Introduction & Two-Page Summary

The Riemann hypothesis is a conjecture first proposed by Professor Georg Friedrich Bernhard Riemann of Göttingen University and submitted in a brief paper (1859) to the Berlin Academy of Sciences, celebrating his recent admittance as a corresponding member of the Academy. With his paper and its conjecture, Riemann completely revolutionized our approach and understanding of the distribution of the primes.

Although it was not necessary for the results of his paper, Riemann conjectured - but was unable to prove - that all the roots of what is known today as the zeta function in the so-called critical strip have real part equal to ½. In the 165 or so years since publication of the paper, the hypothesis has neither been proved nor disproved. In fact, the Riemann hypothesis is perhaps the most important unresolved problem in pure mathematics today.

A resolution of the Riemann hypothesis would have very important consequences - not only regarding the distribution of the primes, but also for a myriad of hypothesis-dependent results in number theory, as well as potentially for quantum physics and encryption technologies.

The infinite series representation of the Riemann zeta function, $\zeta(s)$, is

$$\zeta(s) \equiv \sum_{n=1}^{\infty} n^{-s} \ = \ 1 + 2^{-s} + 3^{-s} + \cdots$$

where the argument $s = \sigma + i \cdot t$, $i = \sqrt{-1}$, and σ and t are real.

It has long been accepted that the infinite series representation of the zeta function diverges everywhere in the critical strip, where $0 < \sigma < 1$, and therefore the series representation is inapplicable for a resolution of the hypothesis.

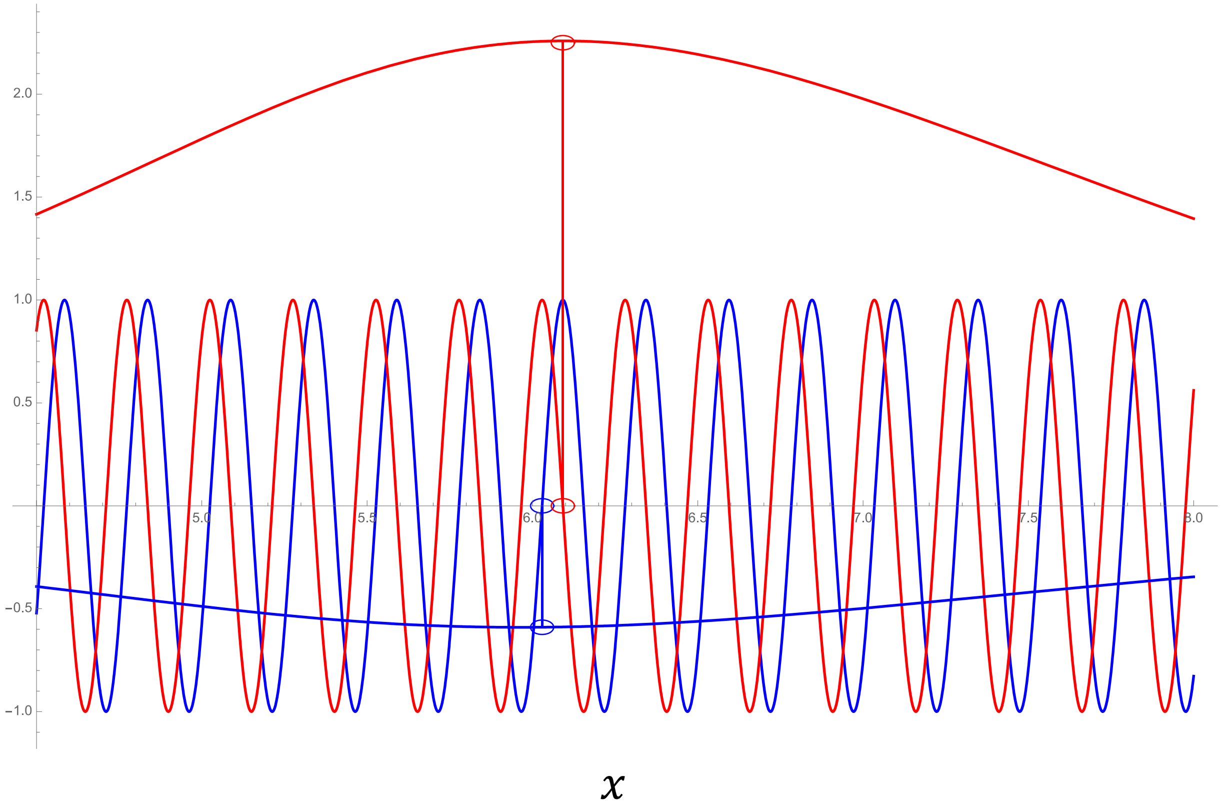
What if this is wrong? What if the infinite series representation of the Riemann zeta function converges at its roots in the critical strip in a very unusual way but diverges everywhere else? This website investigates this question and provides answers.

Consider the next page.

a root of the Riemann zeta function in the critical strip

$\sigma = 1/2,\ t = 25.0108\ldots$, and $m = 51$ (arbitrary)

(the two non-cyclic functions are scaled by $\pm 5\text{x}10^{15}$)



Blue Vertical Line

$$x \approx 6.0292 = \frac{(m-3)\cdot\pi}{t} \quad \Longrightarrow \quad t \approx \frac{(51-3)\cdot\pi}{6.0292} \approx 25.0108$$

Red Vertical Line

$$x \approx 6.0920 = \frac{(2\cdot m-5)\cdot\pi}{2\cdot t} \quad \Longrightarrow \quad t \approx \frac{(2\cdot 51-5)\cdot\pi}{2\cdot 6.0920} \approx 25.0108$$

The value of t from the literature for the root of the Riemann zeta function is approximately 25.010857...

