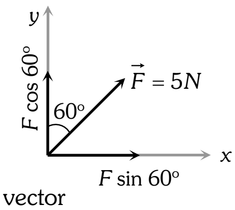
1. (d) Component of vector (force) along vertical axis

$= 5\cos 60° = 5 \times \frac{1}{2} = 2.5N.$



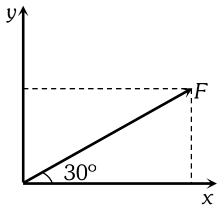
2. (b) Rotation of axis do not change vector magnitude.

3. (c) $\vec{F} = F_x\hat{i} + F_y\hat{j}$

$= (F\cos\theta)\hat{i} + (F\sin\theta)\hat{j}$

$\theta = 30°$

$\vec{F} = \left(\frac{\sqrt{3}}{2}F\right)\hat{i} + \left(\frac{1}{2}F\right)\hat{j}.$



4. (b) $0.4\hat{i} + 0.8\hat{j} + c\hat{k} = \sqrt{(0.4)^2 + (0.8)^2 + c^2} = 1$

$\Rightarrow c^2 = 1 - (0.16 + 0.64)$

$\Rightarrow c = \sqrt{0.2}.$

5. (a) $\vec{r} = \vec{a} + \vec{b} + \vec{c} = 4\hat{i} - \hat{j} - 3\hat{i} + 2\hat{j} - \hat{k} = \hat{i} + \hat{j} - \hat{k}$

Univector $\hat{r} = \frac{\vec{r}}{|r|} = \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{1^2 + 1^2 + (-1)^2}} = \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}.$

6. (a) Vector $\vec{C} = \frac{\vec{A}}{A} \times |\vec{B}|$

$= \frac{(\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{(1)^2 + (2)^2 + (2)^2}} \times \sqrt{3^2 + (6)^2 + (2)^2}$

$= \frac{7}{3}(\hat{i} + 2\hat{j} + 2\hat{k}).$

7. (a) $\vec{A} = (A\cos\alpha)\hat{i} + (A\cos\beta)\hat{j} + (A\cos\gamma)\hat{k} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\cos\alpha = \frac{A_x}{A}$; $\cos\beta = \frac{A_y}{A}$; $\cos\gamma = \frac{A_z}{A}$

Since,

$\vec{A} = 2\hat{i} + 4\hat{j} - 5\hat{k}$; $|\vec{A}| = A = \sqrt{2^2 + 4^2 + (-5)^2} = \sqrt{45}$

$\cos\alpha = \frac{2}{\sqrt{45}}$; $\cos\beta = \frac{4}{\sqrt{45}}$; $\cos\gamma = \frac{-5}{\sqrt{45}}.$

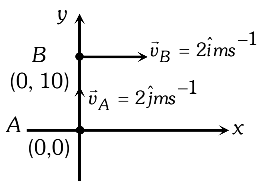
8. (a) Position after time 't'

Position of A is $(0, \upsilon_A t)$

Position of B is $(\upsilon_B t, 10)$

distance between them –

$l^2 = (0 - \upsilon_B t)^2 + (\upsilon_A t - 10)^2$



$\Rightarrow \quad l^2 = 4t^2 + 4t^2 + 100 - 40t = 8t^2 - 40t + 100$

For maxima and minima

$\frac{dl}{dt} = 0 \Rightarrow 16t - 40 = 0 \Rightarrow t = \frac{40}{16} = 2.5\,s$

Now,

$\frac{d^2l}{dt^2} = 16(+ve) > 0$; t value gives minima.

9. (b) $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$

$= (-4i - 5j + 5\hat{k}) + (5\hat{i} + 8\hat{j} + 6\hat{k}) + (-3\hat{i} + 4\hat{j} - 7\hat{k})$

$+ (2\hat{i} - 3\hat{j} - 2\hat{k})$

$= 4\hat{j} + 2\hat{k}$ Move in yz plane.

10. (a) $|\vec{A} - \vec{B}|^2 = |\vec{A}|^2$

$\Rightarrow \quad A^2 + A^2 - 2A^2\cos\theta = A^2$

$\Rightarrow \quad \cos\theta = \frac{A^2 - 2A^2}{-2A^2} = \frac{1}{2} \Rightarrow \theta = 60°.$

11. (b) $A_x = 12m$; $A_y = 8m$ $\quad [\vec{A} = A_x\hat{i} + A_y\hat{j}]$

Magnitude of vector

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{12^2 + 8^2} = \sqrt{208}\,m$

After rotation

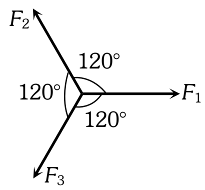
$A = \sqrt{A'^2_x + A'^2_y} \Rightarrow \sqrt{208} = \sqrt{\left(\frac{A_x}{2}\right)^2 + A'^2_y}$

$\Rightarrow \quad \sqrt{208} = \sqrt{(6)^2 + A'^2_y}$

$\Rightarrow \quad A'^2_y = 208 - 36 = 172$

$\Rightarrow \quad A'_y = \sqrt{172} = 13.11m.$

12. (b) If N forces or vector of equal magnitude, act on a point and resultant is zero.



Then angle between any two forces or vector

$= \frac{360}{N} = \frac{360}{3} = 120°.$

13. (c) $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$; given $A = 2B$, $R = A$

$\Rightarrow \quad A^2 = A^2 + \left(\frac{A}{2}\right)^2 + 2A\frac{A}{2}\cos\theta$

$\Rightarrow \quad -\frac{A^2}{4} = A^2\cos\theta \quad \Rightarrow \quad \theta = \cos^{-1}\left(-\frac{1}{4}\right).$

14. (a) $R^2 = A^2 + B^2 + 2AB\cos\theta$

$A = (x + y)$; $B = (x - y)$; $R = \sqrt{x^2 + y^2}$

$x^2 + y^2 = (x + y)^2 + (x - y)^2 + 2(x + y)(x - y)\cos\theta$

$\Rightarrow x^2 + y^2 = 2x^2 + 2y^2 + 2(x^2 - y^2)\cos\theta$

$\cos\theta = \frac{-(x^2 + y^2)}{2(x^2 - y^2)} \Rightarrow \theta = \cos^{-1}\left[\frac{-(x^2 + y^2)}{2(x^2 - y^2)}\right].$

