Introduction & Two-Page Summary

The Riemann hypothesis is a conjecture first proposed by Professor Georg Friedrich Bernhard Riemann of Göttingen University and submitted in a brief paper (1859) to the Berlin Academy of Sciences, celebrating his recent admittance as a corresponding member of the Academy. With his paper and its conjecture, Riemann completely revolutionized our approach and understanding of the distribution of the primes.

Although it was not necessary for the results of his paper, Riemann conjectured - but was unable to prove - that all the roots of what is known today as the zeta function in the so-called critical strip have real part equal to ½. In the 165 or so years since publication of the paper, the hypothesis has neither been proved nor disproved. In fact, the Riemann hypothesis is perhaps the most important unresolved problem in pure mathematics today.

A resolution of the Riemann hypothesis would have very important consequences - not only regarding the distribution of the primes, but also for a myriad of hypothesis-dependent results in number theory, as well as potentially for quantum physics and encryption technologies.

The infinite series representation of the Riemann zeta function, $\zeta(s)$, is

$$\zeta(s) \equiv \sum_{n=1}^{\infty} n^{-s} \ = \ 1 + 2^{-s} + 3^{-s} + \cdots$$

where the argument $s = \sigma + i \cdot t$, $i = \sqrt{-1}$, and σ and t are real.

It has long been accepted that the infinite series representation of the zeta function diverges everywhere in the critical strip, where $0 < \sigma < 1$, and therefore the series representation is inapplicable for a resolution of the hypothesis.

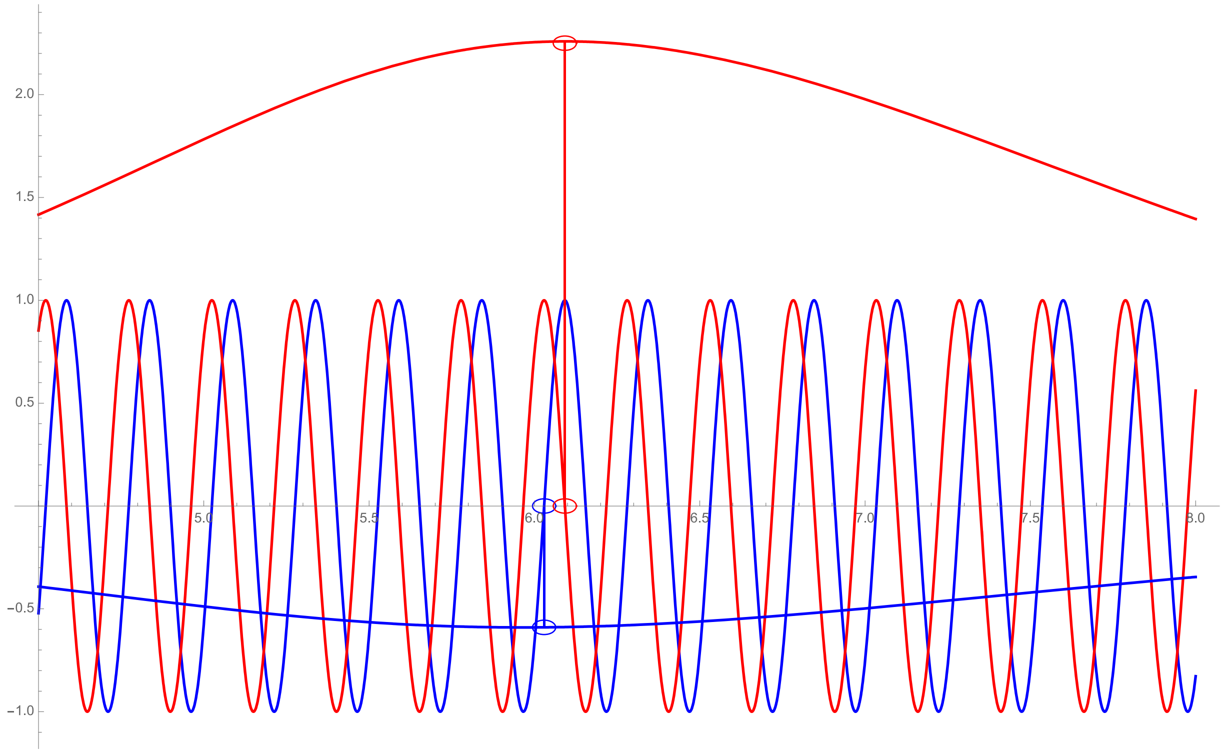
What if this is wrong? What if the infinite series representation of the Riemann zeta function converges at its roots in the critical strip in a very unusual way but diverges everywhere else? This website investigates this question and provides answers.

In the event that you, reader, are short of time and rightly skeptical of the foregoing introduction and the premise of this work, consider the following page.

a root of the Riemann zeta function in the critical strip

$\sigma = 1/2,\ t = 25.0108...$, and $m = 51$ (arbitrary)

(the two non-cyclic functions are scaled by $\pm 5\text{x}10^{15}$)



x

Blue Vertical Line

$$x \approx 6.0292 = \frac{(m-3)\cdot\pi}{t} \quad \Longrightarrow \quad t \approx \frac{(51-3)\cdot\pi}{6.0292} \approx 25.0108$$

Red Vertical Line

$$x \approx 6.0920 = \frac{(2\cdot m-5)\cdot\pi}{2\cdot t} \quad \Longrightarrow \quad t \approx \frac{(2\cdot 51-5)\cdot\pi}{2\cdot 6.0920} \approx 25.0108$$

The value of t from the literature for the root of the Riemann zeta function is approximately 25.010857...

