FLUID MECHANICS

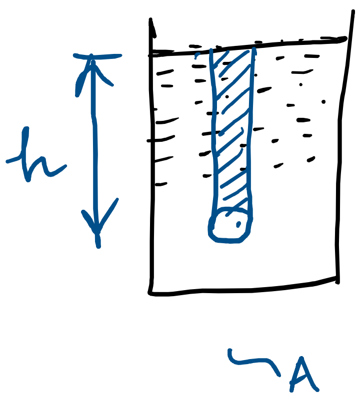
A fluid is a substance that can flow when external force is applied on it. The term fluids include both liquids and gases. Though liquids and gases are termed as fluids, there are marked differences between them. For example, gases are compressible whereas liquids are nearly incompressible. We only use those properties of liquids and gases, which are linked with their ability to flow, while discussing the mechanics of fluids.

Pressure = Force / Area

- MKS system N/m^2

Pressure due to a liquid column

Let h be the height of the liquid column in a cylinder of cross-sectional area A. If **ρ** is the density of the liquid, then weight of the liquid column W is given by-



$$W = m \times g$$
$$= (A h) \rho \cdot g$$
$$= A h \rho g.$$

$$\text{Pressure} = \frac{\text{Force}}{\text{area}} = \frac{\cancel{A} h \rho g}{\cancel{A}} = h \rho g$$

$$\boxed{P = h \rho g}$$

Density

Density = $\dfrac{\text{Mass}}{\text{Volume}}$ $\boxed{\rho = \dfrac{M}{V}}$

Density of water → 1000 kg/m^3 (SI unit)

or 1gm/cc

Relative Density

Relative density of a substance is defined as the ratio between the density of the substance to the density of water at 4°C. Relative density is also known as specific gravity

$$\text{Relative density} = \frac{\text{Density of substance}}{\text{Density of water}}$$

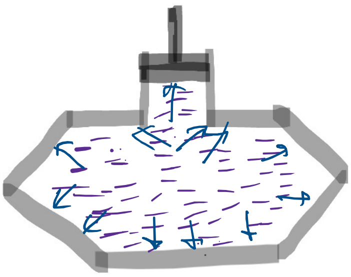
$$\text{R.D} = \frac{\text{Density of Iron}}{\text{Density of water}} = \frac{8500\ kg/m^3}{1000\ kg/m^3} = 8.5$$

(No unit)

Density of water is maximum at 4°C = 1000 kg/m³.

Pascal's law

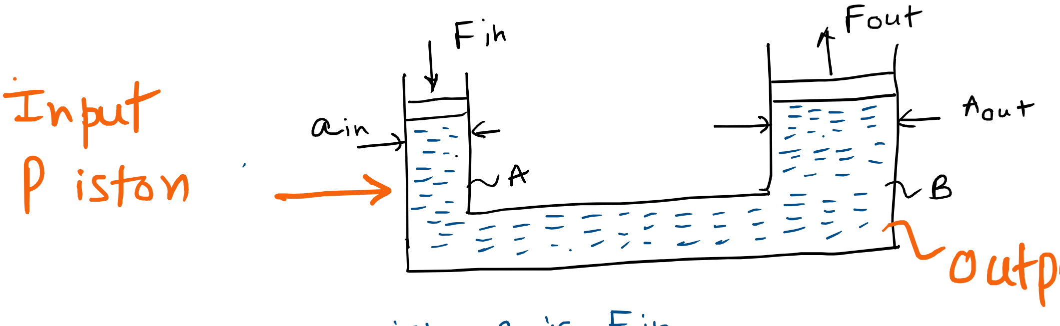
Pascal's law states that if the effect of gravity can be neglected then the pressure in a fluid in equilibrium is the same everywhere.



Application
- Hydraulic Lift
- Hydraulic Press
- Hydraulic brakes.

Hydraulic Lift (Application of Pascal's Law)

The most useful feature of fluid power is the ease with which it is able to multiply force. This is accomplished by using an output piston that is larger than the input piston. Such a system is shown in Fig



$a_{in} \rightarrow$ area of crosssection of piston A

$A_{out} \rightarrow$ Area of cross section of piston B

Input force on piston A is F_{in}

Pressure developed $= \frac{F_{in}}{a_{in}} \approx P.$

This pressure will be transmitted to Piston B as per Pascal's law.

upward force $\approx F_{out} = P \times A_{out}$

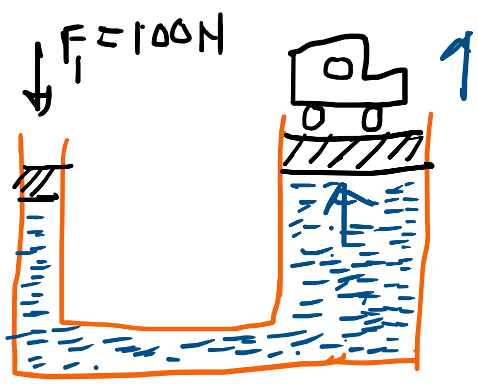
$$= \frac{F_{in}}{a_{in}} * A_{out}$$

$$F_{out} = F_{in}\left(\frac{A_{out}}{A_{in}}\right)$$

if $A \gg a$ then ~ $\boxed{F_{out} \gg F_{in}}$ Imp

Q.1

A hydraulic lift is used to lift cars of up to 3500 kg of mass. The platform of the lift has a mass of 500 kg. The smaller cylinder has a diameter of $d_1 = 25$ cm. If the maximum force that the operator can apply on this cylinder is $F_1 = 100$ N, find the diameter of the larger cylinder.



total mass (lift) ~

$F_{out} = (3500 + 500) * g$

$= 4000 \times 10$ N

we know that,

$$\boxed{F_{out} = F_{in} \frac{A_{out}}{A_{in}}}$$

$$A_{out} = \frac{F_{out}}{F_{in}} * A_{in}$$

$$\cancel{\pi} R^2 = \frac{4000\cancel{0}}{1\cancel{0}\cancel{0}} * \cancel{\pi} r^2$$

$$R^2 = 400 * r^2$$

$$R = \sqrt{400} * r$$

$$D = 20 \times d = 20 \times 25 = 500 \text{ cm Ans}$$

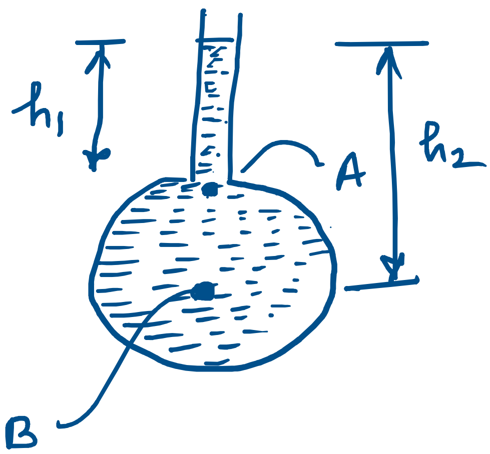
Pressure Measurement by Manometer

The relationship between pressure and head is used to measure pressure with a manometer

The Piezometer Tube Manometer

The simplest manometer is a tube, open at the top, which is attached to the top of a vessel containing liquid at a pressure (higher than atmospheric) to be measured.

An example can be seen in the figure below. This simple device is known as a Piezometer tube. As the tube is open to the atmosphere the pressure measured is relative to atmospheric so is gauge pressure.

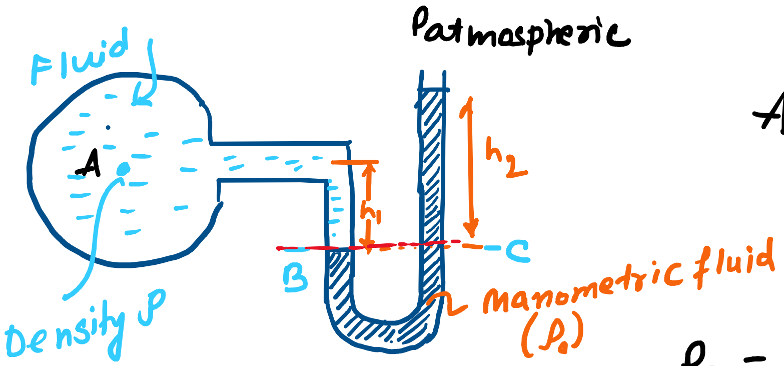


Pressure at A = $p_A = h_1 \rho g$

Pressure at B = $p_B = h_2 \rho g$.