15. (b) $|\vec{A}+\vec{B}|^2 = n^2|\vec{A}-\vec{B}|^2 \Rightarrow |A+B| = n|\vec{A}-\vec{B}|$

$\Rightarrow A^2+B^2+2AB\cos\theta = n^2A^2+n^2B^2-2n^2AB\cos\theta$

$\Rightarrow A^2+A^2+2A^2\cos\theta = n^2A^2+n^2A^2-2n^2A^2\cos\theta$

$\Rightarrow A^2[2+2\cos\theta] = A^2[2n^2-2n^2\cos\theta]$

$\Rightarrow 2-2n^2 = (-2-2n^2)\cos\theta$

$\Rightarrow \cos\theta = \dfrac{1-n^2}{-1-n^2} = \dfrac{n^2-1}{n^2+1}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{n^2-1}{n^2+1}\right).$

16. (b) Net force $\vec{F} = 4\hat{i}+\hat{j}-3\hat{k}+3\hat{i}+\hat{j}-\hat{k}$

$= (7\hat{i}+2\hat{j}-4\hat{k})N$

Displacement $\vec{S} =$ final position – initial position

$= (5\hat{i}+4\hat{j}-\hat{k})-(\hat{i}+2\hat{j}-3\hat{k})$

$= 4\hat{i}+2\hat{j}+2\hat{k}$

Work done $W = \vec{F}.\vec{S}$

$= 28+4-8 = 24J.$

17. (a) Component of $\vec{A}$ along $\vec{B} = \dfrac{\vec{A}.\vec{B}}{|\vec{B}|} = \dfrac{(2\hat{i}+3\hat{j}).(\hat{i}+\hat{j})}{\sqrt{1^2+1^2}}$

$= \dfrac{2+3}{\sqrt{2}} = \dfrac{5}{\sqrt{2}}.$

18. (c) Unit vector perpendicular to $\vec{A}$ & $\vec{B}$

$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$

$\hat{n} = \dfrac{\vec{A}\times\vec{B}}{AB\sin\theta}.$

19. (c) $\vec{A} = 2\hat{i}+2\hat{j}-\hat{k}$; $B = 6\hat{i}-3\hat{j}+2\hat{k}$

$$\vec{C} = \vec{A}\times\vec{B} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 2 & -1\\ 6 & -3 & 2\end{vmatrix} = \hat{i}-10\hat{j}-18\hat{k}$$

Unit vector perpendicular to both $\vec{A}$ and $\vec{B}$

$\hat{C} = \dfrac{\vec{C}}{|\vec{C}|} = \dfrac{\hat{i}-10\hat{j}-18\hat{k}}{\sqrt{1^2+(-10)^2+(-18)^2}}$

$= \dfrac{\hat{i}-10\hat{j}-18\hat{k}}{5\sqrt{17}}.$

20. (d) Vector $\vec{A} = 2\vec{i}+\vec{j}-\vec{k}$ and vector $\vec{B} = \vec{i}+\vec{j}+\vec{k}$

Area of the triangle $= \dfrac{1}{2}\left|(\vec{A}\times\vec{B})\right|$

$$= \frac{1}{2}\left|\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -1\\ 1 & 1 & 1\end{vmatrix}\right|$$

$= \dfrac{1}{2}|(2\hat{i}-3\hat{j}+\hat{k})|$

$= \dfrac{1}{2}\sqrt{4+9+1} = \dfrac{\sqrt{14}}{2}\,sq.unit.$

21. (d) $$\vec{C} = \vec{A}\times\vec{B} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & -1\\ -1 & 3 & 4\end{vmatrix} = 15\hat{i}-7\hat{j}+9\hat{k}$$

Unit vector perpendicular to both $\vec{A}$ and $\vec{B}$

$\hat{C} = \dfrac{\vec{C}}{|\vec{C}|} = \dfrac{15\hat{i}-7\hat{j}+9\hat{k}}{\sqrt{15^2+(-7)^2+(9)^2}} = \dfrac{1}{\sqrt{355}}(15\hat{i}-7\hat{j}+9\hat{k}).$

22. (d) $\vec{A}.\vec{B} = |\vec{A}\times\vec{B}|$

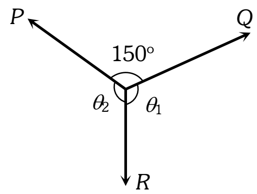
$\Rightarrow AB\cos\theta = AB\sin\theta \Rightarrow \tan\theta = 1 \Rightarrow \theta = 45°$

$|\vec{C}| = \sqrt{A^2+B^2+2AB\cos 45°} = \sqrt{A^2+B^2+2AB\times\dfrac{1}{\sqrt{2}}}$

$|\vec{C}| = \sqrt{A^2+B^2+\sqrt{2}\,AB}.$

23. (c) Minimum four non-zero, non-coplanar vector required to make resultant zero.

24. (c) $\dfrac{P}{\sin\theta_1} = \dfrac{Q}{\sin\theta_2} = \dfrac{R}{\sin 150°}$



$\therefore \quad R = \dfrac{1.9318\times\sin 150}{0.9659}$

$= \dfrac{1.9318\times\frac{1}{2}}{0.9659} = 1.$

25. (d) Since, metal sphere is in equilibrium

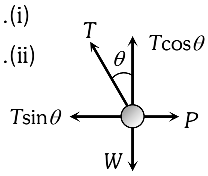
$\vec{T}+\vec{P}+\vec{W} = 0$

Now, $W = T\cos\theta$ ....(i)

$P = T\sin\theta = W\tan\theta$ ....(ii)

by (i)^2 + (ii)^2

$W^2+P^2 = T^2.$



26. (2) If $\vec{P}$ make angle α, β, γ respectively with *x*, *y*, *z-axis*.

