The Roots of the Riemann Zeta Function

The even part of the first translated function from the previous section, or

$$\left(\frac{e^{-\sigma\cdot\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}{e^{e^{-\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}-1}\right) \cdot \left[Re[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right]\cdot e^{-\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}\right) + e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Im[\Gamma(s)] \cdot\left(1-e^{-\left[e^{\frac{(m-1)\cdot\pi}{t}}\right]\cdot e^{-\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}\right)\right]$$

is

$$\left(\frac{1}{2}\right)\cdot\left[\left\{\left(\frac{e^{-\sigma\cdot\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}{e^{e^{-\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}-1}\right) \cdot \left[Re[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right]\cdot e^{-\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}\right) + e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Im[\Gamma(s)] \cdot\left(1-e^{-\left[e^{\frac{(m-1)\cdot\pi}{t}}\right]\cdot e^{-\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}\right)\right]\right\} + \left\{\left(\frac{e^{\sigma\cdot\left[x-\frac{(m-1)\cdot\pi}{t}\right]}}{e^{e^{\left[x-\frac{(m-1)\cdot\pi}{t}\right]}}-1}\right) \cdot \left[Re[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right]\cdot e^{\left[x-\frac{(m-1)\cdot\pi}{t}\right]}}\right) + e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Im[\Gamma(s)] \cdot\left(1-e^{-\left[e^{\frac{(m-1)\cdot\pi}{t}}\right]\cdot e^{\left[x-\frac{(m-1)\cdot\pi}{t}\right]}}\right)\right]\right\}\right]$$

A metric of the difference of the even part of the translated function and the translated function itself is a linear, unweighted average of the ratio of the even part of the translated function to the translated function, with the average taken over equally spaced points along the x axis.

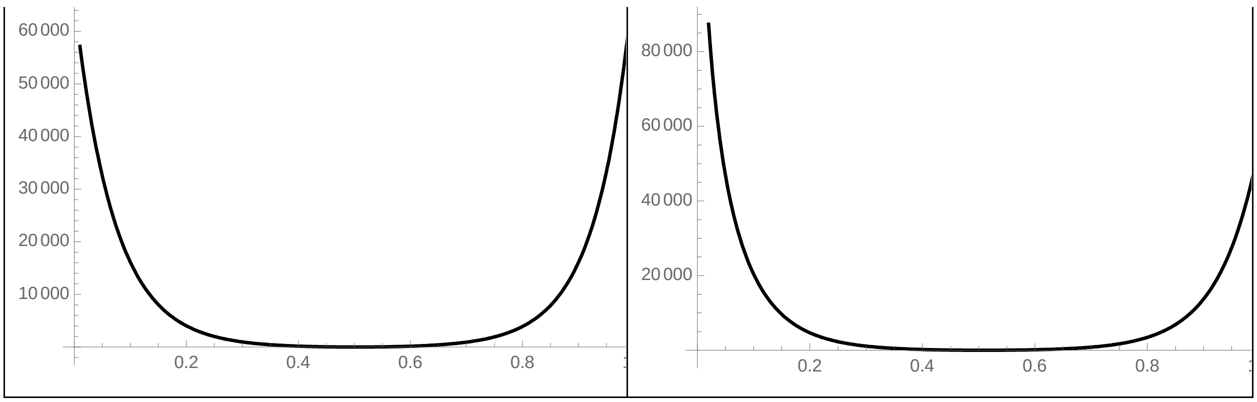
The metric was evaluated using values of t generated with a pseudo-random real number generator, with values of integer m adjusted for computational efficiency.

Higher values of t require higher values of integer m for optimal computation. The minimum in the metric occurs at $\sigma = ½$ for all values of t (Mathematica®).

The partial derivative of the metric taken with respect to σ was calculated analytically and numerically. The derivative exhibits a single minimum of zero at $\sigma = ½$, for every pseudo-random value of t (Mathematica®).

The metric is shown in the graphs below for $t = 14.134725$ and $t = 16$, with $m = 51$.

t = 14.134725 **t = 16**



σ σ

The metric is at its minimum and is equal to zero when $\sigma = ½$ for all values of t.

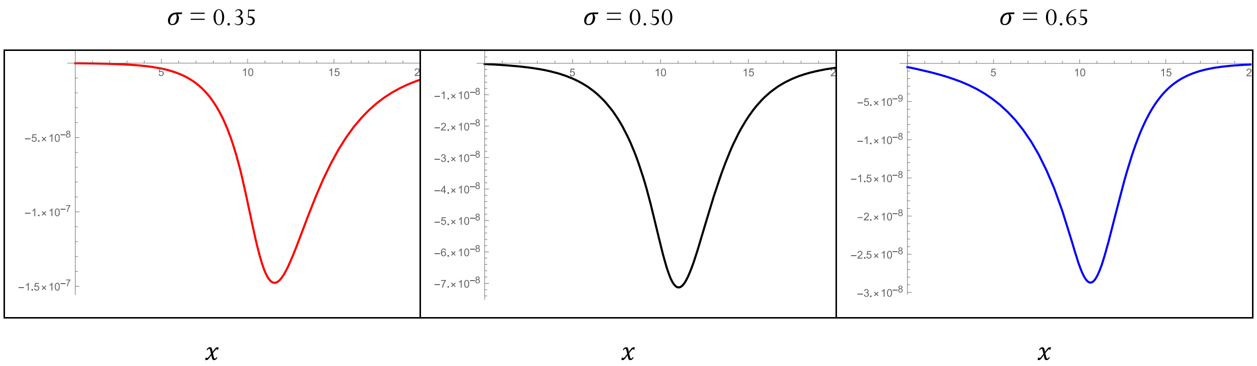
Similarly, if the second function above is translated linearly with respect to the x axis, where the variable x is translated as $x + (2 \cdot m - 1) \cdot \pi/(2 \cdot t)$, the closest approximation of the translated function to the even part of the translated function occurs when $\sigma = ½$, for all values of t.

The second function

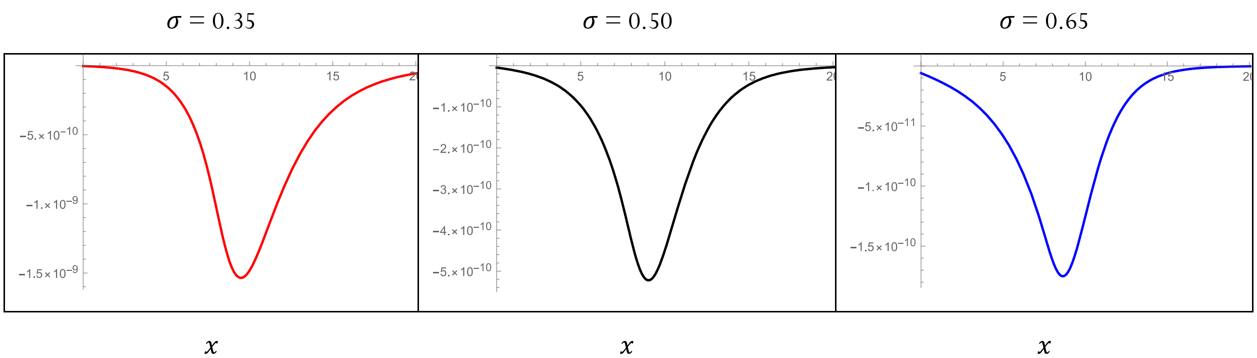
$$\left(\frac{e^{-\sigma \cdot x}}{e^{e^{-x}} - 1}\right) \cdot \left[Im[\Gamma(s)] \cdot \left(1 - e^{-\left[e^{\frac{(2 \cdot m-1) \cdot \pi}{2 \cdot t}}\right] \cdot e^{-x}}\right) - e^{\frac{\pi \cdot (1-\sigma)}{2 \cdot t}} \cdot Re[\Gamma(s)] \cdot \left(1 - e^{-\left[e^{\frac{(m-1) \cdot \pi}{t}}\right] \cdot e^{-x}}\right)\right]$$

is shown in the graphs below for $t = 14.134725\ldots$ and $t = 17$, with $m = 51$, for $\sigma = 0.35$, $\sigma = 0.50$, and $\sigma = 0.65$.