The "U"-Tube Manometer

Using a "U"-Tube enables the pressure of both liquids and gases to be measured with the same instrument. The "U" is connected as in the figure below and filled with a fluid called the manometric fluid. The fluid whose pressure is being measured should have a mass density less than that of the manometric fluid and the two fluids should not be able to mix readily - that is, they must be immiscible.



(P_A = liquid pressure)

At any horizontal level, pressure is same

so, Pressure at B = Pressure at C.

$$P_B = P_C$$

Now,

$$P_B = P_A + h_1 \rho g$$

$$P_C = P_{Atmospheric} + h_2 \rho_0 g$$

$$\therefore \quad P_A + h_1 \rho g = P_{atm} + h_2 \rho_0 g$$

$$\boxed{P_A - P_{atm} = h_2 \rho_0 g - h_1 \rho g}$$

— Practice Questions —

Q.2.

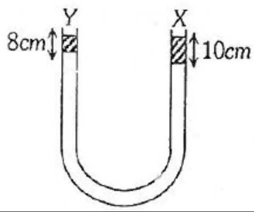
A liquid X of density 3.36g/cm^3 is poured in a U-tube, which contains Hg. Another liquid Y is poured in left arm with height 8 cm, upper levels of X and Y are same. What is density of Y

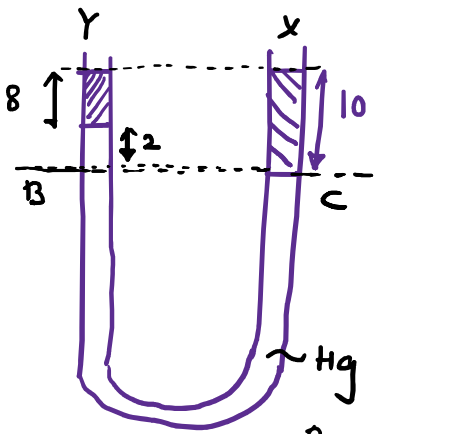
(a) 0.8 g/cc

(b) 1.2 g/cc

(c) 1.4 g/cc

(d) 1.6 g/cc





$\rho_{Hg} = 13.6$

Pressure at B = Pressure at C

$$h_1 \rho_{Hg} \cdot g + h_2 \rho_Y g = h \rho_x g$$

$$2 \times 13.6 \times \cancel{g} + 8 \rho_Y \cdot \cancel{g} = 10 \times 3.36 \times \cancel{g}$$

$$27.2 + 8 \rho_y = 33.6$$

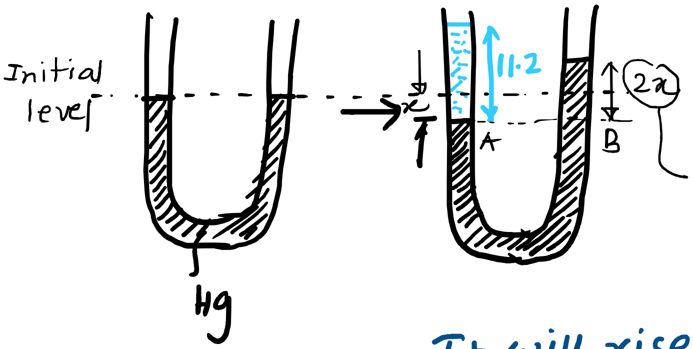
$$\underline{\rho_y = 0.8 \text{ gm/cc}}. \quad \text{Ans (a)}$$

Q.3

An open *U*-tube contains mercury. When 11.2 *cm* of water is poured into one of the arms of the tube, how height does the mercury rise in the other arm form its initial level

(a) 0.56 *cm* (b) 1.35 *cm*

(c) 0.41 *cm* (d) 2.32 *cm*



Due to water Hg level drops by x in left side

In right side it will be ($2x$) from B point

[Pressure at A] = [Pressure At B]

$h\rho g = 2x\, \rho_{Hg}\, g$

$11.2 \times 1 = 2x \times 13.6$

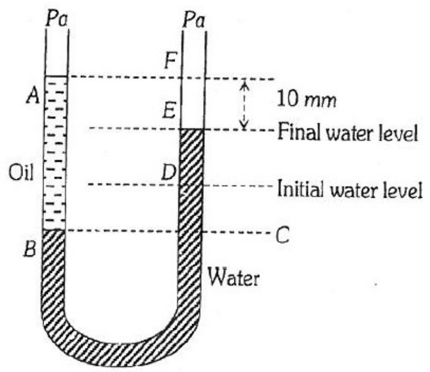
$x = \dfrac{11.2}{2 \times 13.6}$

$= 0.41$ cm Ans

It will rise by 0.41cm from initial level.

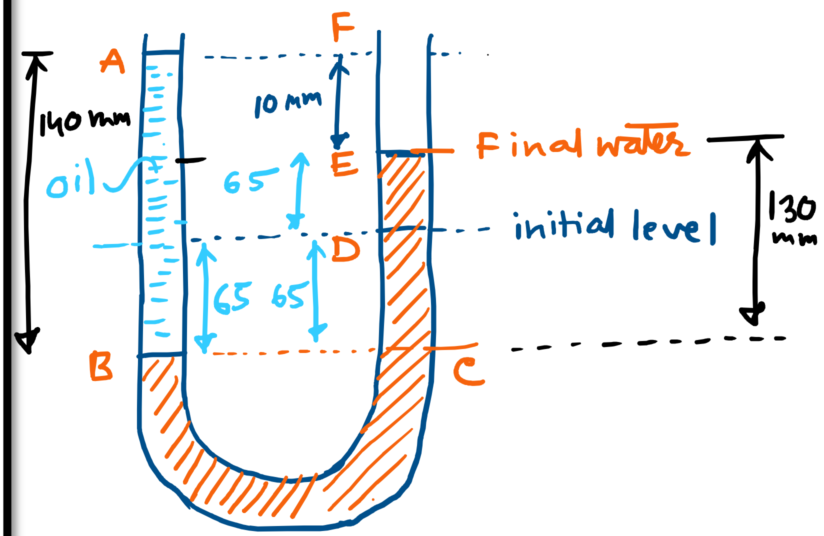
Q.4.

A *U* tube with both ends open to the atmosphere, is partially filled with water. Oil, which is immiscible with water, is poured into one side until it stands at a distance of 10 *mm* above the water level on the other side. Meanwhile the water rises by 65 *mm* from its original level (see diagram). The density of the oil is [2017]



(a) 650 $kg\,m^{-3}$ (b) 425 $kg\,m^{-3}$

(c) 800 $kg\,m^{-3}$ (d) 928 $kg\,m^{-3}$



Pressure at B = Pressure at C

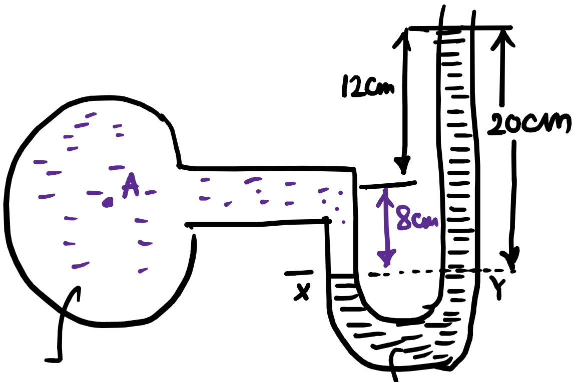
$h_1 \rho_{oil} \cdot \cancel{g} = h_2 \rho_w \ast \cancel{g}$

$140 \ast \rho_{oil} = 130 \ast 1000$

$\rho_{oil} = \dfrac{130 \times 1000}{140}$

$= 928$ kg/m^3

Q.5) The right limb of a simple U tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. Gr. 0.9 is flowing. The center of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury in the two limbs is 20 cm.