There are no co-incidences in mathematics. The graph above and calculations based on the graph are not co-incidences.

It has been said that if an author cannot explain their work on a single page, or perhaps on two pages, then the author cannot explain their work at all.

In that spirit, the author's work is explained on the next two pages. The mathematics are slightly abridged, but details are given elsewhere on this website and in three referenced books.

Turn to the next page…

Two-Page Summary

The roots of the series representation of the Riemann zeta function occur when

$$\zeta(s) \equiv \sum_{n=1}^{\infty} n^{-s} = 1 + 2^{-s} + 3^{-s} + \cdots = 0$$

Since we are considering only roots of the function in the critical strip, this is equivalent to:

$$Re\{\zeta(s)\} = \lim_{N\to\infty} \left\{ Re\left[\sum_{n=1}^{N} n^{-s} \right] \right\} = 0 \quad summable$$

and

$$Im\{\zeta(s)\} = \lim_{N\to\infty} \left\{ Im\left[\sum_{n=1}^{N} n^{-s} \right] \right\} = 0 \quad summable$$

where both series are "summable", but not convergent in a classical, formal sense.

These relationships are also equivalent to:

$$Re\{\zeta(s)\} = \lim_{m\to\infty} \left\{ Re\left[\sum_{n=1}^{\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}} \right\rfloor} n^{-s} \right] \right\} = 0 \quad summable$$

and

$$Im\{\zeta(s)\} = \lim_{m\to\infty} \left\{ Im\left[\sum_{n=1}^{\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}} \right\rfloor} n^{-s} \right] \right\} = 0 \quad summable$$

It will become clear later why it is convenient to replace integer N in the first pair of series with integer m in the second pair of series.

The Borel integral summation method and the Euler-Maclaurin summation formula, or Cauchy's residue theorem can be used to show that

$$Re\left[\sum_{n=1}^{\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}} \right\rfloor} n^{-s} \right] \sim Re\left[\int_{0}^{\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}} \right\rfloor} x^{-s}\, dx \right]$$

and

$$Im\left[\sum_{n=1}^{\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}} \right\rfloor} n^{-s} \right] \sim Im\left[\int_{0}^{\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}} \right\rfloor} x^{-s}\, dx \right]$$

at the roots of the Riemann zeta function in the critical strip, for arbitrarily large values of $m = 1, 2, 3, \ldots$

Partial sums of the zeta function can be represented everywhere in the critical strip with the bi-lateral integral transform

$$\sum_{n=1}^{N} n^{-s} = \int_{-\infty}^{\infty} \frac{e^{-s\cdot x}}{\Gamma(s)} \cdot \left(\frac{1 - e^{e^{-N\cdot x}}}{e^{e^{-x}} - 1} \right) dx$$

and therefore

$$Re\left\{ \sum_{n=1}^{\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}} \right\rfloor} n^{-s} \right\} = \int_{-\infty}^{\infty} Re\left\{ \frac{e^{-s\cdot x}}{\Gamma(s)} \right\} \cdot \left(\frac{1 - e^{e^{-x\cdot\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}} \right\rfloor}}}{e^{e^{-x}} - 1} \right) dx$$

and

$$Im\left\{ \sum_{n=1}^{\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}} \right\rfloor} n^{-s} \right\} = \int_{-\infty}^{\infty} Im\left\{ \frac{e^{-s\cdot x}}{\Gamma(s)} \right\} \cdot \left(\frac{1 - e^{e^{-x\cdot\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}} \right\rfloor}}}{e^{e^{-x}} - 1} \right) dx$$

Combining the formulae above and applying techniques of complex analysis gives one of two dependent asymptotic relationships that define the roots of the Riemann zeta function in the critical strip:

$$\int_{-\infty}^{\infty} sin(t\cdot x) \cdot \left\{ \left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}} - 1} \right) \cdot \left[Re[\Gamma(s)] \cdot \left(1 - e^{-\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}} \right\rfloor \cdot e^{-x}} \right) + e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}} \cdot Im[\Gamma(s)] \cdot \left(1 - e^{-\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}} \right\rfloor \cdot e^{-x}} \right) \right] \right\} dx \sim 0$$

This integral vanishes only when the function in the integrand,

$$\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}} - 1} \right) \cdot \left[Re[\Gamma(s)] \cdot \left(1 - e^{-\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}} \right\rfloor \cdot e^{-x}} \right) + e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}} \cdot Im[\Gamma(s)] \cdot \left(1 - e^{-\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}} \right\rfloor \cdot e^{-x}} \right) \right]$$

asymptotically approximates an even function of the variable of integration, x. This only occurs when $\sigma = 1/2$. Therefore, the roots of the Riemann zeta function in the critical strip all must have real part equal to ½ and the Riemann hypothesis is correct.

That's two pages – exactly two pages.