t = 14.134725



$t = 17.$

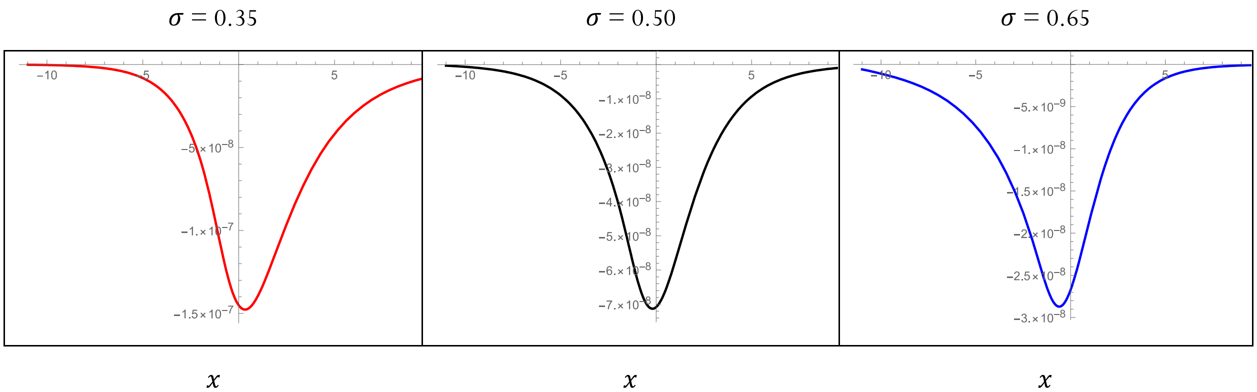


If the function is translated linearly with respect to the x axis, where the variable x is translated so that $x \to x + (2 \cdot m - 1) \cdot \pi/(2 \cdot t)$, the closest approximation of the translated function to the even part of the function occurs when $\sigma = ½$, for all values of t. The translated function

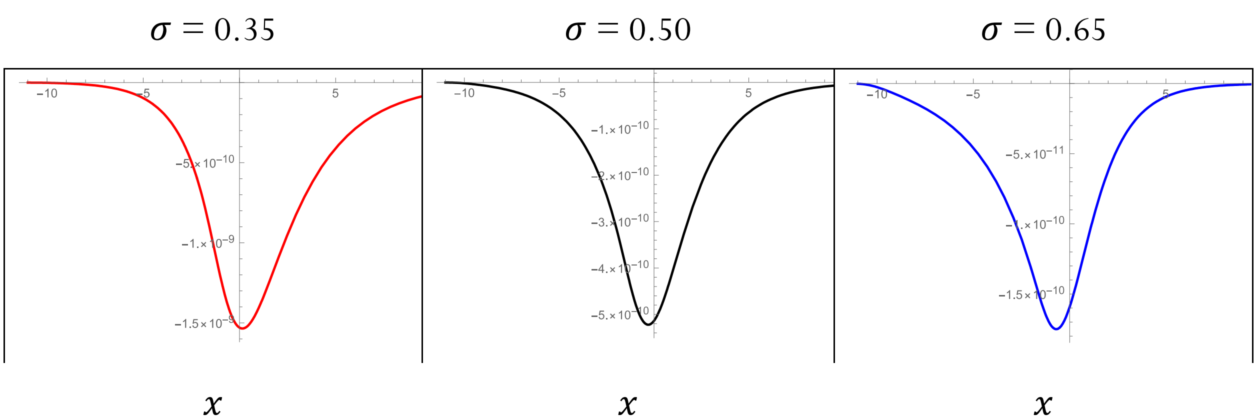
$$\left(\frac{e^{-\sigma\cdot\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}{e^{e^{-\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}-1}\right) \cdot\left[Im[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right]\cdot e^{-\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}\right)-e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Re[\Gamma(s)] \cdot\left(1-e^{-\left[e^{\frac{(m-1)\cdot\pi}{t}}\right]\cdot e^{-\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}\right)\right]$$

is shown in the graphs below for $t = 14.134725$ and $t = 17$, with $m = 51$, for $\sigma = 0.35$, $\sigma = 0.50$, and $\sigma = 0.65$.

$t = 14.134725$



$t = 17$



The even part of the second translated function from the previous section is:

$$
\left(\frac{1}{2}\right)\cdot\left[\left\{\left(\frac{e^{-\sigma\cdot\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}{e^{e^{-\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}-1}\right)\right.\right.
$$

$$
\cdot\left[Im[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right]\cdot e^{-\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}\right)-e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Re[\Gamma(s)]\right.
$$

$$
\left.\left.\cdot\left(1-e^{-\left[e^{\frac{(m-1)\cdot\pi}{t}}\right]\cdot e^{-\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}\right)\right]\right\}
$$

$$
+\left\{\left(\frac{e^{\sigma\cdot\left[x-\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}{e^{e^{\left[x-\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}-1}\right)\right.
$$

$$
\cdot\left[Im[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right]\cdot e^{\left[x-\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}\right)-e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Re[\Gamma(s)]\right.
$$

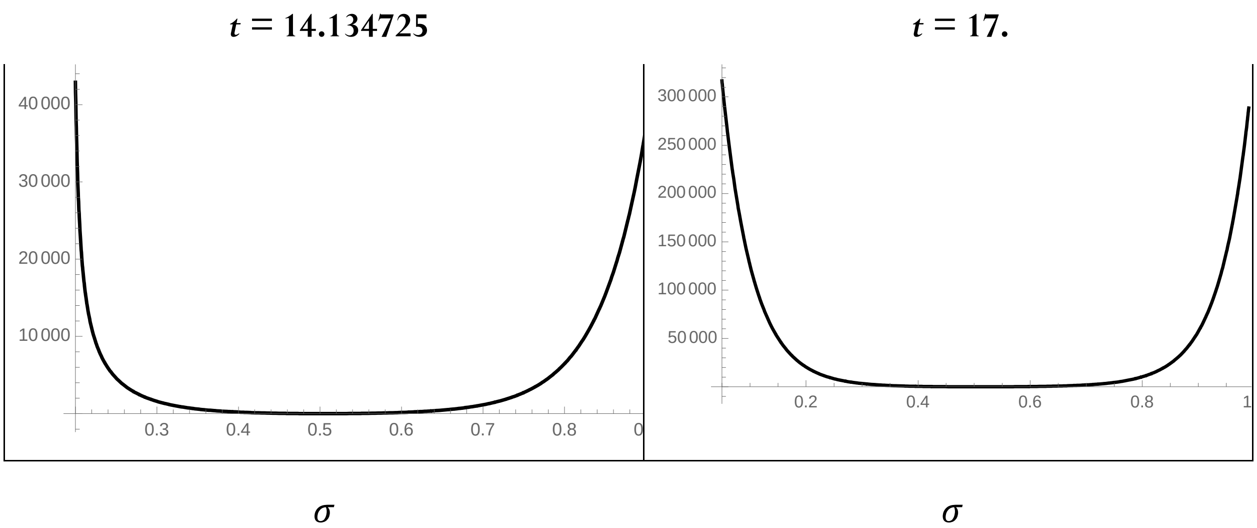
$$
\left.\left.\left.\cdot\left(1-e^{-\left[e^{\frac{(m-1)\cdot\pi}{t}}\right]\cdot e^{\left[x-\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}\right)\right]\right\}\right]
$$

Similarly, a metric of the difference of the even part of the translated function and the translated function itself is a linear, unweighted average of the ratio of the even part of the translated function to the translated function, with the average taken over many equally spaced points along the x axis.

The metric was evaluated using values of t generated with a pseudo-random real number generator, with values of integer m adjusted for computational efficiency. Higher values of t require higher values of integer m for optimal computation. The minimum in the metric occurs at $\sigma = ½$ for all values of t (Mathematica®).

The partial derivative of the metric, taken with respect to the real part of the argument σ, was calculated analytically and numerically. The derivative exhibits a single minimum of zero at $\sigma =½$, for every pseudo-random value of t (Mathematica®).

The metric for the second function above is shown in the graphs below for $t =$ 14.134725 and $t = 16$, with $m = 51$.



The metric is at its minimum and is equal to zero when $\sigma = ½$ for all values of t.

