Bi-lateral Integral Transform: An Illustrative Example

The following example is important because it illustrates the mathematics at the core of the approach in this work. The example given here with a bi-lateral sine integral transform with a skew normal distribution function is much simpler than the bi-lateral sine transforms of the Riemann zeta function, but the concepts are identical. An understanding of these concepts is crucial for an understanding of the remainder of this work.

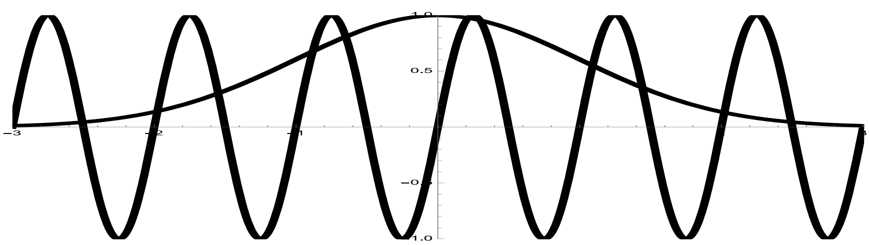
Consider the skew-normal distribution function

$$\left(\frac{5}{2\pi}\right) \cdot e^{-\frac{(x-\mu)^2}{2}} \cdot \int_{-\infty}^{a \cdot x} e^{-\frac{(t-\mu)^2}{2}} dt$$

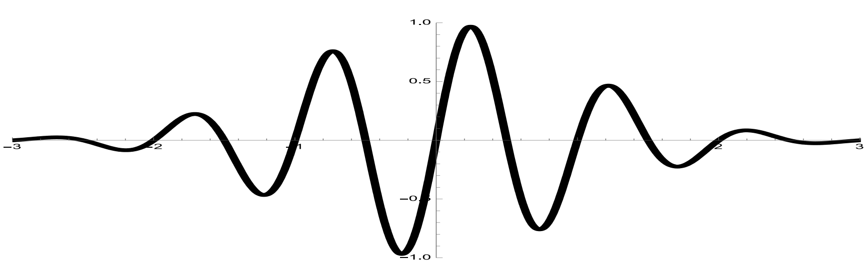
where μ is the mean of the distribution and a is the skew of the distribution. The bi-lateral sine integral transform of the distribution is

$$\int_{-\infty}^{\infty} sin(2\pi x) \cdot \left[\left(\frac{5}{2\pi}\right) \cdot e^{-\frac{(x-\mu)^2}{2}} \cdot \int_{-\infty}^{a \cdot x} e^{-\frac{(t-\mu)^2}{2}} dt\right] dx$$

The kernel of the transform, $sin(2\pi x)$, and the distribution are shown in the following graph with mean and skew equal to 0.

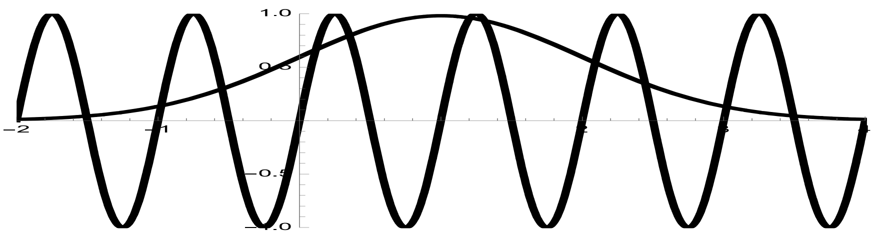


The product of the kernel and the distribution is shown in the following graph:

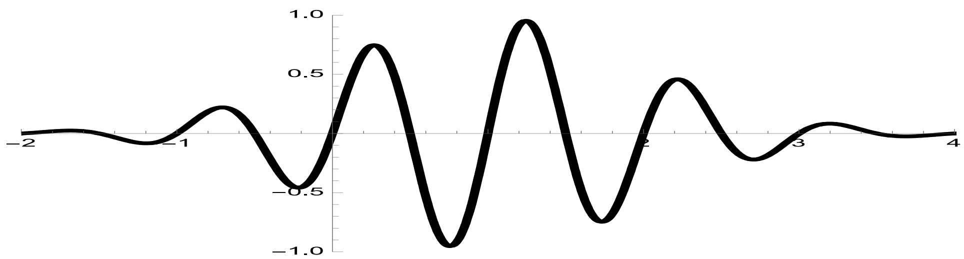


Since the kernel is an odd function of the variable of integration and the distribution is an even function of the variable of integration, the product of the kernel and the distribution is an odd function, and consequently, the value of the transform is zero.

Similarly, the kernel of the transform, $sin(2\pi x)$, and a linearly translated distribution are shown in the following graph with mean equal to 1 and skew equal to 0.

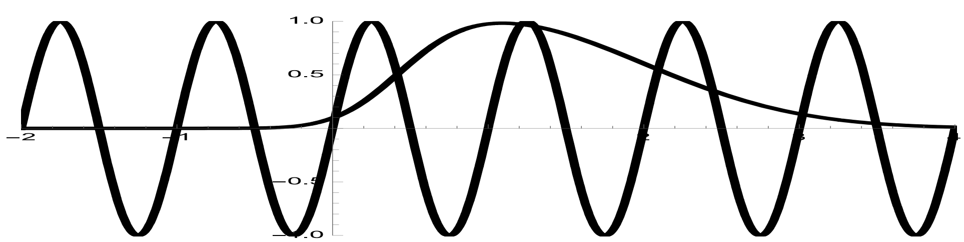


The product of the kernel and the scaled distribution is shown in the following graph:

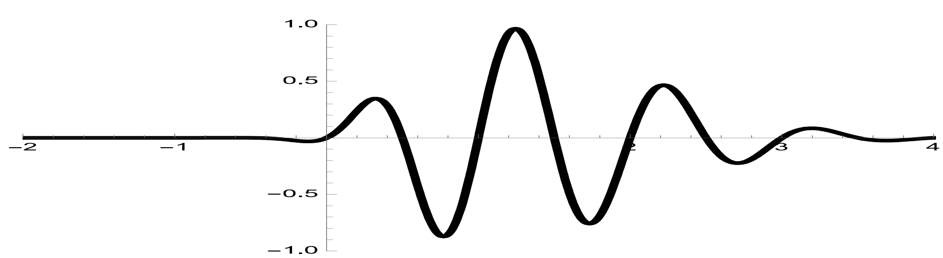


Since the kernel is an odd function of the variable of integration and the distribution is an even function of the translated variable of integration, the product of the kernel and the distribution is an odd function and the value of the transform is also zero.

Finally, the kernel of the transform, $sin(2\pi x)$, and a linearly translated distribution are shown in the following graph with mean equal to 1 and skew equal to 3.



The product of the kernel and the translated distribution is shown in the following graph:



Since the kernel is an odd function of the variable of integration and the skewed distribution clearly is not an even function of the translated variable of integration, the product of the kernel and the skewed distribution is not an odd function of the translated variable, and consequently, the value of the transform is not equal to zero.

In summary, the value of the bi-lateral sine integral transform of the skew-normal distribution

$$\int_{-\infty}^{\infty} sin(2\pi x) \cdot \left[\left(\frac{5}{2\pi} \right) \cdot e^{-\frac{(x-\mu)^2}{2}} \cdot \int_{-\infty}^{a \cdot x} e^{-\frac{(t-\mu)^2}{2}} dt \right] dx$$

with an untranslated or a translated variable of integration is identically zero if and only if the value of skew is zero so that the distribution is an even function and the product of the odd kernel and the distribution is an odd function.