$\rho_1 = 0.9 \times 1000 = 900\ kg/m^3$

$\rho_{Hg} = 13.6 \times 1000\ kg/m^3$

For X-Y section

Pressure X = Pressure Y.

$P_A + h_1 \rho g = h_2 \rho g + P_{atm}$

$\boxed{P_A - P_{atm} = h_2 \rho g - h_1 \rho g}$

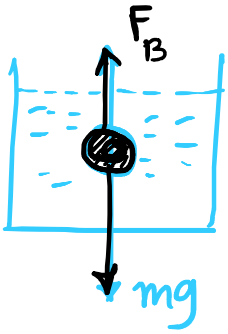
Gauge Pressure $= 0.2 \times 13.6 \times 1000 \times 10 - 900 \times 0.08 \times 10$

$= 25977\ N/m^2$

Archimedes' Principle

Archimedes' Principle states that when an object is immersed fully or partially in a liquid, it experiences an upward force which is equal to the weight of liquid displaced by the object

$F_b = V \rho_e g$

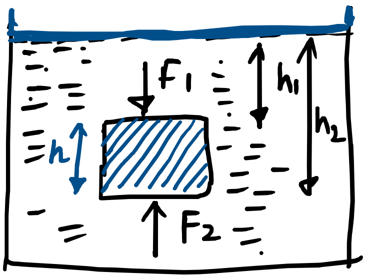


F_b = buoyancy

$= V \rho_e g$

(V) volume of object

Proof



$(h = h_2 - h_1)$

$F_1 = A P_1 = A(h_1 \rho g + P_{atm})$

$F_2 = A P_2 = A(h_2 \rho g + P_{atm})$

Upthrust $= F_2 - F_1 = A(h_2 - h_1)\rho g$

$= A \times h \rho g = V \rho g$

ρ Density of liquid

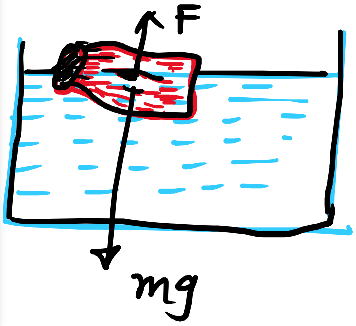
Q.6

A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is [2019]

(a) 0.6 (b) 0.5

(c) 0.8 (d) 0.7

Case 1

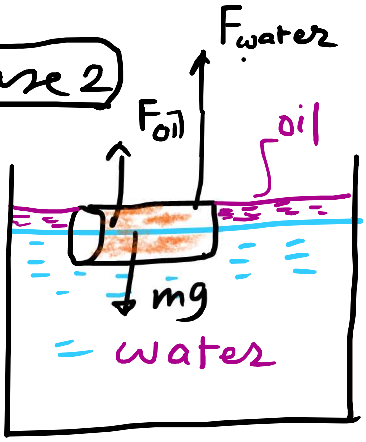


m = mass of wood.
$= V \times \rho_b$.
Density of wood

$F_U = mg$.

$$\left(\frac{4}{5}V\right)\rho_w * g = mg \quad -(1)$$

Case 2



upthrust due to oil + water will balance weight

$$F_{oil} + F_{water} = mg$$

$$\left(\frac{\cancel{V}}{2}\right)\rho_{oil}\, g + \left(\frac{\cancel{V}}{2}\right)\rho_w * g = \left(\frac{4}{5}V\rho_w g\right) \text{[From 1)}$$

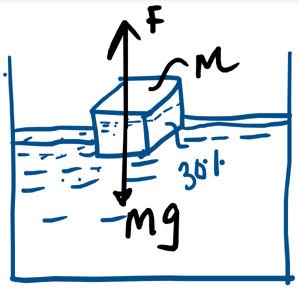
$$\frac{\rho_w}{2} + \frac{\rho_{oil}}{2} = \frac{4}{5}\rho_w \Rightarrow \text{solve } \boxed{\frac{\rho_{oil}}{\rho_w} = \frac{3}{5}}$$

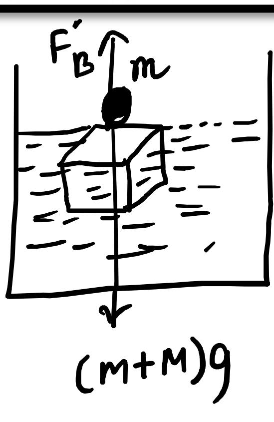
Q.7

A cubical block of side $0.5m$ floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water (Take density of water $= 10^3\ kg/m^3$) [2019]

(a) $87.5kg$ (b) $46.3kg$

(c) $30.1kg$ (d) $65.4kg$





$$Mg = F_B$$

$$Mg = \left(\frac{30}{100}V\right)\rho_l g \quad -(1)$$

$$(m+M)g = F_B'$$

$$(m+M)g = V\rho_l g$$

$$mg + Mg = V\rho_l g$$

$$mg = V\rho_l g - Mg \quad \text{[From 1]}$$

$$= V\, 1000 \times g - 0.3\, V \cdot 1000 \times g$$

$$m\cancel{g} = 0.7 \times (0.5)^3 \cdot 1000 \times \cancel{g}.$$

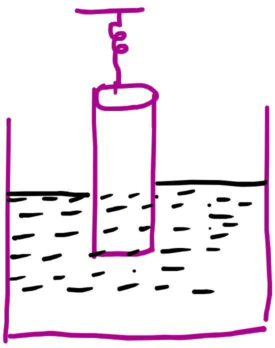
$$m = 87.5\,kg \text{ Ans}$$

8)

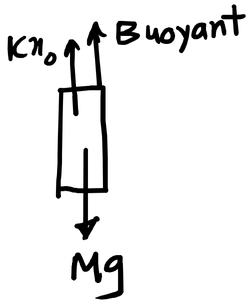
A uniform cylinder of length L and mass M having cross - sectional area A is suspended, with its length vertical, from a fixed point by a massless spring such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is **[2013]**

(a) $\frac{Mg}{k}$ (b) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$

(c) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$ (d) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$



F. B. D



$F_b + Kx_0 = Mg$

$\left\{\left(\frac{L}{2}\right)\cdot A\right\}\rho_l g + Kx_0 = Mg$

$Kx_0 = Mg - \frac{AL}{2}\rho_l g$

$x_0 = \frac{1}{K}\left(Mg - \frac{AL}{2}\rho_l g\right)$

$= \frac{Mg}{K}\left(1 - \frac{AL\rho_l}{2M}\right)$

$\rho_l \rightarrow$ density of liquid $= \sigma$

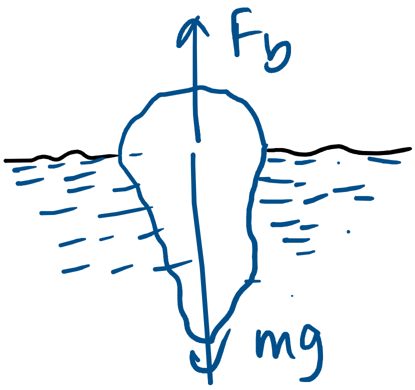
(C)

9

An iceberg of density 900 kg/m^3 is floating in water of density 1000 kg/m^3. The percentage of volume of ice-cube outside the water is

(a) 20% (b) 35%

(c) 10% (d) 25%



$\rho_{ice} = 900\ kg/m^3$.

$\rho_{water} = 1000\ kg/m^3$.

$F_b = V_{in}\,\rho_l * g$

$V_{in}\,\rho_l * g = M \cdot g$.

$V_{in}\,\rho_l * \cancel{g} = V \cdot \rho_{ice}\,\cancel{g}$.

$\frac{V_{in}}{V_{total}} = \frac{\rho_{ice}}{\rho_w} = \frac{900}{1000} = 0.9$.