Therefore, the two functions:

$$\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right)\cdot\left[Re[\Gamma(s)]\cdot\left(1-e^{-\left|e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right|\cdot e^{-x}}\right)+\,e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Im[\Gamma(s)]\cdot\left(1-e^{-\left|e^{\frac{(m-1)\cdot\pi}{t}}\right|\cdot e^{-x}}\right)\right]$$

and

$$\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right)\cdot\left[Im[\Gamma(s)]\cdot\left(1-e^{-\left|e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right|\cdot e^{-x}}\right)-\,e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Re[\Gamma(s)]\cdot\left(1-e^{-\left|e^{\frac{(m-1)\cdot\pi}{t}}\right|\cdot e^{-x}}\right)\right]$$

in the integrands of the asymptotic relationship:

$$\int_{-\infty}^{\infty} sin(t\cdot x)\cdot\left\{\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right)\cdot\left[Re[\Gamma(s)]\cdot\left(1-e^{-\left|e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right|\cdot e^{-x}}\right)+\,e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Im[\Gamma(s)]\cdot\left(1-e^{-\left|e^{\frac{(m-1)\cdot\pi}{t}}\right|\cdot e^{-x}}\right)\right]\right\}dx$$

$$\sim$$

$$-\int_{-\infty}^{\infty} cos(t\cdot x)\cdot\left\{\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right)\cdot\left[Im[\Gamma(s)]\cdot\left(1-e^{-\left|e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right|\cdot e^{-x}}\right)-\,e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Re[\Gamma(s)]\cdot\left(1-e^{-\left|e^{\frac{(m-1)\cdot\pi}{t}}\right|\cdot e^{-x}}\right)\right]\right\}dx$$

when translated, most closely approximate the even parts of the functions when $\sigma = ½$, for all values of t.

Let $f(x,\sigma,t,m)$ be defined by the first function above, with the variable x is again translated so that $x\rightarrow x+(m-1)\cdot\pi/t$, or

$$\left(\frac{e^{-\sigma\cdot\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}{e^{e^{-\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}-1}\right)$$
$$\cdot\left[Re[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right]\cdot e^{-\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}\right)+\ e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Im[\Gamma(s)]\right.$$
$$\left.\cdot\left(1-e^{-\left[e^{\frac{(m-1)\cdot\pi}{t}}\right]\cdot e^{-\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}\right)\right]$$

and let $g(x,\sigma,t,m)$ be defined by the second function above, where the variable x is translated so that $x\rightarrow x+(2\cdot m-1)\cdot\pi/(2\cdot t)$ is

$$\left(\frac{e^{-\sigma\cdot\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}{e^{e^{-\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}-1}\right)$$
$$\cdot\left[Im[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right]\cdot e^{-\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}\right)+\ e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Re[\Gamma(s)]\right.$$
$$\left.\cdot\left(1-e^{-\left[e^{\frac{(m-1)\cdot\pi}{t}}\right]\cdot e^{-\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}\right)\right]$$

Translating the kernels of the integral transforms and substituting gives

$$\int_{-\infty}^{\infty} sin\left[t\cdot x\ +\ \frac{(m-1)\cdot\pi}{t}\right]\cdot f(x,\sigma,t,m)dx\ \ \sim\int_{-\infty}^{\infty} cos\left[t\cdot x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]\cdot g(x,\sigma,t,m)dx$$

and

$$\int_{-\infty}^{\infty} sin\left[t\cdot x\ +\ \frac{(m-1)\cdot\pi}{t}\right]\cdot g(x,\sigma,t,m)dx\ \ \sim -\int_{-\infty}^{\infty} cos\left[t\cdot x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]\cdot f(x,\sigma,t,m)dx$$

Note that

$$cos\left[t\cdot x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]\ =\ -\,sin\left[t\cdot x\ +\ \frac{(m-1)\cdot\pi}{t}\right]$$

and therefore,

$$\int_{-\infty}^{\infty} sin\left[t\cdot x\ +\ \frac{(m-1)\cdot\pi}{t}\right]\cdot f(x,\sigma,t,m)dx\ \ \sim -\int_{-\infty}^{\infty} sin\left[t\cdot x\ +\ \frac{(m-1)\cdot\pi}{t}\right]\cdot g(x,\sigma,t,m)dx$$

and

$$\int_{-\infty}^{\infty} sin\left[t\cdot x\ +\ \frac{(m-1)\cdot\pi}{t}\right]\cdot g(x,\sigma,t,m)dx\ \ \sim\int_{-\infty}^{\infty} sin\left[t\cdot x\ +\ \frac{(m-1)\cdot\pi}{t}\right]\cdot f(x,\sigma,t,m)dx$$

or

$$\int_{-\infty}^{\infty} sin\left[t \cdot x + \frac{(m-1) \cdot \pi}{t}\right] \cdot f(x,\sigma,t,m)dx + \int_{-\infty}^{\infty} sin\left[t \cdot x + \frac{(m-1) \cdot \pi}{t}\right] \cdot g(x,\sigma,t,m)dx \sim 0$$

and

$$\int_{-\infty}^{\infty} sin\left[t \cdot x + \frac{(m-1) \cdot \pi}{t}\right] \cdot g(x,\sigma,t,m)dx - \int_{-\infty}^{\infty} sin\left[t \cdot x + \frac{(m-1) \cdot \pi}{t}\right] \cdot f(x,\sigma,t,m)dx \sim 0$$

The property of linearity for sine transforms gives

$$\int_{-\infty}^{\infty} sin\left[t \cdot x + \frac{(m-1) \cdot \pi}{t}\right] \cdot [f(x,\sigma,t,m) + g(x,\sigma,t,m)]dx \sim 0$$

and

$$\int_{-\infty}^{\infty} sin\left[t \cdot x + \frac{(m-1) \cdot \pi}{t}\right] \cdot [g(x,\sigma,t,m) - f(x,\sigma,t,m)]dx \sim 0$$

Since every real-valued function can be represented as the sum of an odd and an even function,

$$f(x,\sigma,t,m) = f_{odd}(x,\sigma,t,m) + f_{even}(x,\sigma,t,m)$$

and

$$g(x,\sigma,t,m) = g_{odd}(x,\sigma,t,m) + g_{even}(x,\sigma,t,m)$$

where $f_{odd}(x,\sigma,t,m)$ and $g_{odd}(x,\sigma,t,m)$ are odd functions and $f_{even}(x,\sigma,t,m)$ and $g_{even}(x,\sigma,t,m)$ are even functions of variable x.

Substituting and rearranging gives

$$\int_{-\infty}^{\infty} sin\left[t \cdot x + \frac{(m-1) \cdot \pi}{t}\right] \cdot [f_{odd}(x,\sigma,t,m) + g_{odd}(x,\sigma,t,m)]dx + \int_{-\infty}^{\infty} sin\left[t \cdot x + \frac{(m-1) \cdot \pi}{t}\right] \cdot [f_{even}(x,\sigma,t,m) + g_{even}(x,\sigma,t,m)]dx \sim 0$$

and

$$\int_{-\infty}^{\infty} sin\left[t \cdot x + \frac{(m-1) \cdot \pi}{t}\right] \cdot [g_{odd}(x,\sigma,t,m) - f_{odd}(x,\sigma,t,m)]dx + \int_{-\infty}^{\infty} sin\left[t \cdot x + \frac{(m-1) \cdot \pi}{t}\right] \cdot [g_{even}(x,\sigma,t,m) - f_{even}(x,\sigma,t,m)]dx \sim 0$$

Note that

$$sin\left[t \cdot x + \frac{(m-1) \cdot \pi}{t}\right]$$

is an odd function of the variable x. Also, the functions

$$f_{odd}(x,\sigma,t,m) + g_{odd}(x,\sigma,t,m)$$

and

$$g_{odd}(x,\sigma,t,m) - f_{odd}(x,\sigma,t,m)$$

are odd functions of x, and the functions

$$f_{even}(x,\sigma,t,m) + g_{even}(x,\sigma,t,m)$$

and

$$g_{even}(x,\sigma,t,m) - f_{even}(x,\sigma,t,m)$$

are even functions of x.

Furthermore, the products of the odd functions of variable x:

$$sin\left[t \cdot x + \frac{(m-1)\cdot\pi}{t}\right] \cdot [f_{odd}(x,\sigma,t,m) + g_{odd}(x,\sigma,t,m)]$$

and

$$sin\left[t \cdot x + \frac{(m-1)\cdot\pi}{t}\right] \cdot [g_{odd}(x,\sigma,t,m) - f_{odd}(x,\sigma,t,m)]$$

are both even functions of x, and the products of the odd functions of variable x and the even functions of x:

$$sin\left[t \cdot x + \frac{(m-1)\cdot\pi}{t}\right] \cdot [f_{even}(x,\sigma,t,m) + g_{even}(x,\sigma,t,m)]$$

and

$$sin\left[t \cdot x + \frac{(m-1)\cdot\pi}{t}\right] \cdot [g_{even}(x,\sigma,t,m) - f_{even}(x,\sigma,t,m)]$$

are both odd functions of x.

