $\rho_{M0} = -.143\ (-.00122, .325)$ and $\rho_{S0} = .176\ (.0302, .304)$, and coefficients of ED proportion$_i$ are positive, $\rho_{M1} = .176\ (.0302, .304)$ and $\rho_{S1} = .154\ (.0297, .359)$. Given the median and even the minimum values of ED proportion$_i$, respectively .696 and .462, usually $\rho_{M0} + \rho_{M1}(\text{ED proportion}_i)$ and $\rho_{S0} + \rho_{S1}(\text{ED proportion}_i)$ are positive. Even more clearly than do the nonnegative values, the positive incremental frauds magnitudes mean the eforensics-fraudulent votes measure malevolent distortions of electors' intentions.

A question about the model specification of Table 2 is whether the frauds magnitudes associated with the ED proportion_i variable mean that malevolent distortions are directly related to election-day voting or to something else that is related to election-day voting. Obviously election-day voting per se is only a description of the time period to which votes are being attributed, so finding that the ED proportion_i variable is related to the magnitudes of the eforensics-fraudulent votes is not sharply or precisely diagnostic. Indeed the proportion of votes cast that are cast on election day varies slightly by county: .683 for Allegheny; .731 for Erie; and .727 for Philadelphia. So for example other features of the counties that are related to the differences in election day voting may be reasons for the eforensics-fraudulent votes.

The eforensics estimates reported in Table 2 are for a model specification that includes county fixed effects for turnout, vote choice and frauds magnitudes. The ED proportion_i variable continues to be a covariate in the specification for the frauds magnitudes. A question for this specification is whether the coefficients of the ED proportion_i variable continue to have nonzero coefficient estimates for the eforensics-frauds that are active[3] when the county-identifying variables are taken into account. Table 2 reports that only incremental frauds are active, and the 95%-HPD intervals for ρ_{M1} and for ρ_{S1} include zero. So even though the credible interval for ρ_{M1} includes a wider range of positive values than negative values and the credible interval for ρ_{S1} includes a wider range of negative values than positive values, strictly speaking neither coefficient differs statistically from zero. With the county fixed effects added for frauds magnitudes, diagnostics still signal MCMC posterior multimodality for the mixture probabilities, e.g., $D(\pi_2) = 0$ is significant and $M(\pi_2) = .0387$ is large.

In Table 2 all eforensics-frauds are incremental frauds: of $n = 3179$ precincts 186 have incremental frauds. The total of eforensics-fraudulent votes,

[3] I say a fixed effect is active if it is associated with a precinct that has the corresponding type of eforensics-frauds.

Table 2: Pennsylvania 2024 President Three Counties `eforensics` Estimates, County Fixed Effects II

Type	Parameter	Covariate	Mean	lo[a]	up[b]
mixture probabilities	π_1	No Fraud	.890	.856	.910
	π_2	Incremental Fraud	.110	.0900	.143
	π_3	Extreme Fraud	.000317	3.08e-08	.000995
incremental frauds	ρ_{M0}	(Intercept)	$-.0332$	$-.163$	.149
	ρ_{M1}	ED proportion	.149	$-.0693$	.423
	ρ_{S0}	(Intercept)	$-.393$	$-.713$	$-.149$
	ρ_{S1}	ED proportion	$-.362$	$-.645$	.0698
extreme frauds	δ_{M0}	(Intercept)	$-.0870$	$-.335$	.238
	δ_{M1}	ED proportion	$-.0572$	$-.243$	.155
	δ_{S0}	(Intercept)	.0636	$-.139$	.376
	δ_{S1}	ED proportion	.0506	$-.454$	.518

MCMC posterior multimodality diagnostics:
dip test p-values $D(\pi_1) = 0$; $D(\pi_2) = 0$; $D(\pi_3) = 1$.[c]
means difference $M(\pi_1) = ..0387$; $M(\pi_2) = ..0387$; $M(\pi_3) = 6.41\text{e-}05$.[d]

units `eforensics`-fraudulent: (186 incremental, 0 extreme, 2993 not fraudulent)
manufactured votes $F_t = 12411.2\ [11010.9, 13962.8]$[e]
total `eforensics`-fraudulent votes $F_w = 28829.0\ [25545.4, 31939.1]$[e]