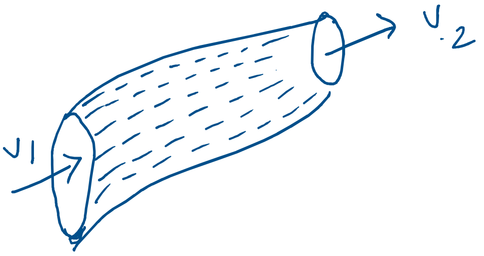
$V_{outside} = 0.1\,V_{total} \approx 10\%$ Ans

$V_{inside} = 0.9\,V_{total}$

Continuity Equations

Consider a non-viscous liquid in streamline flow through a tube AB of varying cross section as shown in Fig. Let a_1 and a_2 be the area of cross section, v1 and v2 be the velocity of flow of the liquid at A and B respectively.

Volume of liquid entering per second at A = a_1v_1. If **ρ** is the density of the liquid, then mass of liquid entering per second at A = $a_1v_1\rho$. Similarly, mass of liquid leaving per second at B = $a_2v_2\rho$.



$$a_1 v_1 \cancel{\rho} = a_2 v_2 \cancel{\rho} \quad \text{[For const mass flow]}$$

$$a_1 v_1 = a_2 v_2$$

$$\boxed{av = \text{Constant}}$$

☞ Bernoulli's theorem

According to Bernoulli's theorem, for the streamline flow of a non-viscous and incompressible liquid, the sum of the pressure energy, kinetic energy and potential energy per unit mass is a constant.

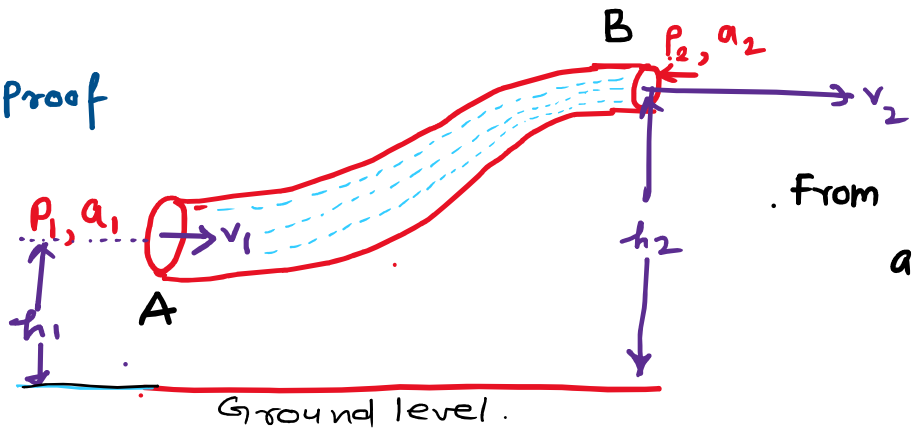
$$\frac{P}{\rho} + \frac{v^2}{2} + hg = \text{constant}$$

This equation is known as Bernoulli's equation.

The above formula can be writtenas ~ $\left[\frac{P}{\rho g} + \frac{v^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2\right]$

$$\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{Constant}$$

Proof



Streamline flow,

From continuity eq^n

$a_1 v_1 = a_2 v_2$ [mass flow constant]

$= V = \frac{m}{\rho}$

Force acting on liquid at A $= P_1 a_1$

work done per sec on liquid at A $= P_1 a_1 v_1 = P_1 V$

Similarly,

work done per sec on liquid at B $= P_2 a_2 v_2 = P_2 V$

Net work done per second on the liquid by the pressure energy in moving the liquid from A to B
$= P_1 V - P_2 V$

Increase in KE/sec of the liquid $= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

Increase in PE $= mgh_2 - mgh_1$

Now by the law of conservation of Mechanical Energy

$$\Delta W = \Delta U + \Delta K$$

$$P_1 V - P_2 V = mg(h_2 - h_1) + \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$P_1 V + mgh_1 + \frac{1}{2} m v_1^2 = P_2 V + mgh_2 + \frac{1}{2} m v_2^2$$

$$\frac{P_1 V}{m} + gh_1 + \frac{1}{2} v_1^2 = \frac{P_2 V}{m} + gh_2 + \frac{1}{2} v_2^2 \qquad \left[\rho = \frac{m}{V}\right]$$

Density

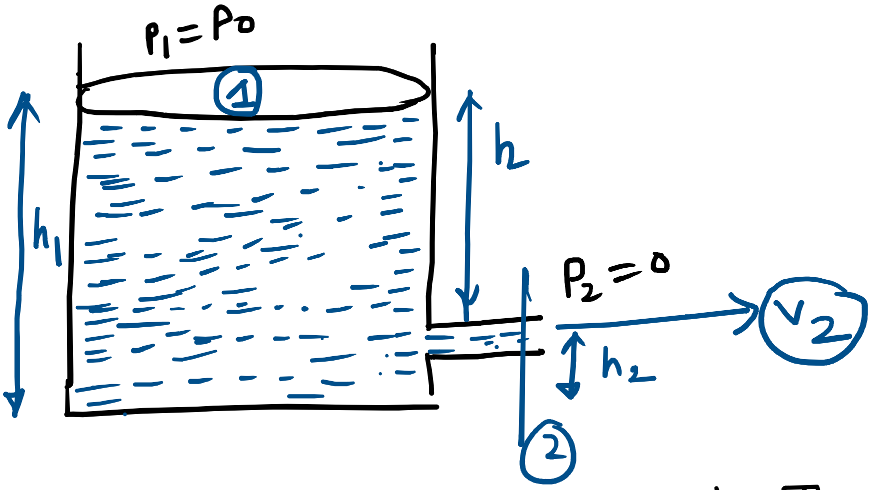
$$\frac{P_1}{\rho} + gh_1 + \frac{v_1^2}{2} = \frac{P_2}{\rho} + gh_2 + \frac{v_2^2}{2}$$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2$$

Bernoulli's Theorem

☞ Torricelli's law

Torricelli's law, also known as Torricelli's theorem, is a theorem in fluid dynamics relating the speed of fluid flowing out of an opening to the height of fluid above the opening



Applying Bernoulis Theorem at Point 1 & 2

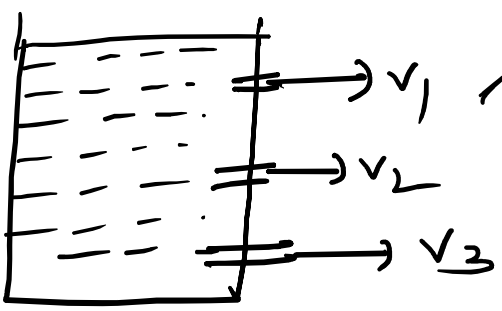
$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2$$

$$\cancel{\frac{P_0}{\rho g}} + \frac{0^2}{2g} + h_1 = \cancel{\frac{P_0}{\rho g}} + \frac{v_2^2}{2g} + h_2$$

[$A_1 >> A_2$ we are assuming $v_1 = 0$ at top]

$$\frac{v_2^2}{2g} = h_1 - h_2 = h$$

$$\boxed{v_2 = \sqrt{2gh}}$$ speed of efflux.



$v_3 > v_2 > v_1$

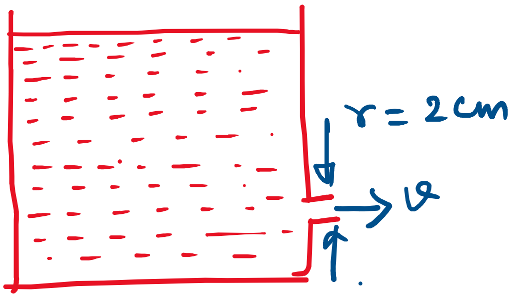
$$\boxed{v \propto \sqrt{h}}$$

Let's understand this concepts by solving few more numerical.....