Recall that

$$\int_{-\infty}^{\infty} sin\left[t \cdot x + \frac{(m-1)\cdot\pi}{t}\right] \cdot [f_{odd}(x,\sigma,t,m) + g_{odd}(x,\sigma,t,m)]dx + \int_{-\infty}^{\infty} sin\left[t \cdot x + \frac{(m-1)\cdot\pi}{t}\right] \cdot [f_{even}(x,\sigma,t,m) + g_{even}(x,\sigma,t,m)]dx \sim 0$$

and

$$\int_{-\infty}^{\infty} sin\left[t \cdot x + \frac{(m-1)\cdot\pi}{t}\right] \cdot [g_{odd}(x,\sigma,t,m) - f_{odd}(x,\sigma,t,m)]dx + \int_{-\infty}^{\infty} sin\left[t \cdot x + \frac{(m-1)\cdot\pi}{t}\right] \cdot [g_{even}(x,\sigma,t,m) - f_{even}(x,\sigma,t,m)]dx \sim 0$$

Since the sine transforms of the even translated functions are zero:

$$\int_{-\infty}^{\infty} sin\left[t \cdot x + \frac{(m-1)\cdot\pi}{t}\right] \cdot [f_{even}(x,\sigma,t,m) + g_{even}(x,\sigma,t,m)]dx = 0$$

and

$$\int_{-\infty}^{\infty} sin\left[t \cdot x + \frac{(m-1)\cdot\pi}{t}\right] \cdot [g_{even}(x,\sigma,t,m) - f_{even}(x,\sigma,t,m)]dx = 0$$

it follows that the sine transforms of the odd translated functions

$$\int_{-\infty}^{\infty} sin\left[t \cdot x + \frac{(m-1)\cdot\pi}{t}\right] \cdot [f_{odd}(x,\sigma,t,m) + g_{odd}(x,\sigma,t,m)]dx$$

and

$$\int_{-\infty}^{\infty} sin\left[t \cdot x + \frac{(m-1)\cdot\pi}{t}\right] \cdot [g_{odd}(x,\sigma,t,m) - f_{odd}(x,\sigma,t,m)]dx$$

must also be zero.

Also, since

$$g_{odd}(x,\sigma,t,m) \neq f_{odd}(x,\sigma,t,m)$$

and

$$g_{odd}(x,\sigma,t,m) \neq -f_{odd}(x,\sigma,t,m)$$

it follows that the odd translated functions must be zero:

$$g_{odd}(x,\sigma,t,m) = f_{odd}(x,\sigma,t,m) = 0$$

Therefore, the translated functions $f(x,\sigma,t,m)$:

$$\left(\frac{e^{-\sigma\cdot\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}{e^{e^{-\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}-1}\right) \cdot \left[Re[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right]\cdot e^{-\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}\right) + e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Im[\Gamma(s)] \cdot\left(1-e^{-\left[e^{\frac{(m-1)\cdot\pi}{t}}\right]\cdot e^{-\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}\right)\right]$$

and $g(x,\sigma,t,m)$,

$$\left(\frac{e^{-\sigma\cdot\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}{e^{e^{-\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}-1}\right) \cdot \left[Im[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right]\cdot e^{-\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}\right) + e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Re[\Gamma(s)] \cdot\left(1-e^{-\left[e^{\frac{(m-1)\cdot\pi}{t}}\right]\cdot e^{-\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}\right)\right]$$

must be even functions of the variable x.

It was previously demonstrated that these two functions most closely approximate even functions of the variable of integration x for all values of t, only when $\sigma = 1/2$.

To review, at the roots of the Riemann zeta function in the critical strip, the real and imaginary components of the partial sums of the zeta function series diverge proportionally and asymptotically, and the bi-lateral sine and cosine integral transforms:

$$\int_{-\infty}^{\infty} sin(t \cdot x) \cdot \left\{\left(\frac{e^{-\sigma \cdot x}}{e^{e^{-x}} - 1}\right) \cdot \left[Re[\Gamma(s)] \cdot \left(1 - e^{-\left|e^{\frac{(2 \cdot m - 1) \cdot \pi}{2 \cdot t}}\right| \cdot e^{-x}}\right) + e^{\frac{\pi \cdot (1-\sigma)}{2 \cdot t}} \cdot Im[\Gamma(s)] \cdot \left(1 - e^{-\left|e^{\frac{(m-1) \cdot \pi}{t}}\right| \cdot e^{-x}}\right)\right]\right\} dx$$

and

$$\int_{-\infty}^{\infty} cos(t \cdot x) \cdot \left\{\left(\frac{e^{-\sigma \cdot x}}{e^{e^{-x}} - 1}\right) \cdot \left[Im[\Gamma(s)] \cdot \left(1 - e^{-\left|e^{\frac{(2 \cdot m - 1) \cdot \pi}{2 \cdot t}}\right| \cdot e^{-x}}\right) - e^{\frac{\pi \cdot (1-\sigma)}{2 \cdot t}} \cdot Re[\Gamma(s)] \cdot \left(1 - e^{-\left|e^{\frac{(m-1) \cdot \pi}{t}}\right| \cdot e^{-x}}\right)\right]\right\} dx$$

vanish simultaneously and asymptotically.

It was shown that this occurs only when the functions in the integral transforms above, or

$$\left(\frac{e^{-\sigma \cdot x}}{e^{e^{-x}} - 1}\right) \cdot \left[Re[\Gamma(s)] \cdot \left(1 - e^{-\left|e^{\frac{(2 \cdot m - 1) \cdot \pi}{2 \cdot t}}\right| \cdot e^{-x}}\right) + e^{\frac{\pi \cdot (1-\sigma)}{2 \cdot t}} \cdot Im[\Gamma(s)] \cdot \left(1 - e^{-\left|e^{\frac{(m-1) \cdot \pi}{t}}\right| \cdot e^{-x}}\right)\right]$$

and

$$\left(\frac{e^{-\sigma \cdot x}}{e^{e^{-x}} - 1}\right) \cdot \left[Im[\Gamma(s)] \cdot \left(1 - e^{-\left|e^{\frac{(2 \cdot m - 1) \cdot \pi}{2 \cdot t}}\right| \cdot e^{-x}}\right) - e^{\frac{\pi \cdot (1-\sigma)}{2 \cdot t}} \cdot Re[\Gamma(s)] \cdot \left(1 - e^{-\left|e^{\frac{(m-1) \cdot \pi}{t}}\right| \cdot e^{-x}}\right)\right]$$

most closely approximate even functions of the translated variable of integration, where $x = (m - 1) \cdot \pi/t$ and $x = (2 \cdot m - 1) \cdot \pi/(2 \cdot t)$, or linear combinations of these general terms, respectively, for all values of t. This occurs only when $\sigma = 1/2$.

Therefore, the roots of the Riemann zeta function in the critical strip all have real part equal to ½ and the Riemann hypothesis is correct.

The graphs below show the three functions:

$$sin(t\cdot x)$$

and

$$\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right)\cdot\left[Re[\Gamma(s)]\cdot\left(1-e^{-\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right\rfloor\cdot e^{-x}}\right)+e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Im[\Gamma(s)]\cdot\left(1-e^{-\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}}\right\rfloor\cdot e^{-x}}\right)\right]$$

and

$$sin(t\cdot x)\cdot\left\{\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right)\cdot\left[Re[\Gamma(s)]\cdot\left(1-e^{-\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right\rfloor\cdot e^{-x}}\right)+e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Im[\Gamma(s)]\cdot\left(1-e^{-\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}}\right\rfloor\cdot e^{-x}}\right)\right]\right\}$$

as blue curves, and the three functions

$$cos(t\cdot x)$$

and

$$\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right)\cdot\left[Im[\Gamma(s)]\cdot\left(1-e^{-\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right\rfloor\cdot e^{-x}}\right)-e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Re[\Gamma(s)]\cdot\left(1-e^{-\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}}\right\rfloor\cdot e^{-x}}\right)\right]$$

and

$$cos(t\cdot x)\cdot\left\{\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right)\cdot\left[Im[\Gamma(s)]\cdot\left(1-e^{-\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right\rfloor\cdot e^{-x}}\right)-e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Re[\Gamma(s)]\cdot\left(1-e^{-\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}}\right\rfloor\cdot e^{-x}}\right)\right]\right\}$$

as red curves, for five roots and of the Riemann zeta function. The first graph for each root is at a higher resolution than the second graph.