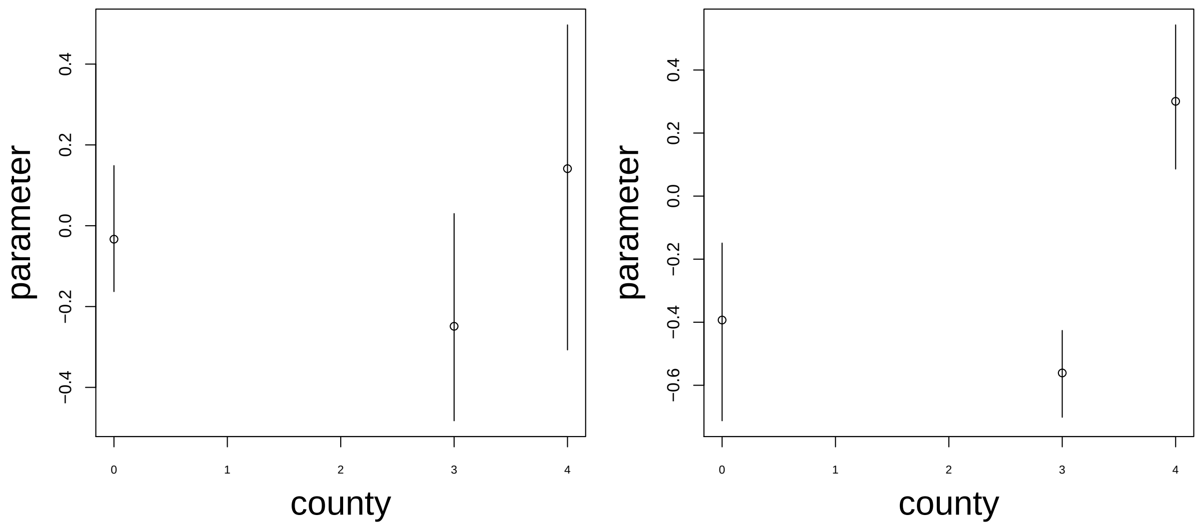
Note: selected `eforensics` model parameter estimates (posterior means and credible intervals). County fixed effects for turnout, vote choice and `eforensics`-frauds magnitudes are not shown (see Figure 2 for active frauds magnitudes fixed effects). $n = 3179$ precincts. Electors, votes cast and votes for the leader: $\sum_{i=1}^{n} N_i = 2253256$; $\sum_{i=1}^{n} V_i = 1560271$; $\sum_{i=1}^{n} W_i = 496505$. [a] 95% HPD lower bound. [b] 95% HPD upper bound. [c] dip test for unimodality null hypothesis over all MCMC chains. [d] difference between largest and smallest chain-specific posterior means. [e] posterior mean [99.5% credible interval].

$F_w = 28829.0\ [25545.4, 31939.1]$, is a posterior mean proportion of .240 of the statewide gap of 120266 votes between Trump and Harris: not enough to change the election outcome, but also not all that small. Both the number of precincts that have `eforensics`-frauds and the number of `eforensics`-fraudulent votes greatly exceed the number for the model specification that omits county fixed effects for frauds magnitudes. Active frauds magnitudes fixed effects are shown in Figure 2.[4] Taking into account the boundaries of the fixed effects' credible intervals, Philadelphia has fixed effects for stolen frauds magnitudes

[4]Only three counties exist in the data. Places for five counties appear along the x-axis in Figure 2 due to an artifact in my plotting code that I did not correct.

Figure 2: Pennsylvania 2024 President Three CountiesL: `eforensics`-frauds Magnitude Fixed Effect Parameters

(a) incremental manufactured: ρ_{Mj} (b) incremental stolen: ρ_{Sj}



Note: active fixed effects parameters (posterior means and 95% HPD intervals) for frauds magnitude (ρ_{Mj}, ρ_{Sj}) parameters in the `eforensics` model reported in Table 2. Counties: 0 Allegheny; 2 Erie; 3 Philadelphia.

