Bi-lateral Integral Transform: An Illustrative Example

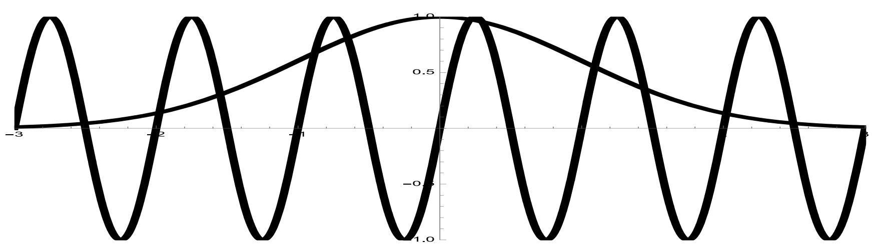
Consider the bi-lateral sine integral transform of the skew-normal distribution function

$$\left(\frac{5}{2\pi}\right) \cdot e^{-(x-\mu)^2/2} \cdot \int_{-\approx}^{a \cdot x} e^{-(t-\mu)^2/2}\, dt$$

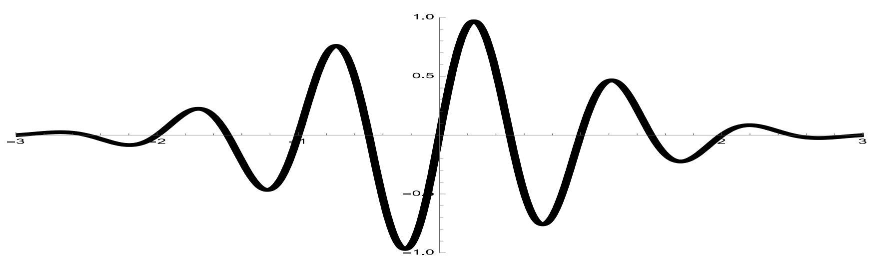
where μ is the mean of the distribution and a is the skew of the distribution. The bi-lateral sine integral transform of the distribution is

$$\int_{-\infty}^{\infty} sin(2\pi x) \cdot \left[\left(\frac{5}{2\pi}\right) \cdot e^{-\frac{(x-\mu)^2}{2}} \cdot \int_{-\approx}^{a \cdot x} e^{-\frac{(t-\mu)^2}{2}}\, dt\right] dx$$

The kernel of the transform, $sin(2\pi x)$, and the distribution are shown in the following graph with both mean and skew equal to 0.

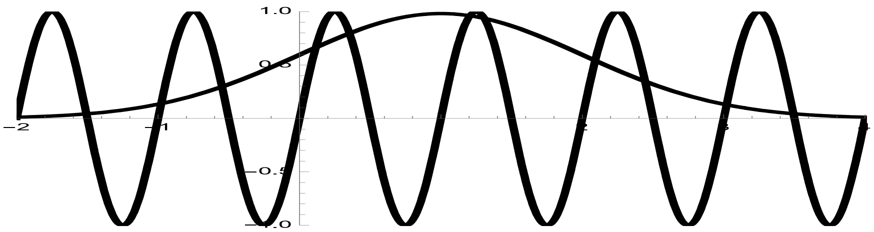


The product of the kernel and the distribution is shown in the following graph:

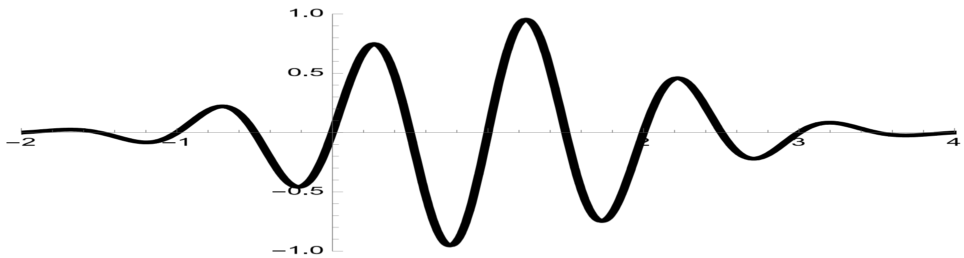


Since the kernel is an odd function of the variable of integration and the distribution is an even function of the variable of integration, the product of the kernel and the distribution is an odd function, and consequently, the value of the transform is identically zero.

Similarly, the kernel of the transform, $sin(2\pi x)$, and a linearly translated distribution are shown in the following graph with mean equal to 1 and skew equal to 0.

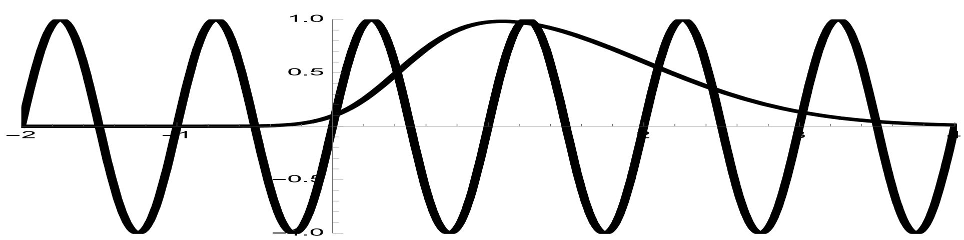


The product of the kernel and the scaled distribution is shown in the following graph:

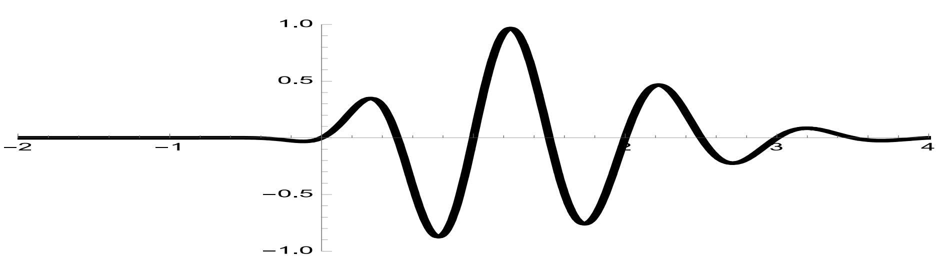


Since the kernel is an odd function of the variable of integration and the distribution is an even function of the translated variable of integration, the product of the kernel and the distribution is an odd function and the value of the transform is also identically zero.

Finally, the kernel of the transform, $sin(2\pi x)$, and a linearly translated distribution are shown in the following graph with mean again equal to 1 and with skew equal to 3.



The product of the kernel and the translated distribution is shown in the following graph:



Since the kernel is an odd function of the variable of integration and the skewed distribution clearly is not an even function of the translated variable of integration, the product of the kernel and the skewed distribution is not an odd function of the translated variable, and consequently, the value of the transform is not equal to zero.

In summary, the value of the bi-lateral sine integral transform of the skew-normal distribution

$$\int_{-\infty}^{\infty} sin(2\pi x)\cdot\left[\left(\frac{5}{2\pi}\right)\cdot e^{-\frac{(x-\mu)^2}{2}}\cdot\int_{-\approx}^{a\cdot x} e^{-\frac{(t-\mu)^2}{2}}\,dt\right]dx$$

with an untranslated or with a translated variable of integration is identically zero if and only if the value of skew is zero so that the distribution is an even function and the product of the kernel and the distribution is an odd function.