At each root, the functions

$$\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right)\cdot\left[Re[\Gamma(s)]\cdot\left(1-e^{-\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right\rfloor\cdot e^{-x}}\right)+e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Im[\Gamma(s)]\cdot\left(1-e^{-\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}}\right\rfloor\cdot e^{-x}}\right)\right]$$

and

$$\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right)\cdot\left[Im[\Gamma(s)]\cdot\left(1-e^{-\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right\rfloor\cdot e^{-x}}\right)-e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Re[\Gamma(s)]\cdot\left(1-e^{-\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}}\right\rfloor\cdot e^{-x}}\right)\right]$$

are not exactly even functions of the translated variable of integration. As a result, the roots of $sin(t \cdot x)$ and $cos(t \cdot x)$ do not align precisely at the respective maxima or minima of the two functions above. However, at the roots of the Riemann zeta function in the critical strip, (1.) the functions in the integrands of the transforms most closely approximate even translated functions of the variable of integration, (2.) the maxima or minima in the functions occur when $x = (m-1) \cdot \pi/t$ and $x = (2 \cdot m - 1) \cdot \pi/(2 \cdot t)$, or linear combinations of these general terms, and (3.) the bi-lateral integral transforms

$$\int_{-\infty}^{\infty} sin(t \cdot x) \cdot \left\{\left(\frac{e^{-\sigma \cdot x}}{e^{e^{-x}}-1}\right) \cdot \left[Re[\Gamma(s)] \cdot \left(1 - e^{-\left[e^{\frac{(2 \cdot m-1) \cdot \pi}{2 \cdot t}}\right] \cdot e^{-x}}\right) + e^{\frac{\pi \cdot (1-\sigma)}{2 \cdot t}} \cdot Im[\Gamma(s)] \cdot \left(1 - e^{-\left[e^{\frac{(m-1) \cdot \pi}{t}}\right] \cdot e^{-x}}\right)\right]\right\} dx$$

and

$$\int_{-\infty}^{\infty} cos(t \cdot x) \cdot \left\{\left(\frac{e^{-\sigma \cdot x}}{e^{e^{-x}}-1}\right) \cdot \left[Im[\Gamma(s)] \cdot \left(1 - e^{-\left[e^{\frac{(2 \cdot m-1) \cdot \pi}{2 \cdot t}}\right] \cdot e^{-x}}\right) - e^{\frac{\pi \cdot (1-\sigma)}{2 \cdot t}} \cdot Re[\Gamma(s)] \cdot \left(1 - e^{-\left[e^{\frac{(m-1) \cdot \pi}{t}}\right] \cdot e^{-x}}\right)\right]\right\} dx$$

vanish asymptotically and simultaneously.

The functions above are shown in the following two graphs for a root of the Riemann zeta function with $\sigma = 1/2$, $t = 14.134725$, and $m = 51$.

The functions $sin(t \cdot x)$ and

$$\left(\frac{e^{-\sigma \cdot x}}{e^{e^{-x}}-1}\right) \cdot \left[Re[\Gamma(s)] \cdot \left(1 - e^{-\left[e^{\frac{(2 \cdot m-1) \cdot \pi}{2 \cdot t}}\right] \cdot e^{-x}}\right) + e^{\frac{\pi \cdot (1-\sigma)}{2 \cdot t}} \cdot Im[\Gamma(s)] \cdot \left(1 - e^{-\left[e^{\frac{(m-1) \cdot \pi}{t}}\right] \cdot e^{-x}}\right)\right]$$

are shown in blue.

The functions $cos(t \cdot x)$ and

$$\left(\frac{e^{-\sigma \cdot x}}{e^{e^{-x}}-1}\right) \cdot \left[Im[\Gamma(s)] \cdot \left(1 - e^{-\left[e^{\frac{(2 \cdot m-1) \cdot \pi}{2 \cdot t}}\right] \cdot e^{-x}}\right) - e^{\frac{\pi \cdot (1-\sigma)}{2 \cdot t}} \cdot Re[\Gamma(s)] \cdot \left(1 - e^{-\left[e^{\frac{(m-1) \cdot \pi}{t}}\right] \cdot e^{-x}}\right)\right]$$

are shown in red.

The blue vertical lines connect the point $x = (m-1) \cdot \pi/t$ where $sin(t \cdot x) = 0$ with the approximate minimum (in this case) of the function

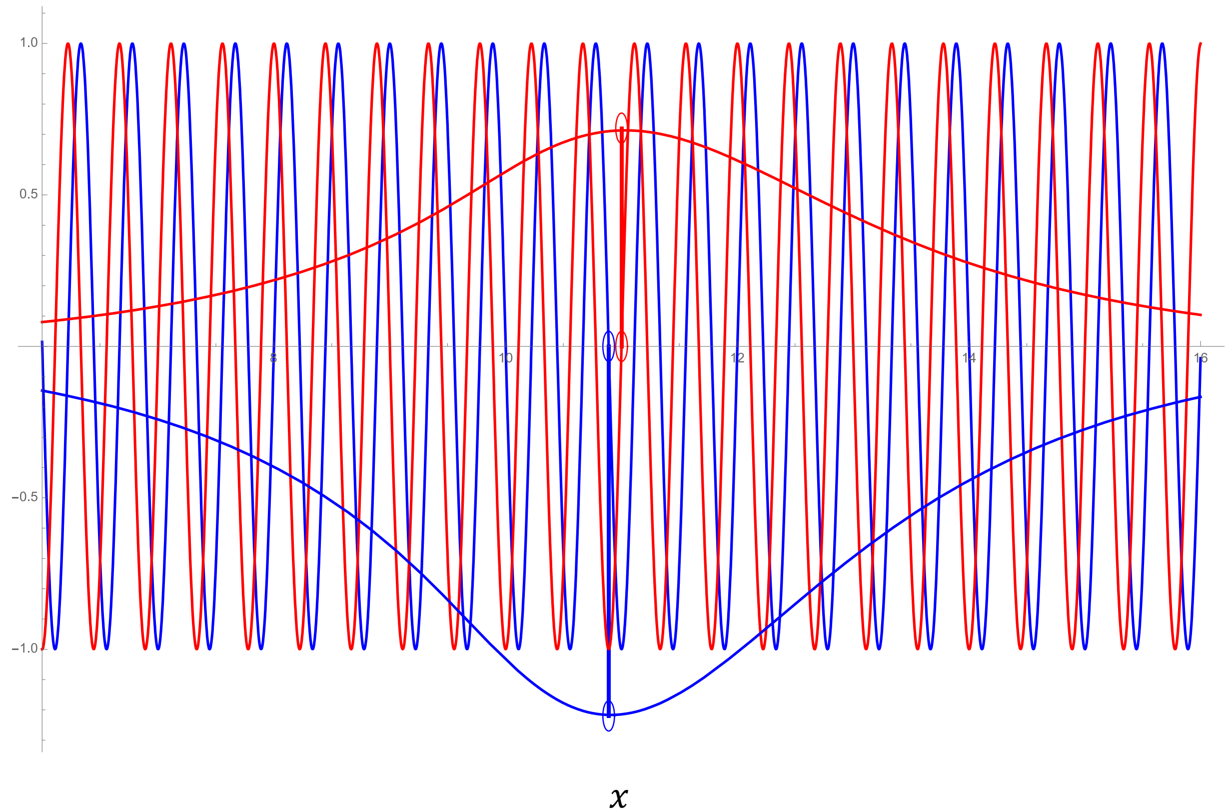
$$\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right)\cdot\left[Re[\Gamma(s)]\cdot\left(1-e^{-\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right\rfloor\cdot e^{-x}}\right)+\ e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Im[\Gamma(s)]\cdot\left(1-e^{-\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}}\right\rfloor\cdot e^{-x}}\right)\right]$$

The red vertical lines connect the point $x = (2\cdot m-1)\cdot\pi/(2\cdot t)$ where $cos(t\cdot x) = 0$ with the approximate maximum (in this case) of the function

$$\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right)\cdot\left[Im[\Gamma(s)]\cdot\left(1-e^{-\left\lfloor e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right\rfloor\cdot e^{-x}}\right)-\ e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Re[\Gamma(s)]\cdot\left(1-e^{-\left\lfloor e^{\frac{(m-1)\cdot\pi}{t}}\right\rfloor\cdot e^{-x}}\right)\right]$$

a root of the Riemann zeta function with $\sigma = 1/2$, $t = 14.1347$, and $m = 51$

(functions are scaled by $\pm 1\text{x}10^7$)



Notes:

Blue Vertical Lines

$$x \approx 10.8908 = \frac{(m-2)\cdot\pi}{t} \quad \Longrightarrow \quad t \approx \frac{(51-2)\cdot\pi}{10.8908} \approx 14.1347$$

Red Vertical Lines

$$x \approx 11.0019 = \frac{(2\cdot m-3)\cdot\pi}{2\cdot t} \quad \Longrightarrow \quad t \approx \frac{(2\cdot 51-3)\cdot\pi}{2\cdot 11.0019} \approx 14.1347$$

The value of t from the literature for the root of the Riemann zeta function is approximately 14.134725...