10) The top of a water tank is open to air and its water level is maintained. It is giving out $0.74 m^3/min$ water per minute through a circular opening of $2cm$ radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to [2019]

(a) $2.9m$ (b) $9.6m$

(c) $4.8m$ (d) $6.0m$



$Q = 0.74 \, m^3/min$

$= \frac{0.74}{60} \, m^3/sec.$

$Q = A \cdot v$

$\frac{0.74}{60} = \pi r^2 \cdot v$

$= \pi \cdot \left(\frac{2}{100}\right)^2 \cdot v$

velocity = 9.81 m/s

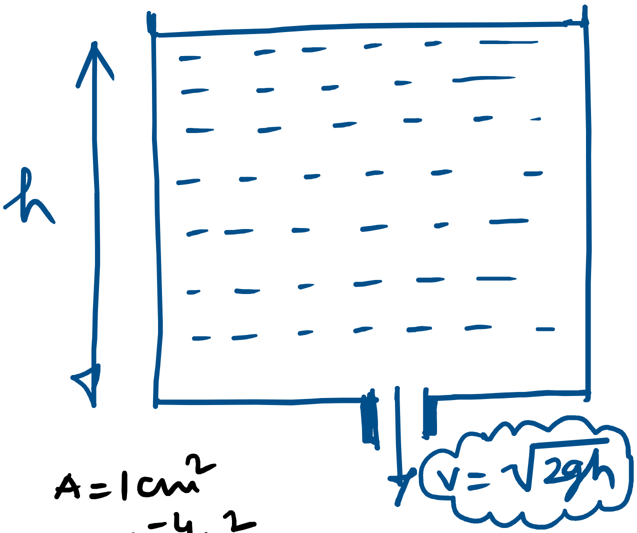
we know $v = \sqrt{2gh}$

$h = \frac{v^2}{2g} = \frac{9.81^2}{2 * 9.81} = \underline{4.8m}$

11) Water flows into a large tank with flat bottom at the rate of $10^{-4} m^3 s^{-1}$. Water is also leaking out of a hole of area $1cm^2$ at its bottom. If the height of the water in the tank remains steady, then this height is [2019]

(a) $1.7cm$ (b) $4cm$

(c) $2.9cm$ (d) $5.1cm$



z

$A = 1 cm^2$

$= 10^{-4} m^2$

Volume flow,

$Q = \text{area} \times \text{velocity}.$

$= A * \sqrt{2gh}$

$h = \frac{Q^2}{A^2 \, 2g} = \frac{(10^{-4})^2}{(10^{-4})^2 \, 2 \times 9.81}$

$= \underline{5.1 \, cm}$ Ans

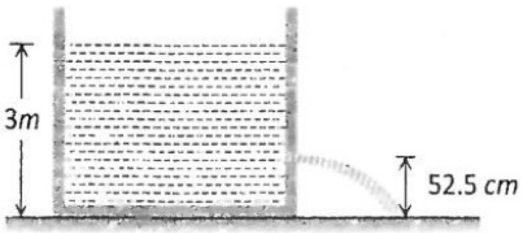
12) Water is filled in a cylindrical container to a height of 3m. The ratio of the cross-sectional area of the orifice and the beaker is 0.1. The square of the speed of the liquid coming out from the orifice is ($g = 10\ m/s^2$) **[2005]**

(a) 50 m^2/s^2

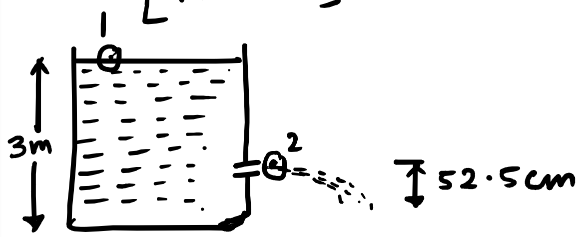
(b) 50.5 m^2/s^2

(c) 51 m^2/s^2

(d) 52 m^2/s^2



Given $\left[\frac{a}{A} = 0.1\right]$



$A v_1 = a v_2$ [continuity eqn.]

$v_1 = \frac{a}{A} \cdot v_2$

Applying Bernoulis theorem for (1) & (2)

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2$$ [$P_1 = P_2$ = atmospheric pressure Both are open]

$$\left(\frac{a}{A}\right)^2 \frac{v_2^2}{2g} + h_1 = \frac{v_2^2}{2g} + h_2$$

$$h_1 - h_2 = \frac{v_2^2}{2g} - \left(\frac{a}{A}\right)^2 \cdot \frac{v_2^2}{2g}$$

$$h = \frac{v_2^2}{2g}\left\{1 - \left(\frac{a}{A}\right)^2\right\}$$

$$v_2^2 = \frac{2gh}{1 - \left(\frac{a}{A}\right)^2} = \frac{2 \times 10 \times (3 - 0.525)}{1 - (0.1)^2}$$

$v^2 = 50\ (m/s)^2$ Ans

$\boxed{\text{Note}}$

if $a << A$

Then $v = \sqrt{2gh}$ (use)

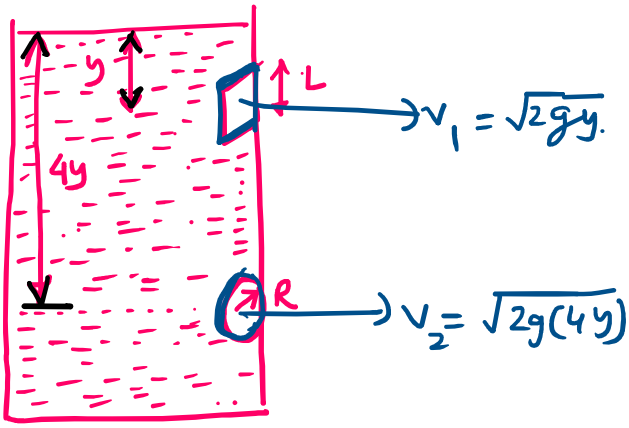
if $\frac{a}{A} \approx$ some const

$$\left[v = \sqrt{\frac{2gh}{1 - \left(\frac{a}{A}\right)^2}}\right] \leftarrow \text{use This}$$

13) A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water the quantities of water flowing out per second from both the holes are the same. Then R is equal to **[2000]**

(a) $2\pi L$ (b) $\frac{L}{\sqrt{2\pi}}$

(c) L (d) $\frac{L}{2\pi}$



Rate flow is same,

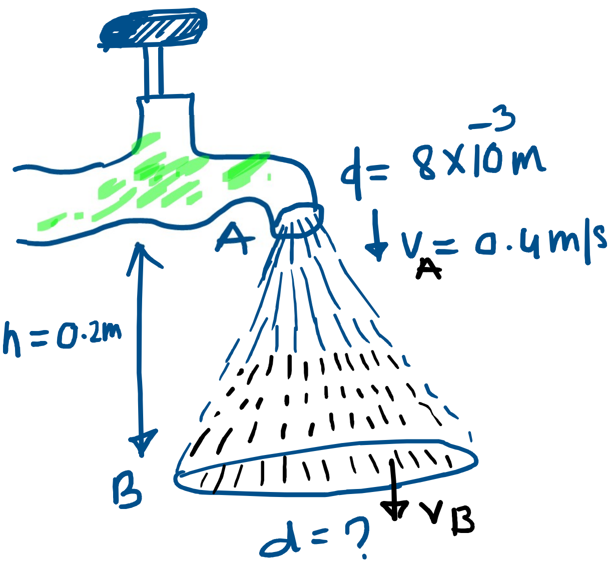
$\Rightarrow \sqrt{2gy} * L^2 = \sqrt{2g(4y)} * \pi R^2$

$\Rightarrow L^2 = 2\pi R^2 \longrightarrow \boxed{R = \frac{L}{\sqrt{2\pi}}}$

14) Water is flowing continuously from a tap having an internal diameter $8\times10^{-3}m$. The water velocity as it leaves the tap is $0.4\,ms^{-1}$. The diameter of the water stream at a distance $2\times10^{-1}m$ below the tap is close to **[2011]**