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma = 1$

$\Rightarrow \quad 1-\sin^2\alpha+1-\sin^2\beta+1-\sin^2\gamma = 1$

$\Rightarrow \quad \sin^2\alpha+\sin^2\beta+\sin^2\gamma = 2.$

27. (13) $R_{max} = A+B = 17$; $\theta = 0$

$R_{min} = A-B = 7$; $\theta = 180°$

$A = 12, \quad B = 5$

if $\theta = 90°$; $\quad R = \sqrt{A^2+B^2} = \sqrt{12^2+5^2} = 13.$

28. (3) $\vec{C} = \vec{A}+\vec{B}$

Since, $\vec{C}\perp\vec{A}$ and $|\vec{C}| = |\vec{A}|$

$\tan\alpha = \dfrac{B\sin\theta}{A+B\cos\theta} \Rightarrow \tan 90° = \dfrac{B\sin\theta}{A+B\cos\theta}$

$\Rightarrow \quad A+B\cos\theta = 0 \Rightarrow \cos\theta = -\dfrac{A}{B}$

$C^2 = A^2+B^2+2AB\cos\theta$

$\Rightarrow \quad A^2 = A^2 + B^2 + 2AB\left(\frac{-A}{B}\right) \Rightarrow -B^2 = -2A^2$

$\Rightarrow \quad A = +\frac{1}{\sqrt{2}}B \quad \Rightarrow \frac{A}{B} = \frac{1}{\sqrt{2}}$

$\cos\theta = -\frac{1}{\sqrt{2}} = 135° = \frac{3\pi}{4}$ radian

Aliter: Since, $\vec{C} \perp \vec{A}$ [Right angle triangle]

$B^2 = A^2 + C^2 \Rightarrow B^2 = 2A^2 \Rightarrow B = \sqrt{2}A$

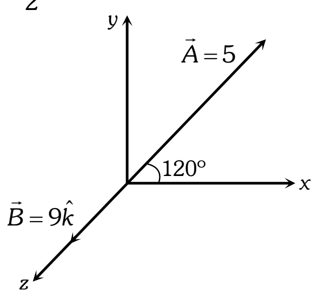
Now, $A^2 + B^2 + 2AB\cos\theta = C^2 = A^2$

$2A\sqrt{2}A\ \cos\theta = -A^2 \Rightarrow \cos\theta = -\frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{3\pi}{4}\ rad.$

29. (45) $\vec{A} = (5\cos 120)\hat{i} + (5\sin 120)\hat{j}$

$= \frac{-5}{2}\hat{i} + \frac{5\sqrt{3}}{2}\hat{j}$

$\vec{B} = 9\hat{k}$



$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{-5}{2} & \frac{5\sqrt{3}}{2} & 0 \\ 0 & 0 & 9 \end{vmatrix} = -\left(\frac{-45}{2}\right)\hat{j} + \left(\frac{45\sqrt{3}}{2}\right)\hat{i}$$

$= \frac{45\sqrt{3}}{2}\hat{i} + \frac{45}{2}\hat{j}$

$|\vec{A}\times\vec{B}| = \sqrt{\left(\frac{45\sqrt{3}}{2}\right)^2 + \left(\frac{45}{2}\right)^2} = \frac{45}{2}\sqrt{4} = 45.$

30. (3) We know that the volume (V) of the parallelepiped is the scalar triple product of three given vectors,

$\vec{A} = (2\hat{i} + 3\hat{j} + 4\hat{k}); \vec{B} = 4\hat{j}$

and $\quad \vec{C} = (5\hat{j} + m\hat{k})$.

Here, $\vec{B} = 0\hat{i} + 4\hat{j} + 0\hat{k}; \vec{C} = 0\hat{i} + 5\hat{j} + m\hat{k}$;

Volume V = 24.

Volume of parallelepiped is given by

$$V = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \begin{vmatrix} 2 & 3 & 4 \\ 0 & 4 & 0 \\ 0 & 5 & m \end{vmatrix}$$

= 2 [4m − 0] + 3 [0 − 0] + 4 [0 − 0]

= 8m

$\therefore$ 24 = 8m or m = 3

1. (d) $F = \eta A \frac{\Delta v}{\Delta r} \Rightarrow \eta = \frac{kgms^{-2} \times m}{m^2 \times ms^{-1}} = kgm^{-1}s^{-1}$.

2. (c) $EMF(e) = L\frac{di}{dt} \Rightarrow L = \frac{e}{di/dt}$

$\Rightarrow L = \frac{volt \times \sec}{Ampere} = ohm \sec$.

3. (a) Energy (E) = $F \times d \Rightarrow F = \frac{E}{d}$

So, Erg/metre can be the unit of force.

4. (c) $[x] = [bt^2] \Rightarrow [b] = [x/t^2] = \text{km/s}^2$

5. (a) $\frac{L}{R} = \left(\frac{volt \times \sec}{A}\right).\frac{A}{volt} = \sec$

It is also called time constant in L-R circuit.

6. (b) $F = \frac{1}{4\pi\varepsilon_0}.\frac{q_1 q_2}{r^2}$ and $1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{volt}}$

$\Rightarrow \varepsilon_0 = \frac{q_1 q_2}{4\pi F r^2} = \frac{\text{Coulomb}}{\left(\frac{Nm}{\text{Coulomb}}\right)m} = \frac{1 \text{coulomb}}{1 \text{volt}.1 \text{meter}}$.

or $C = \varepsilon_0 \frac{A}{d} \Rightarrow \varepsilon_0 = Fm^{-1}$.

7. (b) $B = \mu_0 \frac{N}{l} I \Rightarrow \frac{F}{I.l} = \frac{\mu_0 NI}{l} \Rightarrow \mu_0 = \frac{F}{I^2}$.

8. (d) Thermal Conductance $= \frac{1}{\text{Thermal resistance}} = \frac{1}{R}$

$R = \frac{L}{KA} = \frac{\text{metre}}{(\text{watt metre}^{-1}\text{ kel}^{-1})(\text{metre})^2} = \frac{\text{kelvin}}{\text{watt}}$

$K \rightarrow$ coefficient of thermal conductivity

9. (d) Angular momentum of $e = n\frac{h}{2\pi}$, $n \rightarrow$ integer.

10. (c) $ct^3 = x \Rightarrow [c][T^3] = [L] \Rightarrow [c] = [LT^{-3}]$.

11. (a) $\frac{L}{R}$ = time constant , Frequency $= \frac{R}{L}$.

12. (d) Volume Elasticity $= \frac{\frac{F}{A}}{\frac{\Delta V}{V}} = \frac{\Delta P}{\frac{\Delta V}{V}} = [ML^{-1}T^{-2}]$.