The behavior of the functions in the integrands of the bi-lateral transforms in the vicinity of the root in the critical strip for the Riemann zeta function is shown in the following graph.

The graph below shows the function

$$sin\left\{t\cdot\left[x-\frac{(m-1)\cdot\pi}{t}\right]\right\}$$

and

$$\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right)\cdot\left[Re[\Gamma(s)]\cdot\left(1-e^{-\left|e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right|\cdot e^{-x}}\right)+\ e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Im[\Gamma(s)]\cdot\left(1-e^{-\left|e^{\frac{(m-1)\cdot\pi}{t}}\right|\cdot e^{-x}}\right)\right]$$

as a set of green, blue and purple curves.

The blue vertical line in the graph connects the point x on the abscissa that satisfies

$$sin\left\{t\cdot\left[x-\frac{(m-1)\cdot\pi}{t}\right]\right\}\ =\ 0$$

with the point at the maximum of the function

$$\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right)\cdot\left[Re[\Gamma(s)]\cdot\left(1-e^{-\left|e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right|\cdot e^{-x}}\right)+\ e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Im[\Gamma(s)]\cdot\left(1-e^{-\left|e^{\frac{(m-1)\cdot\pi}{t}}\right|\cdot e^{-x}}\right)\right]$$

when σ is equal 0.50.

The function

$$\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right)\cdot\left[Re[\Gamma(s)]\cdot\left(1-e^{-\left|e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right|\cdot e^{-x}}\right)+\ e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Im[\Gamma(s)]\cdot\left(1-e^{-\left|e^{\frac{(m-1)\cdot\pi}{t}}\right|\cdot e^{-x}}\right)\right]$$

is shown as green, blue and purple curves with values of σ equal to 0.35, 0.50, and 0.65, respectively.

The graph also shows the function

$$cos\left\{t\cdot\left[x-\frac{(m-1)\cdot\pi}{t}\right]\right\}$$

and

$$\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right)\cdot\left[Im[\Gamma(s)]\cdot\left(1-e^{-\left|e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right|\cdot e^{-x}}\right)-\ e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Re[\Gamma(s)]\cdot\left(1-e^{-\left|e^{\frac{(m-1)\cdot\pi}{t}}\right|\cdot e^{-x}}\right)\right]$$

as a set of green, red and purple curves.

The red vertical line in the graph connects the point x on the abscissa that satisfies

$$cos\left\{t\cdot\left[x-\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]\right\} = 0$$

with the point at the maximum of the function

$$\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right)\cdot\left[Im[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right]\cdot e^{-x}}\right)-e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Re[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(m-1)\cdot\pi}{t}}\right]\cdot e^{-x}}\right)\right]$$

when σ is equal 0.50.

The function

$$\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right)\cdot\left[Im[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right]\cdot e^{-x}}\right)-e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Re[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(m-1)\cdot\pi}{t}}\right]\cdot e^{-x}}\right)\right]$$

is shown as green, red and purple curves with values of σ equal to 0.35, 0.50, and 0.65, respectively.

As σ is decreased below $\sigma = ½$, the maxima and minima in the functions shift to higher values of x, and no longer occur at the roots of the equations

$$sin\left\{t\cdot\left[x-\frac{(m-1)\cdot\pi}{t}\right]\right\} = 0$$

and

$$cos\left\{t\cdot\left[x-\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]\right\} = 0$$

Conversely, as σ is increased above $\sigma = ½$, the maxima and minima in the functions shift to lower values of x, and again, no longer occur at the roots of the equation two equations above. Therefore, roots of the Riemann zeta function occur when $\sigma = ½$.

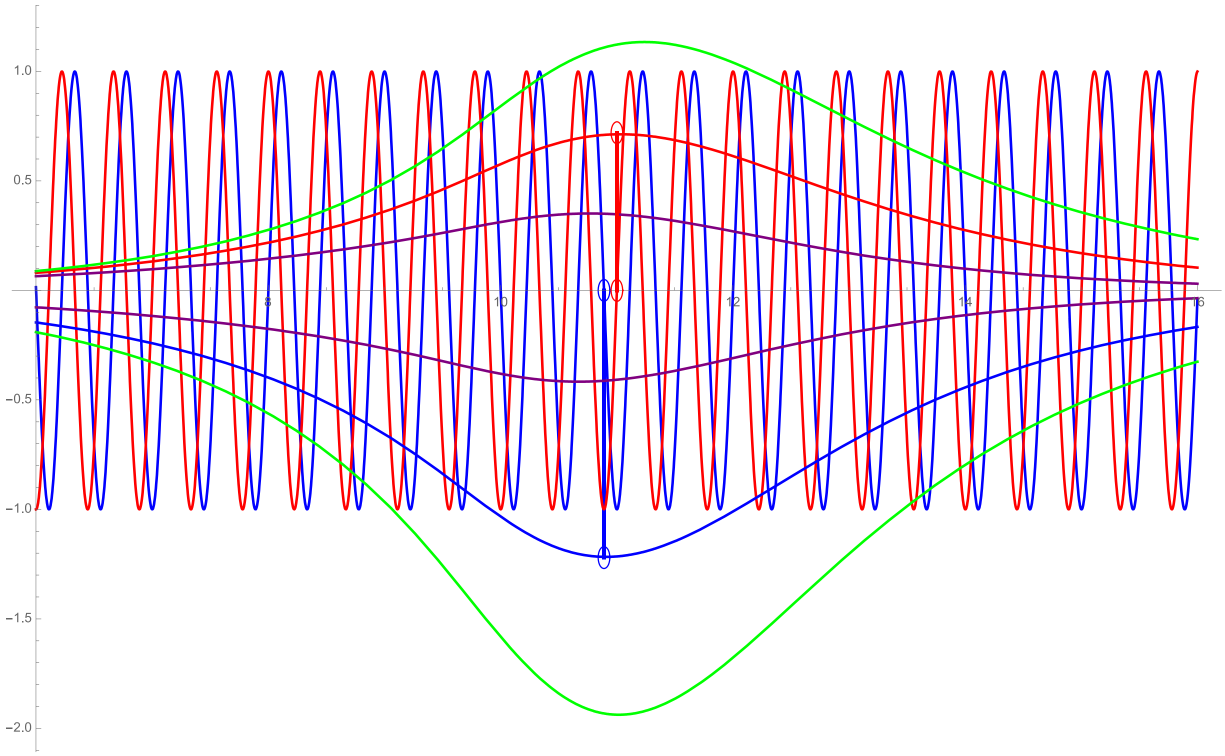
a root of the Riemann zeta function with $t = 14.1347$ and $m = 51$

(functions are scaled by $\pm 1\text{x}10^7$)