(a) $5.0\times10^{-3}m$ (b) $7.5\times10^{-3}m$

(c) $9.6\times10^{-3}m$ (d) $3.6\times10^{-3}m$



Let at B velocity of water is v_1

$v_B^2 = v_A^2 + 2gh$

$v_B = \sqrt{v_A^2 + 2gh}$

$= \sqrt{(0.4)^2 + 2*10\times0.2}$

$= \underline{2\,m/s}$

Now applying continuity eq^n

$a_1 v_1 = a_2 v_2$

$\pi\left(\frac{8\times10^{-3}}{2}\right)^2 \times 0.4 = \pi\frac{d^2}{4} * 2$

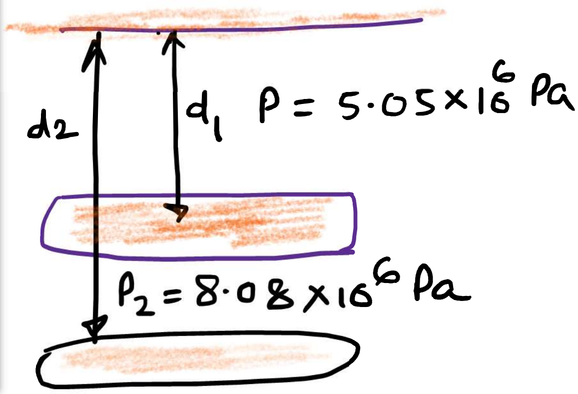
$\boxed{d = 3.6\times10^{-3}m}$ Ans

15)

A submarine experiences a pressure of $5.05\times10^{6}\,Pa$ at a depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of $8.08\times10^{6}\,Pa$. Then d_2-d_1 is approximately (density of water $=10^3\,kg/m^3$ and acceleration due to gravity $=10ms^{-2}$) **[2019]**

(a) $400m$ (b) $500m$

(c) $600m$ (d) $300m$

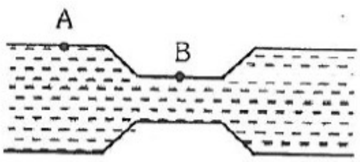


$$P_2 - P_2 = h_2\rho g - h_1\rho g$$
$$= (h_2 - h_1)\rho g$$

$\therefore \boxed{h_2 - h_1 = 303m}$ Ans.

16)

Water flows in a horizontal tube (see figure). The pressure of water changes by 700 Nm^{-2} between A and B where the area of cross section are 40 cm^2 and 20 cm^2, respectively. Find the rate of flow of water through the tube. (density of water = 1000 kgm^{-3}) **[2020]**

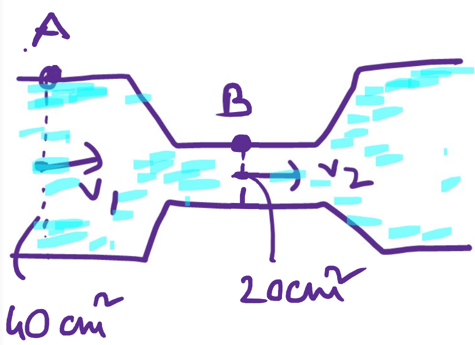


(a) 2720 cm^3/s

(b) 2420 cm^3/s

(c) 3020 cm^3/s

(d) 1810 cm^3/s



$$A_1 v_1 = A_2 v_2$$
$$40 * v_1 = 20 v_2$$
$$2v_1 = v_2$$
$$\boxed{v_1 = v_2/2}$$

Applying. Bernoulis eqn betn ① & ②

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + \cancel{h} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + \cancel{h}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$\frac{700}{1000\times10} = \frac{v_2^2}{2g} - \frac{v_2^2}{4*2g}$$

$$\frac{7\cancel{0}\cancel{0}}{10\cancel{0}\cancel{0}\times1\cancel{0}} = \frac{v_2^2}{2\times\cancel{10}}\left(1-\frac{1}{4}\right)$$

$$\frac{7}{\cancel{10}_5} = \frac{v_2^2}{\cancel{2}} * \frac{3}{4}$$

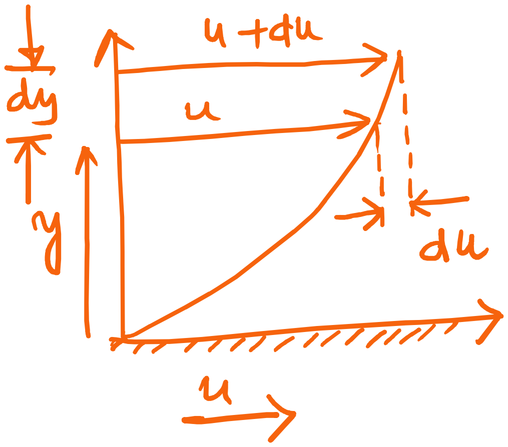
$$\frac{7\times4}{15} = v_2^2$$

$$v_2 = \sqrt{\frac{28}{15}} = 1.36 m/s.$$

$Q = A_2 v_2 = 136 \times 20 = 2732$ $\underline{cm^3/sec}$ A

Viscosity

Viscosity is defined as the property of fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of fluid. When two layers of fluid, a distance 'dy' apart, move one over other a different velocity, say u and u+du as shown in figure, the viscosity together with relative velocity causes a shear stress acting between two fluids.



$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$

[μ = coefficient of dynamic viscosity]

$$\boxed{\mu = \frac{\tau}{\frac{du}{dy}}}$$

Viscosity is also defined as the shear stress required to produce unit rate of shear strain

$$\tau = \mu \cdot \frac{du}{dy}$$

$$\frac{F}{A} = \mu \frac{du}{dy}.$$

$$F = \mu A \frac{du}{dy}. \quad \text{if } A = 1\,m^2$$

$$\frac{du}{dy} = 1\,s^{-1}$$

Then $\boxed{F = \mu}$.

The coefficient of viscosity of a liquid is numerically equal to the viscous force acting tangentially between two layers of liquid having unit area of contact and unit velocity gradient normal to the direction of flow of liquid.

Units of Viscosity

$$\mu = \frac{F}{A\,\frac{du}{dy}} = \frac{N}{m^2 \cdot \frac{m/s}{m}} = \frac{N\text{-}s}{m^2} \quad [SI]$$

CGS unit ~ $\frac{dyne\text{-}s}{cm^2}$

$$1 \text{ Poise} = \frac{1}{10} \frac{N\text{-}s}{m^2}$$

Viscosity

Dynamic Viscosity

$$F = \mu A \cdot \frac{du}{dy}$$

Dynamic viscosity.

Kinematic Viscosity

$$\nu = \frac{\mu}{\rho}$$

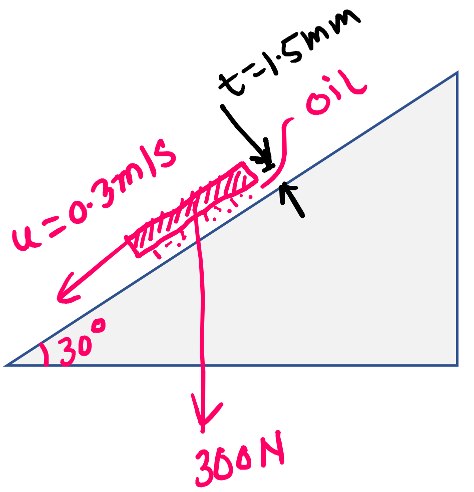
Kinematic viscosity

Newton's Law (Mathematical form)

$$\boxed{F = \mu A \frac{du}{dy}}$$

(17)

Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0.8m x0.8m and an inclined plane with angle of inclination 30 degree. The weight of the square plate is 300 N and it slides down the inclined plane with uniform velocity of 0.3 m/s. the thickness of oil film is 1.5 mm



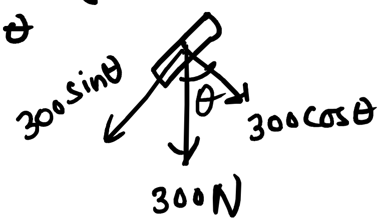
$A = 0.8 \times 0.8 = 0.64 m^2$

Force which is causing shear is $mg \sin\theta$

$\therefore F = 300 \sin\theta$

$= 300 \sin 30$

$= 150 N$



$F = \mu A \cdot \frac{du}{dy} \Rightarrow 150 = \mu * (0.64) * \frac{0.3}{1.5 \times 10^{-3}}$

$\mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3}$

$= 1.17 N\text{-}s/m^2$

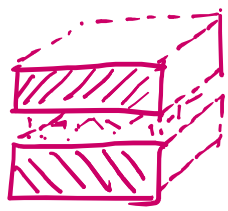
$= 11.7$ poise Ans

(18)

A square plate of 0.1 m side moves parallel to a second plate with a velocity of 0.1 m/s, both plates being immersed in water. If the viscous force is 0.002 N and the coefficient of viscosity is 0.01 *poise*, distance between the plates in m is

(a) 0.1 (b) 0.05

(c) 0.005 (d) 0.0005



Sol^n

$\mu = 0.01$ poise

$= 0.001 NS/m^2$

$\tau = \mu \frac{du}{dy}$

$dy = \frac{\mu}{\tau} du = \frac{\mu * A}{F} \cdot du$

$= \frac{0.001 * (0.1 \times 0.1)\, 0.1}{0.002}$

$= 0.0005 m$

Stokes Law

When a body falls through a highly viscous liquid, it drags the layer of the liquid immediately in contact with it. This results in a relative motion between the different layers of the liquid.