13. (b) Pressure $= \frac{\text{Force}}{\text{Area}} = \frac{\text{Energy}}{\text{Volume}}$.

14. (b) Specific Heat (s) $= \frac{\Delta Q}{m\Delta T} = \frac{[ML^2T^{-2}]}{[M][\theta]} = [L^2T^{-2}\theta^{-1}]$.

15. (d) [Planck constant] $= [ML^2T^{-1}]$,

[Energy] $= [ML^2T^{-2}]$.

Aliter : Energy $= hf$, $f \rightarrow$ frequency.

16. (d) Surface tension $= \frac{F}{l} = [MT^{-2}]$

Spring constant $= \frac{F}{x} = [MT^{-2}]$.

17. (a) Angle of banking $\Rightarrow \tan\theta = \frac{v^2}{rg}$.

18. (d) $n_2 = n_1 \left[\frac{L_1}{L_2}\right]^1 \left[\frac{T_1}{T_2}\right]^{-2} = 10\left[\frac{m}{10^3 m}\right]^1 \left[\frac{\sec}{3600 \sec}\right]^{-2}$

Aliter : $10ms^{-2} = 10\left(\frac{1}{1000}km\right)\left[\frac{1}{3600}hr\right]^{-2}$

$= 129600 \, km/hr^2$.

19. (a) $T = 2\pi\sqrt{\frac{R^3}{GM}} \Rightarrow \left[\frac{L^3}{M^{-1}L^3T^{-2}M}\right]^{\frac{1}{2}} = T$.

20. (a) $\frac{e^2}{4\pi\varepsilon_0 hc} = \frac{[AT]^2}{[M^{-1}L^{-3}T^4A^2][ML^2T^{-1}][LT^{-1}]}$

$= [M^0L^0T^0]$.

21. (c) $m \propto C^a G^b h^c \Rightarrow m \propto C^{1/2} G^{-1/2} h^{1/2}$

$[M] = [LT^{-1}]^a [M^{-1}L^3T^{-2}]^b [ML^2T^{-1}]^c$

$= [M^{-b+c} L^{a+3b+2c} T^{-a-2b-c}]$

$-b + c = 1$ ...(i)

$a + 3b + 2c = 0$ ...(ii)

$-a - 2b - c = 0$ ...(iii)

By equation (ii) and (iii)

$b + c = 0$ ...(iv)

By equation (iv) and (i)

$c = \frac{1}{2}$; $b = -\frac{1}{2}$

From equation (iii) $-a + 1 - \frac{1}{2} = 0 \Rightarrow a = \frac{1}{2}$.

22. (a) $v \propto \eta^x r^y l^z p^w$

Poiseuille's formula

Volume of liquid coming out of the tube per second

$\Rightarrow V = \frac{\pi p r^4}{8\eta l}$.

23. (b) In a correct equation, both sides should have same dimensions. Kinetic energy which has the SI unit kg-m^{2}s^{-2} should have the dimensions,

Right hand side of the option has the dimensions,

$\left[\frac{1}{2}mv^2\right] = [M]\left[LT^{-1}\right]^2 = \left[ML^2T^{-2}\right]$

These dimensions match with the dimensions of kinetic energy.

24. (b) $g = \frac{GM}{R^2} : K = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$

Further, L will remain constant.

$\therefore K \propto \frac{1}{I}$ or $K \propto \frac{1}{\frac{2}{5}MR^2}$ or $K \propto R^{-2}$ and $g \propto R^{-2}$

25. (c) m is mass per unit length.

26. (50) 1 C.G.S unit of density $= 1g/cm^3$

$$= \frac{1}{1000}kg \times \frac{1}{\left(\frac{1}{100}m\right)^3}$$

$= 1000kg/m^3$ in S.I or M.K.S. system

$0.5g/cc = 0.5 \times 1000\,kg/m^3$

$\Rightarrow 0.5\,g/cm^3 = 500\,kg/m^3.$

27. (50) $M_e = 6.64 \times 10^{24}kg$

$n_1u_1 = n_2u_2$

$\Rightarrow \quad n \times 40amu = 6.64 \times 10^{24}kg$

$\Rightarrow \quad n \times 40 \times 1.6 \times 10^{-27}kg = 6.64 \times 10^{29}kg$

$$n = \frac{6.64 \times 10^{29}}{40 \times 1.6 \times 10^{-27}} \approx 10^{50}.$$

28. (2) Dimension of $B = [M^0L^0T^0]$

Dimensions e $= [AT]$

Dimensions $\varepsilon_0 = [A^2M^{-1}L^{-3}T^4]$

Dimensions h $= [ML^2T^{-1}]$

Dimension c $= [LT^{-1}]$

$$\therefore \quad [M^0L^0T^0] = \frac{[AT]^n}{[A^2M^{-1}L^{-3}T^4][ML^2T^{-1}][LT-1]}$$

or $\quad [M^0L^0T^0] = [A^{n-2}L^0T^{n-2}]$

$\therefore \quad$ n − 2 = 0

$\Rightarrow \quad$ n = 2

29. (3) $[K] = \frac{[U]}{[X]^n} = \frac{[ML^2T^{-2}]}{[L^n]} = [ML^{2-n}T^{-2}]$

$T \propto (mass)^X(amp)^Y(K)^Z$

$[M^0L^0T] = [M]^X[L]^Y[ML^{2-n}T^{-2}]^Z$

$= [M^{X+Z}L^{Y+2Z-nZ}T^{-2Z}]$

So, $\;-2z = 1$ or $Z = -\frac{1}{2}$

and X + Z = 0 or X = −Z = $\frac{1}{2}$

and Y+ 2Z – Nz = 0

or Y = 1 – $\frac{n}{2}$ [from (i) and (ii)]

So, $\;T \propto (mass)^{1/2}(amp)^{1-n/2}(K)^{-1/2}$

Given $T \propto (\text{amp})^{-1/2}$

So $\;1 - \frac{n}{2} = -\frac{1}{2} \Rightarrow n = 3$

30. (2) $E = KT^a\rho^bp^c$

LHS = $[ML^2T^{-2}]$

RHS = $[T^a][ML^{-3}]^b[ML^{-1}T^{-2}]^c$

$= [M^{b+c}L^{-3b-c}T^{a-2c}]$

$[ML^2T^{-2}] = [M^{b+c}L^{-3b-c}T^{a-2c}]$

According to homogeneity principle

LHS = RHS

$\therefore$ b + c = 1 …(i)