$^1/_2 < \sigma < 1$ ← purple $\sigma = 0.60$

$\sigma = {}^1/_2$ ↓ red $\sigma = 0.50$

$0 < \sigma < {}^1/_2$ → green $\sigma = 0.43$



$^1/_2 < \sigma < 1$ ← purple $\sigma = 0.59$

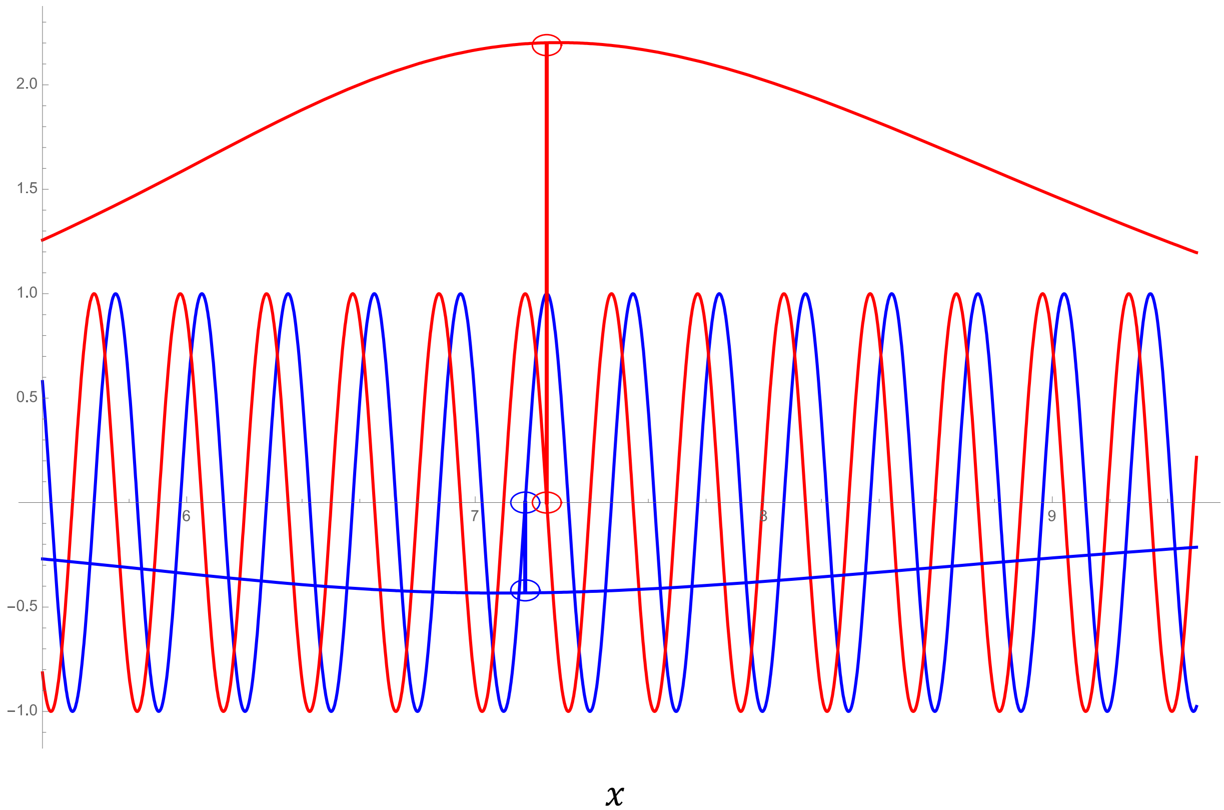
$\sigma = {}^1/_2$ ↑ blue $\sigma = 0.50$

$0 < \sigma < {}^1/_2$ → green $\sigma = 0.46$

The following graphs and calculations illustrate four additional roots of the Riemann zeta function.

a root of the Riemann zeta function with $\sigma = 1/2$, $t = 21.0220$, and $m = 51$

(functions are scaled by $\pm 5\text{x}10^{12}$)



Notes:

Blue Vertical Lines

$$x \approx 7.1732 = \frac{(m-3)\cdot\pi}{t} \quad \Longrightarrow \quad t \approx \frac{(51-3)\cdot\pi}{7.1732} \approx 21.0220$$

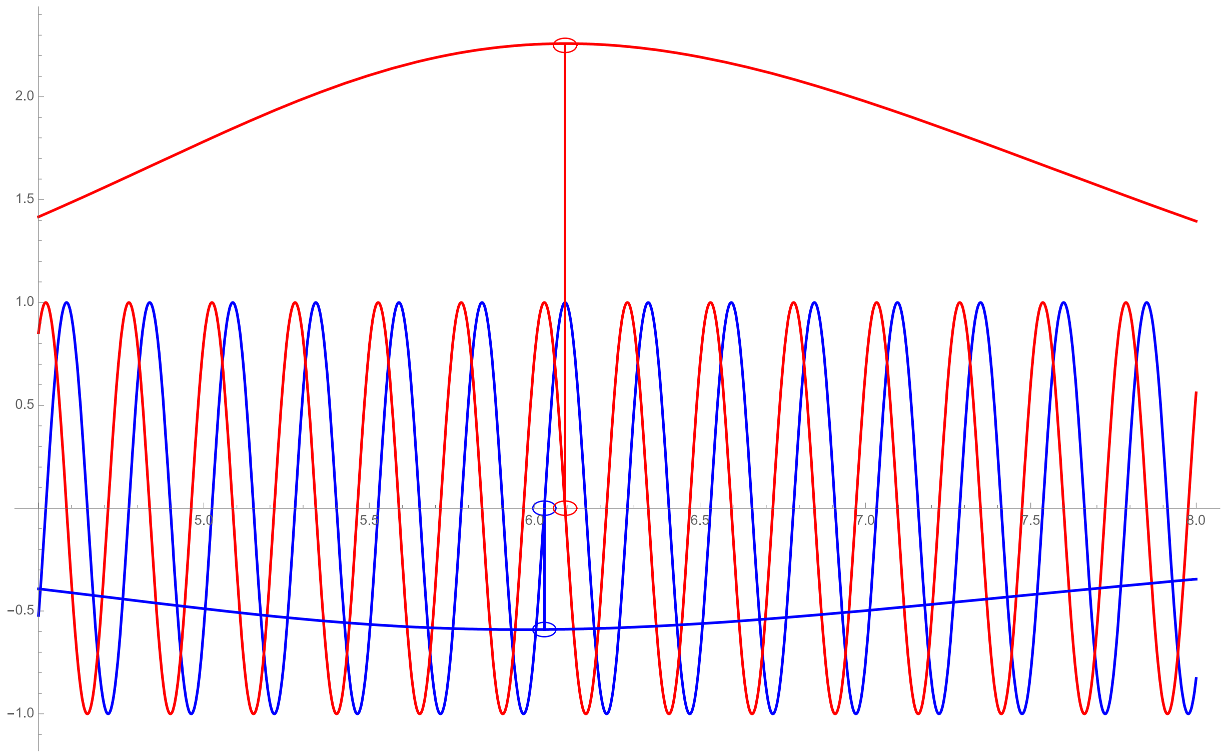
Red Vertical Lines

$$x \approx 7.2480 = \frac{(2\cdot m-5)\cdot\pi}{2\cdot t} \quad \Longrightarrow \quad t \approx \frac{(2\cdot 51-5)\cdot\pi}{2\cdot 7.2480} \approx 21.0220$$

The value of t from the literature for the root of the Riemann zeta function is approximately 21.022039...

a root of the Riemann zeta function with $\sigma = 1/2$, $t = 25.0108$, and $m = 51$

(functions are scaled by $\pm 5\text{x}10^{15}$)



Notes:

Blue Vertical Lines

$$x \approx 6.0292 = \frac{(m-3)\cdot\pi}{t} \quad \Longrightarrow \quad t \approx \frac{(51-3)\cdot\pi}{6.0292} \approx 25.0108$$