As a result of this, the falling body experiences a viscous force F. Stoke performed many experiments on the motion of small spherical bodies in different fluids and concluded that the viscous force F acting on the spherical body depends on

(i) Coefficient of viscosity **μ** of the liquid

(ii) Radius a of the sphere and

(iii) Velocity v of the spherical body.

Dimensionally it can be proved that

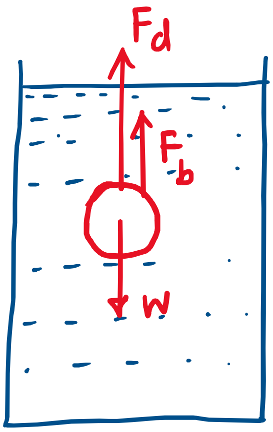
by: $F_d = 6\pi\mu a v$

$$\boxed{F_d = 6\pi\mu a v}$$

Stoke's Drag Formula

Terminal Velocity

Consider a metallic sphere of radius 'a' and density **ρl** to fall under gravity in a liquid of density **ρb**. The viscous force F acting on the metallic sphere increases as its velocity increases. A stage is reached when the weight W of the sphere becomes equal to the sum of the upward viscous force F_d and the upward thrust F_b due to buoyancy. Now, there is no net force acting on the sphere and it moves down with a constant velocity v called terminal velocity



$$W = F_d + F_b$$

$$mg = 6\pi\mu a \underset{\text{velocity}}{v} + \overset{\text{volume}}{V}\rho_l g$$

$$\frac{4}{3}\pi a^3 \cdot \rho_b \cdot g = 6\pi\mu a v + \frac{4}{3}\pi a^3 \rho_l g$$

$$\frac{4}{3}\cancel{\pi} a^3 g(\rho_b - \rho_l) = 6\cancel{\pi}\mu a v$$

$$v = \frac{2}{9} * \frac{a^2(\rho_b - \rho_l)g}{\mu}$$

Terminal velocity

19) If the terminal speed of a sphere of gold (density = 19.5 kg/m^3) is 0.2 m/s in a viscous liquid (density = 1.5 kg/m^3), find the terminal speed of a sphere of silver (density = 10.5 kg/m^3) of the same size in the same liquid **[2006]**

(a) 0.133 m/s (b) 0.1 m/s

(c) 0.2 m/s (d) 0.4 m/s

Sol^n

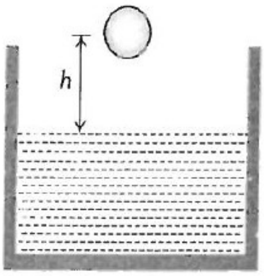
$$v = \frac{2}{9}\frac{r^2 g(\rho_b - \rho_l)}{\eta}$$

$$v \propto (\rho_b - \rho_l)$$

$$\frac{v_G}{v_S} = \frac{19.5-1.5}{10.5-1.5} = \frac{18}{9} = 2$$

$$v_S = \frac{v_G}{2} = \underline{0.1 \, m/s}$$

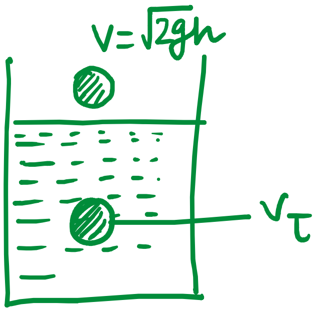
20) A ball of radius r and density ρ falls freely under gravity through a distance h before entering water. Velocity of ball does not change even on entering water. If viscosity of water is η, the value of h is given by



(a) $\frac{2}{9}r^2\left(\frac{1-\rho}{\eta}\right)g$ (b) $\frac{2}{81}r^2\left(\frac{\rho-1}{\eta}\right)g$

(c) $\frac{2}{81}r^4\left(\frac{\rho-1}{\eta}\right)^2 g$ (d) $\frac{2}{9}r^4\left(\frac{\rho-1}{\eta}\right)^2 g$

$u=0$



velocity of ball before hitting $v = \sqrt{2gh}$.

$$v_T = \frac{2}{9}\frac{r^2 g(\rho_b - 1)}{\eta}.$$

$\therefore$ velocity does not change

$$\sqrt{2gh} = \frac{2}{9}\frac{r^2 g(\rho_b - 1)}{\eta}$$

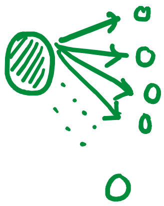
$$h = \frac{2}{81} r^4 \left(\frac{\rho_b - 1}{\eta}\right)^2 \cdot g.$$

Ans

21) A solid sphere of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity v_2 when falling through the same fluid, the ratio (v_1/v_2) equals [2019]

(a) 27 (b) 1/27

(c) 1/9 ✓(d) 9



$M = \rho \cdot \frac{4}{3}\pi R^3$

$\frac{4}{3}\pi R^3 = 27\left(\frac{4}{3}\pi r^3\right)$

Radius of small ball $\sim \boxed{r = \frac{R}{3}}$

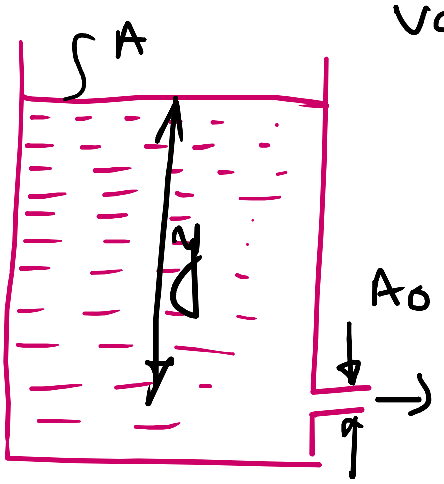
$v_T \propto (\text{Radius})^2$

$\boxed{\text{JEE - 2019}}$

$$\frac{v_T}{v_{new}} = \frac{R^2}{(R/3)^2} = 9.$$

$$v_{new} = \frac{v_T}{9} \quad \text{Ans}$$

Time Required to empty a tank



$\rightarrow v = \sqrt{2gy}$

volume coming out per second,

$$\frac{dv}{dt} = A_0 v$$

$$\frac{dv}{dt} = A_0\sqrt{2gy}$$

Now $dv = A\,dy$

$$A \cdot \frac{dy}{dt} = A_0\sqrt{2gy}.$$

$$dt = \frac{A}{A_0}\frac{1}{\sqrt{2g}}\, y^{-1/2} dy$$

$$\int dt = \frac{A}{A_0}\frac{1}{\sqrt{2g}}\int y^{-1/2} dy.$$

time taken to fall height H_i to H_f

$$t = \frac{A}{A_0}\frac{1}{\sqrt{2g}} \cdot \int_{H_i}^{H_f} y^{-1/2} dt = \frac{A}{A_0}\sqrt{\frac{2}{g}}\left(\sqrt{H_i} - \sqrt{H_f}\right) \quad \text{Ans}$$

If the hole is is at the bottom of the tank, then time required to empty the tank is given by

$$t = \frac{A}{A_0}\sqrt{\frac{2}{g}}\cdot(\sqrt{H}) \qquad \boxed{H_f = 0}$$

$$\boxed{t = \frac{A}{A_0}\sqrt{\frac{2H}{g}}} \quad \text{Ans}$$

Remember

22) A large tank filled with water to a height 'h' is to be emptied through a small hole at the bottom. The ratio of time taken for the level of water to fall from h to $\frac{h}{2}$ and from $\frac{h}{2}$ to zero is

(a) $\sqrt{2}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{2}-1$

(d) $\frac{1}{\sqrt{2}-1}$

Soln

$$t = \frac{A}{A_0}\sqrt{\frac{2}{g}}\cdot\left(\sqrt{H} - \sqrt{H'}\right)$$

$$\frac{t_1}{t_2} = \frac{\sqrt{h} - \sqrt{\frac{h}{2}}}{\sqrt{\frac{h}{2}} - 0} = \frac{1 - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \underline{\underline{\sqrt{2} - 1}} \quad \text{Ans}$$

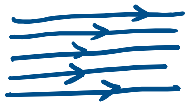
Units.