−3b – c = 2 …(ii)

and a – 2c = 2 …(iii)

Adding Eqs. (i) & (ii)

−2b = 3

$\Rightarrow \; b = \frac{-3}{2}$

$\therefore$ c = 1 – b = 1 +3/2 = $\frac{5}{2}$

From Eq. (iii), a – 2c = −2

or a = −2 + 2c = −2 + 2 ×5/2 =3

$\therefore \; \frac{a^2c}{5b^2} = 2$

1. (b) $KE = \frac{1}{2}mv^2$

$\Rightarrow \left(\frac{\Delta KE}{KE}\right)\% = \left(\frac{\Delta m}{m} + 2\frac{\Delta v}{v}\right)\%$

$\Rightarrow \left(\frac{\Delta KE}{KE}\right)\% = 2 + 2\times 3 = 8\%.$

2. (d) $T = 2.5\,\text{sec},\ \Delta T = \frac{1}{2}\,\text{sec} = 0.5\,\text{sec}$

The permissible error

$\frac{\Delta T}{T}\times 100 = \frac{0.5}{2.5}\times 100 = 20\%.$

3. (c) $\%\ \text{error} = \frac{\Delta x}{x}\times 100\%$ [unit less].

4. (a) $\left(\frac{\Delta l}{l}\right)\times 100 = 2\%$

area $(A) = l^2$

$\Rightarrow \left(\frac{\Delta A}{A}\right)\times 100 = 2\left(\frac{\Delta l}{l}\times 100\right) = 4\%.$

5. (c) $r = (5.3 \pm 0.1)\,cm$

Volume $(V) = \frac{4}{3}\pi r^3$

$\frac{\Delta V}{V} = 3\frac{\Delta r}{r}$

$\frac{\Delta V}{V}\times 100 = 3\left(\frac{0.1}{5.3}\right)\times 100.$

6. (d) $P = \frac{F}{A} = \frac{F}{l^2}$

$\left(\frac{\Delta P}{P}\right)\times 100 = \left(\frac{\Delta F}{F} + 2\frac{\Delta l}{l}\right)\times 100$

$= 4 + 2\times 2 = 8\%.$

7. (b) $T_{mean} = \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5}$

$= 2.62\,sec$

$\Delta T_{mean} = \frac{|\Delta T_1| + |\Delta T_2| + |\Delta T_3| + |\Delta T_4| + |\Delta T_5|}{5}$

$= \frac{0.01 + 0.06 + 0.20 + 0.09 + 0.18}{5}$

$= \frac{0.54}{5} = 0.11\,sec.$

8. (b) $H = I^2Rt$

$\Rightarrow \frac{\Delta H}{H}\times 100 = \left(\frac{2\Delta I}{I} + \frac{\Delta R}{R} + \frac{\Delta t}{t}\right)\times 100$

$= 2\times 3 + 4 + 6 = 16\%$

Hence the error in the measured of H is 16%.

9. (a) $X = M^aL^bT^C$

$\Rightarrow \left(\frac{\Delta x}{x}\right)\times 100 = \left(a\frac{\Delta m}{m} + b\frac{\Delta L}{L} + c\frac{\Delta T}{T}\right)\times 100$

$= (a\alpha + b\beta + c\gamma)\%.$

10. (a) $\rho = \frac{\text{Mass}}{\text{Volume}} \Rightarrow \frac{\Delta\rho}{\rho}\times 100 = \left(\frac{\Delta m}{m} + \frac{\Delta V}{V}\right)\times 100$

$= \left(\frac{0.05}{5} + \frac{0.05}{1.0}\right)\times 100$

$= (0.01 + 0.05)\times 100 = 6\%.$

11. (b) $\text{Distance}(d) = (13.8 \pm 0.2)m$

$\text{Time}\,(t) = (4.0 \pm 0.3)\,sec$

$\text{Velocity}(v) = \frac{d}{t} = \frac{13.8}{4.0} = 3.45\,m/s$

$\frac{\Delta v}{v} = \frac{\Delta d}{d} + \frac{\Delta t}{t}$

$\Rightarrow \Delta v = \left[\frac{0.2}{13.8} + \frac{0.3}{4.0}\right]\times 3.45 = 0.27 \approx 0.3$

$\therefore v = (3.45 \pm 0.3)\,m/s.$

12. (c) $Y = \frac{4MgL}{\pi D^2 l}$

$\left(\frac{\Delta Y}{Y}\right)\times 100 = \left(\frac{\Delta m}{m} + \frac{\Delta g}{g} + \frac{\Delta L}{L} + \frac{2\Delta D}{D} + \frac{\Delta l}{l}\right)\times 100$

$= \left(\frac{0.01}{3.00} + \frac{0.01}{9.81} + \frac{0.001}{2.820} + 2\times\frac{0.001}{0.041} + \frac{0.001}{0.087}\right)\times 100$

$= 0.065\times 100 = 6.5\%.$

13. (c) $R = \frac{R_1R_2}{(R_1 + R_2)} = \frac{10\times 6}{10 + 6} = \frac{60}{16}\,K\Omega = 3.75\,K\Omega$

$\Rightarrow \frac{\Delta R}{R} = \frac{\Delta R_1}{R_2} + \frac{\Delta R_2}{R_2} + \frac{\Delta(R_1 + R_2)}{(R_1 + R_2)}$

$= \frac{0.3}{6} + \frac{0.2}{10} + \frac{(0.3\times 0.2)}{10 + 6} = 0.10125$

$\frac{\Delta R}{R} = 10.125\%$

14. (d) The number of significant figures in given numbers is 4.

15. (b) Volume $(V) = l\times b\times t$

$\Rightarrow 12\times 6\times 2.45 = 176.4\,cm^3$

$= 1.764\times 10^2 cm^3 \approx 2\times 10^2 cm^3$

When two number multiplied or divided. Then result is in least significant figure.