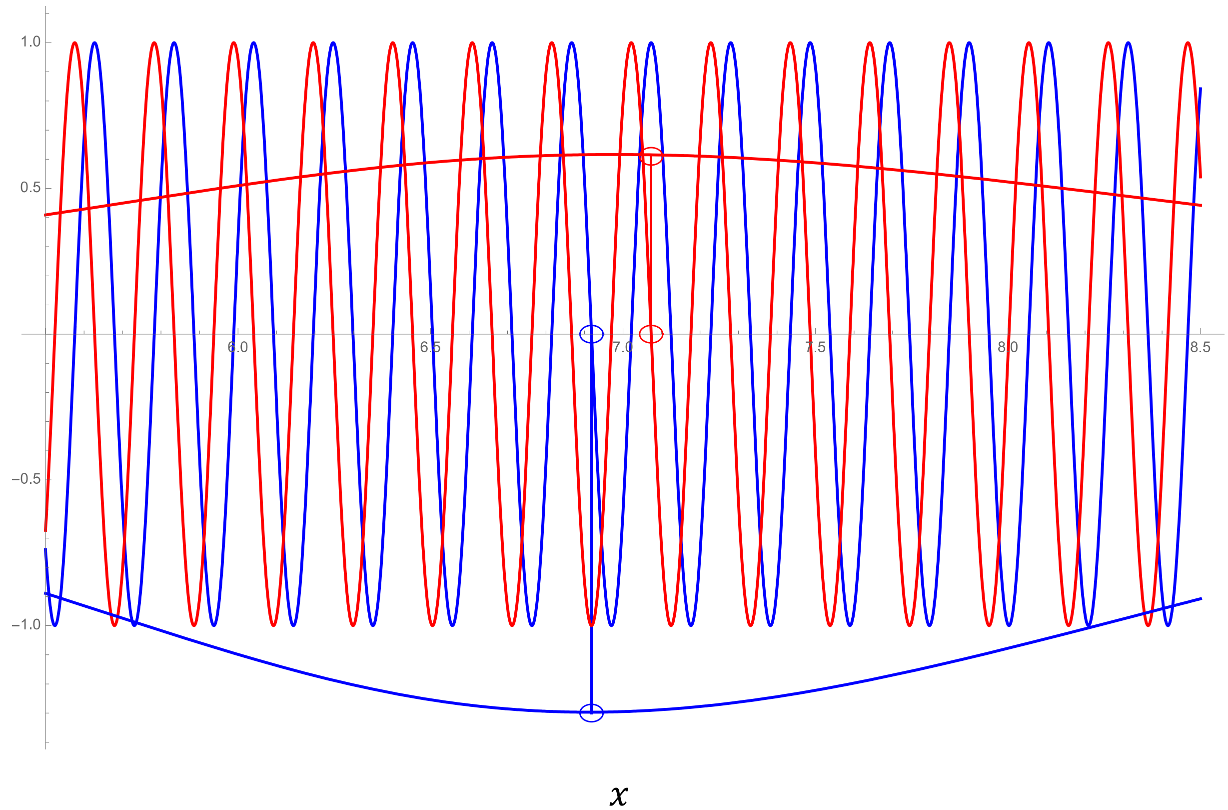
Red Vertical Lines

$$x \approx 6.0920 = \frac{(2\cdot m-5)\cdot\pi}{2\cdot t} \quad \Longrightarrow \quad t \approx \frac{(2\cdot 51-5)\cdot\pi}{2\cdot 6.0920} \approx 25.0108$$

The value of t from the literature for the root of the Riemann zeta function is approximately 25.010857…

a root of the Riemann zeta function with $\sigma = 1/2$, $t = 30.4249$, and $m = 70$

(functions are scaled by $\pm 1\text{x}10^{19}$)



Notes:

Blue Vertical Lines

$$x \approx 6.9182 = \frac{(m-3)\cdot\pi}{t} \quad\Longrightarrow\quad t \approx \frac{(70-3)\cdot\pi}{6.9182} \approx 30.4249$$

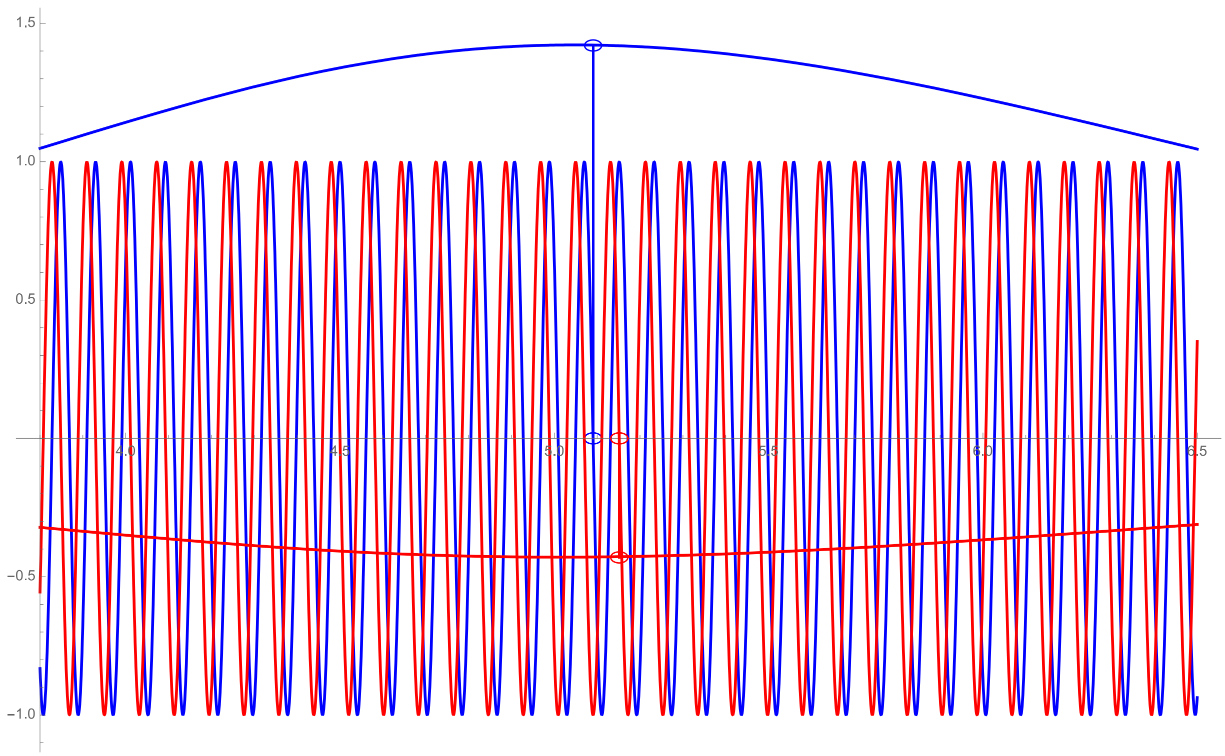
Red Vertical Lines

$$x \approx 7.0731 = \frac{(2\cdot m-3)\cdot\pi}{2\cdot t} \quad\Longrightarrow\quad t \approx \frac{(2\cdot 70-3)\cdot\pi}{2\cdot 7.0731} \approx 30.4249$$

The value of t from the literature for the root of the Riemann zeta function is approximately 30.424876…

a root of the Riemann zeta function with $\sigma = 1/2$, $t = 77.1448$, and $m = 130$

(functions are scaled by $\pm 2\text{x}10^{51}$)



x

Notes:

Blue Vertical Lines

$$x \approx 5.0904 = \frac{(m-5)\cdot\pi}{t} \implies t \approx \frac{(130-5)\cdot\pi}{5.0904} \approx 77.1448$$

Red Vertical Lines

$$x \approx 5.1515 = \frac{(2\cdot m-7)\cdot\pi}{2\cdot t} \implies t \approx \frac{(2\cdot 130-7)\cdot\pi}{2\cdot 5.1515} \approx 77.1448$$

The value of t from the literature for the root of the Riemann zeta function is approximately 77.144840...

The preceding theory and graphs make it clear that roots of the Riemann zeta function occur in the critical strip if and only if (1.) $\sigma = ½$ and the translated functions:

$$\left(\frac{e^{-\sigma\cdot\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}{e^{e^{-\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}-1}\right) \cdot \left[Re[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right]\cdot e^{-\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}\right)+ e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Im[\Gamma(s)] \cdot\left(1-e^{-\left[e^{\frac{(m-1)\cdot\pi}{t}}\right]\cdot e^{-\left[x+\frac{(m-1)\cdot\pi}{t}\right]}}\right)\right]$$

and

$$\left(\frac{e^{-\sigma\cdot\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}{e^{e^{-\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}-1}\right) \cdot \left[Im[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right]\cdot e^{-\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}\right)+ e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Re[\Gamma(s)] \cdot\left(1-e^{-\left[e^{\frac{(m-1)\cdot\pi}{t}}\right]\cdot e^{-\left[x+\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}\right]}}\right)\right]$$

are the best approximations of even functions of the variable of integration, x, (2.) the maxima or minima in the functions occur at $x = (m-1)\cdot\pi/t$ and $x = (2\cdot m-1)\cdot\pi/(2\cdot t)$, or at linear combinations of these general terms, so that the bi-lateral integral transforms

$$\int_{-\infty}^{\infty} sin(t\cdot x)\cdot\left\{\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right)\cdot\left[Re[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right]\cdot e^{-x}}\right)+ e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Im[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(m-1)\cdot\pi}{t}}\right]\cdot e^{-x}}\right)\right]\right\}dx$$

and

$$\int_{-\infty}^{\infty} cos(t\cdot x)\cdot\left\{\left(\frac{e^{-\sigma\cdot x}}{e^{e^{-x}}-1}\right) \cdot\left[Im[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(2\cdot m-1)\cdot\pi}{2\cdot t}}\right]\cdot e^{-x}}\right)- e^{\frac{\pi\cdot(1-\sigma)}{2\cdot t}}\cdot Re[\Gamma(s)]\cdot\left(1-e^{-\left[e^{\frac{(m-1)\cdot\pi}{t}}\right]\cdot e^{-x}}\right)\right]\right\}dx$$

vanish asymptotically and simultaneously.