- $1\,Pa = 1\,N/m^2$
- $1\,psi = 7.015 \times 10^3\,Pa$
- $1\,atm = 760\,mm\text{ of }Hg = 1.013 \times 10^5\,Pa$
- $1\,Bar = 10^5\,Pa$
- $1\,Torr = 1\,mm\text{ of }Hg$

Note

Laminar Flow

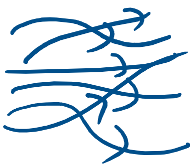
The flow is said to be steady, stream line or laminar if every particle of the liquid follows exactly the path of its preceding particle and has the same velocity of its preceding particle at every point



Critical velocity: Velocity up to which it is streamlined

Turbulent

When the velocity of a liquid exceeds the critical velocity. The path and velocities of the liquid become disorderly. At this stage the flow loses all its orderliness and is called turbulent.



Turbulent

Reynold's Number (Re)

It is a dimensionless number comprised of the physical characteristics of the flow. An increasing Reynolds number indicates an increasing turbulence of flow.

It is defined as:

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho V D}{\mu}$$

- ρ – density
- V – velocity.
- D – Dia of pipe
- μ – coeff of viscosity

Laminar flow: For practical purposes, if the Reynolds number is less than 2000, the flow is laminar. The accepted transition Reynolds number for flow in a circular pipe is Re = 2300.

Transitional flow: At Reynolds numbers between about 2000 and 4000 the flow is unstable as a result of the onset of turbulence. These flows are sometimes referred to as transitional flows.

Turbulent flow: If the Reynolds number is greater than 3500, the flow is turbulent. Most fluid systems in nuclear facilities operate with turbulent flow.

23) Water from a pipe is coming at a rate of 100 liters per minute. If the radius of the pipe is $5\,cm$, the Reynolds number for the flow is the order of (density of water $= 1000\,kg/m^3$, coefficient of viscosity of water $= 1\,mPa\,s$) [2019]

(a) 10^6
(b) 10^4
(c) 10^3
(d) 10^2

$\mu = 1\,mPas$

$\tau = \mu \frac{du}{dy}$

$\frac{NS}{m^2} = \mu$.

Sol^n

Rate of volume flow $= \pi r^2 v = 100\ l/min$

$v = \frac{100 \times 10^{-3}}{60} \cdot \frac{1}{3.14 \times 25 \times 10^{-4}}$

$= 2/3\pi\ m/s$

$Re = \frac{\rho v D}{\mu}$

$= \frac{1000 \times}{10^{-3}} \frac{2}{3\pi} \cdot \underline{10 \times 10^{-2}}$

$= 10^4 \times 2$ order 10^4

$\mu = 10^{-3}\ \frac{N\text{-}S}{m^2}$ Imp

24) The water flows from a tap of diameter 1.25 cm with a rate of $5\times10^{-5} m^3s^{-1}$. The density and coefficient of viscosity of water are $10^3 kg\, m^{-3}$ and 10^{-3} Pas, respectively. The flow of water is

(a) Steady with Reynolds number 5100

(b) Turbulent with Reynolds number 5100 ✓

(c) Steady with Reynolds number 3900

(d) Turbulent with Reynolds number 3900

$$Re = \frac{\rho V D}{\mu} \qquad Q = \frac{\pi D^2}{4} * V$$

$$V = \frac{4Q}{\pi D^2}$$

$$= \frac{\rho}{\mu} \frac{4Q}{\pi D^2} . D$$

$$= \frac{4Q\rho}{\pi D \mu}$$

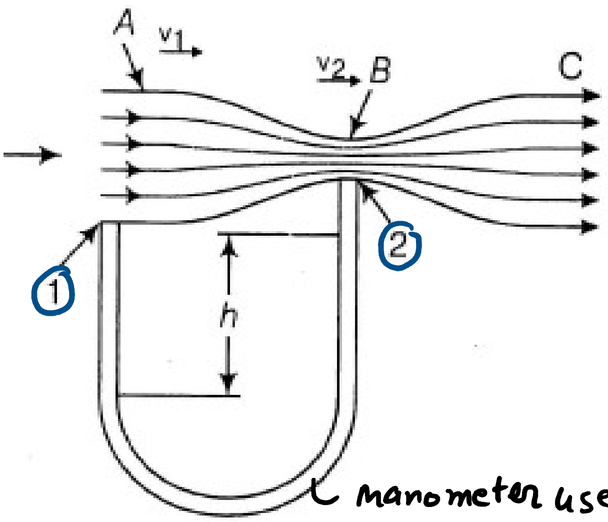
$$= \frac{4 * 5\times10^{-5} \times 1000}{3.14 * 1.25\times10^{-2} * 10^{-3}}$$

$$= 5100$$

Turbulent (b)

Venturimeter

It is used to measure flow of liquid in tube



Find Velocity at 2

For 1 & 2 using continuity eq^n

$$A v_1 = a v_2$$

$$v_1 = \frac{a}{A} v_2$$

manometer used to measure pr. diff. bet^n 1 & 2

$[\Delta P = h \rho_m g]$

For 1 & 2

or $$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + \cancel{h_1} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + \cancel{h_2} \qquad [h_1 = h_2 = h]$$

or $$\frac{P_1 - P_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad — 1$$

or $$\frac{P_1 - P_2}{\rho \cancel{g}} = \frac{v_2^2}{2g} - \frac{1}{2g}\left(\frac{a}{A}\right)^2 v_2^2 = \frac{v_2^2}{2\cancel{g}}\left[1 - \left(\frac{a}{A}\right)^2\right]$$

ρ – density of flowing fluid
ρ_m – manometric fluid density

or $$\frac{2(P_1-P_2)}{\rho\left\{1-\left(\frac{a}{A}\right)^2\right\}} = v_2^2$$

or $$v_2 = \sqrt{\frac{2(P_1-P_2)}{\left\{1-\left(\frac{a}{A}\right)^2\right\}\rho}} = \sqrt{\frac{2(gh\rho_m)}{\left(1-\left(\frac{a}{A}\right)^2\right)\rho}}$$

Similarly,

$$\frac{P_1-P_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$= \left(\frac{A}{a}\right)^2\frac{v_1^2}{2g} - \frac{v_1^2}{2g}$$

$$\frac{h\rho_m g}{\rho g} = \frac{v_1^2}{2g}\left\{\left(\frac{A}{a}\right)^2-1\right\}$$

$$v_1 = \sqrt{\frac{2h\rho_m g}{\rho}}\left\{\left(\frac{A}{a}\right)^2-1\right\}^{-1/2}$$ Ans

25\. The heart of man pumps 5 litres of through the arteries per minute at a pressure of 150 mm of mercury. If the density of mercury be $13.6\times10^3 kg/m^3$ and $g=10m/s^2$ the power of heart in watt is [2015]

(a) 2.35 (b) 3.0

(c) 1.50 (d) 1.70

$$\text{Power} = \frac{W}{t} = \frac{F.S}{t} = \frac{F}{A}\cdot\frac{AS}{t} = P * \frac{V}{t} \approx P.\frac{dV}{dt}$$

Pumping rate $\frac{dv}{dt} = \frac{5\times10^{-3}}{60}$

$$\text{Power} = P.\frac{dv}{dt} = h\rho g.\frac{dv}{dt}$$

$$= \frac{0.15 * 13.6\times10^3 * 10 * 5\times10^{-3}}{60}$$

$= 1.7$ watt Ans