There are no co-incidences in mathematics. The graph above and calculations based on the graph are not co-incidences.

It has been said that if an author cannot explain their work on a single page, or perhaps on two pages, then the author cannot explain their work at all.

In that spirit, this author’s work is explained on the next two pages. The explanation is slightly abridged, but details are given elsewhere on this website and in the author’s three published books.

Turn to the next page…

Two-Page Summary

The roots in the critical strip of the infinite series representation of the Riemann zeta function occur when

$$\zeta(s) \equiv \sum_{n=1}^{\infty} n^{-s} \; = \; 1 + 2^{-s} + 3^{-s} + \cdots = 0$$

or equivalently, when

$$Re\{\zeta(s)\} \; = \lim_{N\to\infty} \left\{ Re\left[\sum_{n=1}^{N} n^{-s} \right] \right\} \; = \; 0 \qquad summable$$

and

$$Im\{\zeta(s)\} \; = \lim_{N\to\infty} \left\{ Im\left[\sum_{n=1}^{N} n^{-s} \right] \right\} \; = \; 0 \qquad summable$$

where both series are "summable", but not convergent in a classical, formal sense.

These relationships are also equivalent to:

$$Re\{\zeta(s)\} \; = \lim_{m\to\infty} \left\{ Re\left[\sum_{n=1}^{\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}} \right\rfloor} n^{-s} \right] \right\} \; = \; 0 \qquad summable$$

and

$$Im\{\zeta(s)\} \; = \lim_{m\to\infty} \left\{ Im\left[\sum_{n=1}^{\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}} \right\rfloor} n^{-s} \right] \right\} \; = \; 0 \qquad summable$$

It will become clear later why it is convenient to replace integer N in the first pair of series above with functions of integer m in the second pair of series.

The Borel integral summation method and the Euler-Maclaurin summation formula, or Cauchy's residue theorem can be used to show that

$$Re\left[\sum_{n=1}^{\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}} \right\rfloor} n^{-s} \right] \sim Re\left[\int_{0}^{\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}} \right\rfloor} x^{-s}\, dx \right]$$

and

$$Im\left[\sum_{n=1}^{\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}} \right\rfloor} n^{-s} \right] \sim Im\left[\int_{0}^{\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}} \right\rfloor} x^{-s}\, dx \right]$$

at the roots of the Riemann zeta function in the critical strip, for arbitrarily large values of $m = 1, 2, 3, \ldots$

Partial sums of the zeta function can be represented everywhere in the critical strip with the bi-lateral integral transform

$$\sum_{n=1}^{N} n^{-s} = \int_{-\infty}^{\infty} \frac{e^{-s\cdot x}}{\Gamma(s)} \cdot \left(\frac{1 - e^{e^{-N\cdot x}}}{e^{e^{-x}} - 1} \right) dx$$

so that

$$Re\left\{ \sum_{n=1}^{\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}} \right\rfloor} n^{-s} \right\} = \int_{-\infty}^{\infty} Re\left\{ \frac{e^{-s\cdot x}}{\Gamma(s)} \right\} \cdot \left(\frac{1 - e^{e^{-x\cdot\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}} \right\rfloor}}}{e^{e^{-x}} - 1} \right) dx$$

and

$$Im\left\{ \sum_{n=1}^{\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}} \right\rfloor} n^{-s} \right\} = \int_{-\infty}^{\infty} Im\left\{ \frac{e^{-s\cdot x}}{\Gamma(s)} \right\} \cdot \left(\frac{1 - e^{e^{-x\cdot\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}} \right\rfloor}}}{e^{e^{-x}} - 1} \right) dx$$

Combining the formulae above and applying techniques of complex analysis gives two dependent asymptotic relationships that define the roots of the Riemann zeta function in the critical strip. One of the two equations is:

$$\int_{-\infty}^{\infty} sin(t\cdot x)\cdot$$

$$\left\{ \left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}} - 1} \right) \cdot \left[Re[\Gamma(s)] \cdot \left(1 - e^{-\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}} \right\rfloor \cdot e^{-x}} \right) + e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}} \cdot Im[\Gamma(s)] \cdot \left(1 - e^{-\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}} \right\rfloor \cdot e^{-x}} \right) \right] \right\} dx \sim 0$$

This integral can only vanishes when the function in the integrand, or

$$\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}} - 1} \right) \cdot \left[Re[\Gamma(s)] \cdot \left(1 - e^{-\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}} \right\rfloor \cdot e^{-x}} \right) + e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}} \cdot Im[\Gamma(s)] \cdot \left(1 - e^{-\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}} \right\rfloor \cdot e^{-x}} \right) \right]$$

asymptotically is an even function of the variable of integration, x, and the function is even only when $\sigma = 1/2$. The roots of the Riemann zeta function in the critical strip can only occur when the function in the integrand of the transform is an even function of the variable of transformation and the sine kernel of the transform exhibits a root at the minimum or maximum of the function. When these two criteria are met, the integral is zero and roots occur in the Riemann zeta function.

Since the two criteria are met only when $\sigma = 1/2$, the Riemann hypothesis is correct.

That’s two pages – exactly two pages.