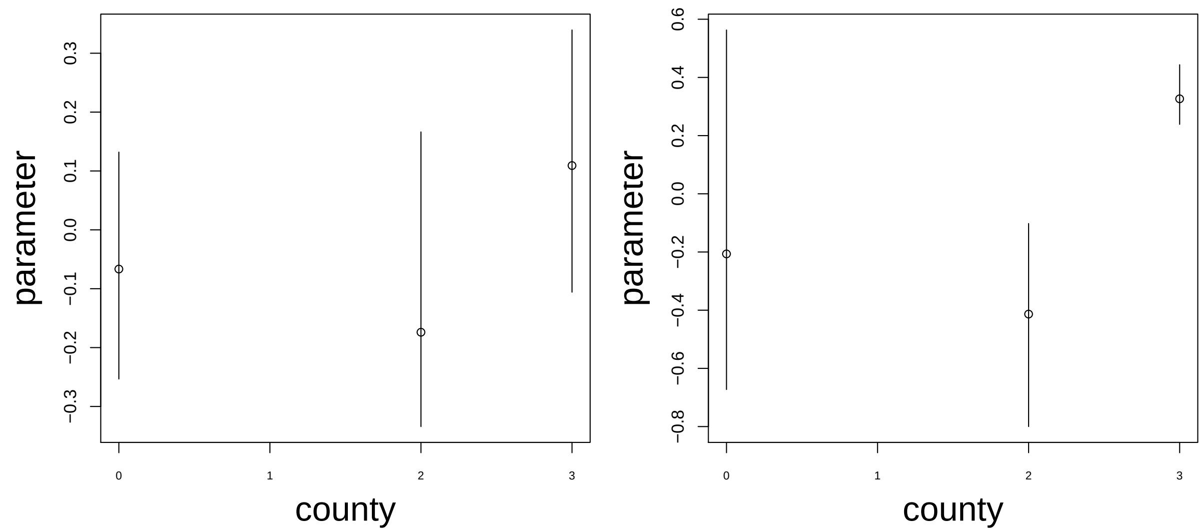
that differ significantly in size from the other two counties, which do not differ all that much from one another.[5] Stolen frauds magnitudes for Philadelphia are positive, even if contributions from the ED proportion$_i$ variable are ignored, while the stolen frauds magnitudes for the other two counties, ignoring the ED proportion$_i$ variable, are negative.

The pattern in Figure 2 does not suggest that only for Philadlphia do the `eforensics`-fraudulent votes stem from malevolent distortions. If the ED proportion$_i$ variable is omitted from the `eforensics` model specification, then 92 precincts have `eforensics`-frauds of which all are incremental, and there are $F_w = 14465.1$ $[5197.0, 18364.7]$ `eforensics`-fraudulent votes. As Figure 3 shows, the incremental active manufactured frauds magnitudes for all counties have indeterminate

[5] A caveat is that for all fixed effects except any displayed in position zero, which corresponds to the intercept, I simply add the posterior mean of the intercept to the fixed effects' coefficient and to the limits of its 95% HPD interval, without adjusting for how these intervals should change to represent the full variation of the combined fixed effects. So pending implementation of such corrected credible intervals, the displays in Figure 2 should be viewed merely as informally illustrative.

Figure 3: Pennsylvania 2024 President Three CountiesL: `eforensics`-frauds Magnitude Fixed Effect Parameters

(a) incremental manufactured: ρ_{Mj} (b) incremental stolen: ρ_{Sj}



Note: active fixed effects parameters (posterior means and 95% HPD intervals) for frauds magnitude (ρ_{Mj}, ρ_{Sj}) parameters. Table 2. Counties: 0 Allegheny; 2 Erie; 3 Philadelphia.

signs, as does the fraud magnitude for stolen incremental frauds for Allegheny. However the active fraud magnitude for stolen incremental frauds for Erie is negative and for Philadelphia is positive. Likely all counties' incremental frauds magnitudes would appear with indeterminate signs if the full variation of the combined fixed effects were correctly represented.[6]

[6]Recall note 5.

References

Mebane, Jr., Walter R. 2023. "Lost Votes and Posterior Multimodality in the `eforensics` Model." Presented at PolMeth 2023, Stanford University, Palo Alto, CA, July 9-11, 2023. URL: http://www.umich.edu/~wmebane/pm23.pdf.