16. (d) Given, A = 1.0 m ± 0.2 m, = 2.0 m ± 0.2 m

Let, $Y = \sqrt{AB} = \sqrt{(1.0)(2.0)} = 1.414\,m$

Rounding off to two significant digits Y = 1.4m

$\frac{\Delta Y}{Y} = \frac{1}{2}\left[\frac{\Delta A}{A} + \frac{\Delta A}{B}\right] = \frac{1}{2}\left[\frac{0.2}{1.0} + \frac{0.2}{2.0}\right] = \frac{0.6}{2\times 20}$

$\Rightarrow \Delta Y = \frac{0.6Y}{2\times 2.0} = \frac{0.6\times 1.4}{2\times 20} = 0.212$

Rounding off to one significant digit, $\Delta Y = 0.2m$

Thus, correct value for $\sqrt{AB} = Y + \Delta Yr = 1.4 \pm 0.2M$

17. (a) All given measurements are correct up to two decimal places. As here 5.00 mm has the smallest unit and the error in 5.00 mm is least (commonly taken as 0.01 mm if not specified), hence 5.00 mm is most precise.

Note : In solving these types of questions, we should be careful about units although their magnitude is same.

18. (a) Given length, $l = 5$ cm

Now, checking the errors with each option one by one, we get

$\Delta l_1 = 5 - 4.9 = 0.1\text{cm}$, $\Delta l_2 = 5 - 4.805 = 0.195\text{cm}$

$\Delta l_3 = 5.25 - 5 = 0.25\text{cm}$, $\Delta l_4 = 5.4 - 5 = 0.4\text{cm}$

Error Δl_1 is least.

Hence, 4.9 cm is most precise.

19. (c) By question, it is given that 50 VSD – 49 MSD

$$1\text{ VSD} = \frac{49}{50}\text{MSD}$$

Minimum inaccuracy = 1 MSD – 1 VSD

$$= 1\text{ MSD} - \frac{49}{50}\text{MSD} = \frac{1}{50}\text{MSD}$$

Given, 1 MSD = 0.5 mm

Hence, minimum inaccuracy = $\frac{1}{50} \times 0.5\text{mm}$

$$= \frac{1}{100} = 0.01\text{mm}$$

20. (b) Given, $t_1 = 39.6$ s, $t_2 = 39.9$ and $t_3 = 39.5$ s

Least count of measuring instrument = 0.1 s

(As measurements have only one decimal place) precision in the measurement = Least count of the measuring instrument = 0.1 s

Mean value of time for 20 oscillations is given by

$$t = \frac{t_1 + t_2 + t_3}{3} = \frac{39.6 + 39.9 + 39.5}{3} = 39.7\text{s}$$

Absolute errors in the measurements

$$\Delta t_1 = t - t_1 = 39.7 - 39.6 = 0.1\text{s}$$

$$\Delta t_2 = t - t_2 = 39.7 - 39.9 = -0.2\text{ s}$$

$$\Delta t_3 = t - t_3 = 39.7 - 39.5 = 0.2\text{ s}$$

Mean absolute error = $\frac{|\Delta t_1| + |\Delta t_2| + |\Delta t_3|}{3}$

$$= \frac{0.1 + 0.2 + 0.2}{3}$$

$$= \frac{0.5}{3} = 0.17 = 0.2$$

(rounding off up to one decimal place)

$\therefore$ Accuracy of measurement $= \pm 0.2$ s

21. (c) $p = \frac{A^3 B^{1/2}}{C^{-4} D^{3/2}}$, Because C has maximum power

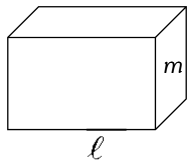
22. (d) $f = \frac{uv}{u+v}, \frac{\Delta f}{f} = \frac{\Delta u}{u} + \frac{\Delta u}{v} + \frac{\Delta(u+v)}{u+v}$

$$= \frac{\Delta u}{u} + \frac{\Delta v}{v} + \frac{\Delta u}{u+v} + \frac{\Delta v}{u+v}$$

23. (b) $\rho = \frac{m}{V} = \frac{m}{\ell^3}$

Given: $\frac{\Delta m}{m} = \pm 2\% = \pm 2 \times 10^2$

$\frac{\Delta \ell}{\ell} = \pm 1\% = 1 \times 10^{-2}$



$\frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} + 3\frac{\Delta \ell}{\ell}$

$= 2 \times 10^{-2} + 3 \times 10^{-2}$

$= 5 \times 10^{-2} = 5\%$

24. (a) $g = 4\pi^2 \frac{L}{T^2}$

$\frac{\Delta L}{L} = \pm 2\% = \pm 2 \times 10^{-2}$

$\frac{\Delta T}{T} = \pm 3\% = \pm 3 \times 10^{-2}$

$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T} = 2 \times 3 \times 10^{-2}$

$= 8 \times 10^{-2} = \pm 8\%$

25. (d) $\frac{\Delta x}{x} = 1\% = 10^{-2}$

$\frac{\Delta y}{y} = 3\% = 3 \times 10^{-2}$

$\frac{\Delta z}{z} = 2\% = 2 \times 10^{-2}$

$\Rightarrow t = \frac{xy^2}{z^3}$

$\frac{\Delta t}{t} = \frac{\Delta x}{x} + \frac{2\Delta y}{y} + \frac{3\Delta z}{z}$

$= 10^{-2} + 2 \times 3 \times 10^{-2} + 3 \times 2 \times 10^{-2}$

$= 13 \times 10^{-2} = 13\%$

26. (8) $z = ax^2 y^{1/2}$

$\Rightarrow \quad x^2 = \frac{z}{ay^{1/2}}$

$\Rightarrow \quad 2\frac{\Delta x}{x} = \frac{\Delta z}{z} + \frac{1}{2}\frac{\Delta y}{y}$

$\Rightarrow \quad \frac{\Delta x}{x} = \frac{1}{2}\left[10 + \frac{1}{2} \times 12\right]$

$= 8\%$.

27. (5) $\because \quad g = \frac{GM}{R^2}$

$\therefore \quad \frac{\Delta g}{g} = \frac{\Delta M}{M} + \frac{2\Delta R}{R}$

Percentage error in g is

$\frac{\Delta g}{g} \times 100 = \frac{\Delta M}{M} \times 100 + \frac{2\Delta R}{R} \times 100$

$= 1 + 4 = 5\%$

28. (7) $\because \ T = \frac{P^2}{2m} \, or \, \frac{\Delta T}{T} = 2\frac{\Delta p}{p} + \frac{\Delta m}{m}$

Percentage error in kinetic energy is

$\frac{\Delta T}{T} \times 100 = 2\frac{\Delta p}{p} \times 100 + \frac{\Delta m}{m} \times 100$

$= 2 \times 2 + 3 = 7\%$

$\therefore$ n = 7

29. (1) $\because \quad y = 2.21 \times 0.3 = 0.663 = 0.7$

$\therefore$ The number of significant digit is 1.

30. (2) $x = 0.72 + 0.8 + 3.87 - 1.089$

$\Rightarrow 0.7 + 0.8 + 3.9 - 1.1 = 4.3$

Thus, the number of significant digit is 2.

1. (d) Velocity = slope of displacement – time graph $= \tan\theta$

$$\frac{v_A}{v_B} = \frac{\tan\theta_A}{\tan\theta_B} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}.$$

2. (b) average speed $\geq$ average velocity
Because distance $\geq$ displacement.

3. (b) Here velocity is decreasing, so acceleration will be opposite to the direction of motion and hence, it will be negative w.r.t the direction of velocity till it comes to rest

4. (a) Uniform velocity means no acceleration

5. (d) $v_{avg} = \frac{\text{Total distance}}{\text{Total time}} = \frac{x}{\frac{2x}{5v_1} + \frac{3x}{5v_2}} = \frac{5v_1 v_2}{2v_2 + 3v_1}.$

6. (a) $v_{avg} = \frac{200}{\frac{100}{60} + \frac{100}{v}}$

$\Rightarrow 40 = \frac{200 \times 60 \times v}{100v + 6000}$

$\Rightarrow 400v + 24000 = 1200v$

$\Rightarrow v = \frac{24000}{800} = 30 km/hr.$

7. (d) Total distance = x
Time taken by the particle to travel first half of the distance. i.e., $t_1 = \frac{x/2}{3} = \frac{x}{6}$ sec
Let t_2 be the time taken by the particle to travel other half of the distances

$$\frac{x}{2} = 4.5 \times \frac{t_2}{2} + 7.5 \times \frac{t_2}{2}$$

$$\Rightarrow t_2 = \frac{x}{12} \text{ sec}$$

$$v_{avg} = \frac{x}{\frac{x}{6} + \frac{x}{12}} = \frac{6 \times 12}{6 + 12} = \frac{72}{18}$$

$$= 4 m/s.$$

8. (a) Speed of the body at the mid-point

$$v = \sqrt{\frac{v_1^2 + v_2^2}{2}} = \sqrt{\frac{(20)^2 + (30)^2}{2}}$$

$$v = \sqrt{650} = 25.5 m/s.$$

9. (d) Since, with respect of time, position does not change

10. (a) $v^2 = u^2 + 2as \Rightarrow a = \frac{v^2 - u^2}{2s} = \frac{(640)^2}{2 \times 1.2}$

$v = u + at \Rightarrow t = \frac{v}{a} = \frac{640}{(640)^2} 2 \times 1.2 = 3.75 \times 10^{-3}$ sec

$= 4 ms.$

11. (b) $u = 0,\ v = 180 \times \frac{5}{18} = 50 m/s$

$a = \frac{v - u}{t} = \frac{50 - 0}{10} = 5 m/s^2$

Distance $(S) = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2} \times 5 \times (10)^2 = 250 m.$

12. (b) $u = 10 m/s,\ v = -2 m/s\ :\ t = 4$ sec

Acceleration $= \frac{v - u}{t} = \frac{-2 - 10}{4} = -3 m/s^2.$

13. (a) $u = 0,\ a = 5 m/s^2$

$v = u + at \Rightarrow v = 0 + 5 \times (10) = 50 m/s.$

14. (b) Distance travelled by car A$=40t$

distance travelled by car B $= \frac{1}{2} \times 4 \times t^2$

$40t = 2t^2 \Rightarrow t(2t - 40) = 0 \Rightarrow t = 0$ and $t = 20$ sec.

15. (c) $S = ut + \frac{1}{2}at^2$

First $5\ sec : t = 5$ sec

$40 = 5u + \frac{1}{2}a(5)^2 \Rightarrow 16 = 2u + 5a$ ....(i)

$t = 10$ sec, $S = 40 + 65 = 105 m$

$105 = 10u + \frac{1}{2}a(10)^2 \Rightarrow 21 = 2u + 10a$ ....(ii)

Multiply eq.(i) by 2 then subtract it from eq. (2)

$21 - 32 = 2u - 4u + 10a - 10a$

$-11 = -2u \Rightarrow u = 5.5 m/s.$

16. (c) $a = \frac{v - u}{t} = \frac{27.5 - 0}{10} = 2.75 m/s^2$

distance travelled in next 10 sec

$$S = (27.5 \times 10) + \frac{1}{2} \times 2.75 \times (10)^2$$

$$= 275 + 137.5 = 412.5.$$

17. (a) Distance covered in n^{th} second

$S_n = u + \frac{a}{2}(2n - 1) \Rightarrow S_5 = 7 + \frac{4}{2}(10 - 1) = 25 m.$

18. (d) $\because v = 0 + na \Rightarrow a = v/n$

Now, distance travelled in $nsec.$

$\Rightarrow S_n = \frac{1}{2}an^2$ and, distance travelled in $(n-2)$sec

$\Rightarrow S_{n-2} = \frac{1}{2}a(n-2)^2$

$\therefore$ Distance travelled in last two seconds,

$= S_n - S_{n-2} = \frac{1}{2}an^2 - \frac{1}{2}a(n-2)^2$

$= \frac{a}{2}[n^2 - (n-2)^2] = \frac{a}{2}[n + (n-2)][n - (n-2)]$

$= a(2n - 2) = \frac{v}{n}(2n - 2) = \frac{2v(n-1)}{n}.$

19. (b) $v = \frac{dx}{dt} = 32 - 8t^2$

$v = 0$ at $t = 2$s $\qquad a = \frac{dv}{dt} = -16t$

At 2s, $\qquad a = -32$m/s^2

20. (d) Integrate twice to convert a-t equation into s-t equation.

21. (c) $x = 3t^3$ | $y = 4t^3$

$\Rightarrow \quad v_x = \frac{dx}{dt} = 9t^2$ | $v_y = \frac{dy}{dt} = 12t^2$

$v = \sqrt{v_x^{\ 2} + v_y^{\ 2}} = \sqrt{(9t^2)^2 + (12t^2)^2} = 15t^2.$

22. (b) Distance travelled = area under $v-t$ graph in time interval 20 to 40 sec

$= \frac{1}{2} \times 20 \times 3 + 20 \times 1 = 50\,m.$

23. (b) Total distance $= \frac{1}{2} \times 10 \times 2 + 2 \times 10 + \frac{1}{2} \times 2 \times 10 = 40m$

Distance travelled in last 2sec $= \frac{1}{2} \times 2 \times 10 = 10m$

$\frac{S_{(last\ 2\ sec)}}{S_7} = \frac{10}{40} = \frac{1}{4}.$

24. (a) $v = gt \quad ; \quad u = 0.$

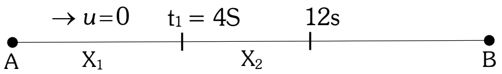
25. (a) $S = ut + \frac{1}{2}at^2$, if $u = 0,\ S = \frac{1}{2}at^2.$

26. (8) $\frac{da}{dt} = 1 \Rightarrow a = t$

Let at t = t_1, a = 4 $\Rightarrow t_1$ = 4s

$\rightarrow a = 0$

$\rightarrow u = 0$



$\frac{dv}{dt} = t \Rightarrow v = \frac{t^2}{2}$

$At\ t = t_1, v_1 = \frac{4^2}{2} = 8m/s$

$\frac{dx}{dt} = \frac{t^2}{2} \Rightarrow x = \frac{t^3}{6}$

Let at t = t_2, v_2 = 144 km/hr = 40 m/s

$v_2 = v_1 + at_2 \Rightarrow 40 = 8 + 4\,t_2$

$\Rightarrow t_2 = 8s\ x_1 = \frac{4^3}{6} = \frac{32}{8}m$

$x_2 = 8 \times 8 + \frac{1}{2} \times 4(8)^2 = 192$ Km

Distance for which it runs with uniform velocity:

S = 2000 – 2 (x_1 + x_2) $t_3 = \frac{s}{40}$

Total time $T = t_1 + t_2 + t_3 + t_2 + t_1 = 64$ s

27. (1) $\vec{r} = (t^2 - 4t + 6)\hat{i} + t^2\hat{j}\,;\vec{v} = \frac{d\vec{r}}{dt}$

$= (2t-4)\hat{i} + 2t\,\hat{j}.\vec{a} = \frac{d\vec{v}}{dt} = 2\hat{i} + 2\hat{j}$

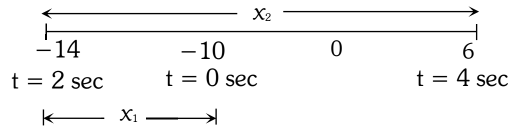
If the vectors are perpendicular

$\vec{a}, \vec{v} = 0$

$(2\hat{i} + 2\hat{j}).((2t-4)\hat{i} + 2t\,\hat{j}) = 0$

8t – 8 = 0, t = 1 sec

28. (4)



$x = t^3 - 3t^2 - 10$

$v = \frac{dx}{dt} = 3t^2 - 6t$

v = 0 gives

t = 0 and t = 2 sec

Velocity will become zero at t = 2 sec, so particle will change direction after t = 2 sec.

At t = 0

$x_{(0\,sec)} = -10$

At t = 2 sec

$x_{(2sec)} = 2^3 - 3(2)^2 - 10$

= 8 – 12 – 10 = –14

At t = 4 sec

$x_{(4sec)} = 4^3 - 3(4)^2 - 10$

= 64 – 48 – 10 = 6

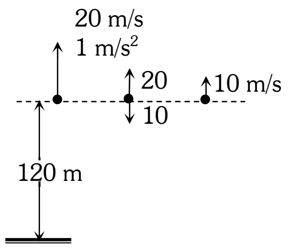
Distance travelled = $x_1 + x_2$

$= |-14 - (-10)| + |6 - (-14)|$

= 4 + 20 = 24

Distance travelled = 24 units = 6 × 4 units.

29. (6)

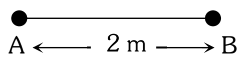


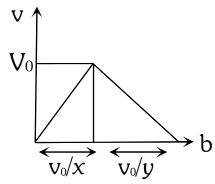
$-120 = 10t\frac{1}{2} - 10 \times t^2$

$t^2 - 2t - 24 = 0$

t = 6 sec

30. (4)





Total time taken = 4 min

(i) $\frac{v_0}{x} + \frac{v_0}{y} = 4$ min

(ii) Total distance travelled = 2 km

$\Rightarrow$ Area under $v-t$ graph = 2 km

$\frac{1}{2} \times \frac{v_0}{x} \times v_0 + \frac{1}{2} \times \frac{v_0}{y} \times v_0 = 2$ km

From (i) and (ii), $\frac{1}{x} + \frac{1}{y} = 4$

1. (d) Relative velocity of A w.r.t. $B = 20 - (-30) = 50\,m/s$

$$\text{time} = \frac{120+130}{50} = 5\,\text{sec}.$$

2. (a) When two particle moves towards each other

$$v_1 + v_2 = 6$$

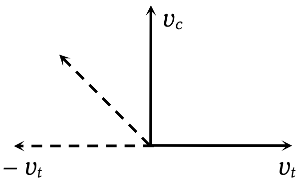
When two particle moves in same direction

$$v_1 - v_2 = 4$$

$$\Rightarrow v_1 = 5m/s,\ v_2 = 1m/s.$$

3. (b) Relative velocity car with respect to train

$$\vec{v}_{ct} = v_c\hat{j} - v_t\hat{i}$$



Hence, direction is 'North-West'.

4. (a) Velocity of car $A\,w.r.t\,C = 45 \times \frac{5}{18} + 36 \times \frac{5}{18} = 22.5$

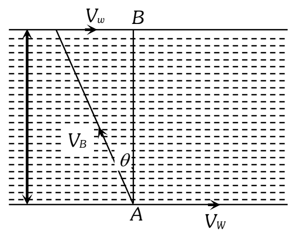
Distance $= 22.5 \times 5 \times 60 = 6.75km.$

5. (d) For, shortest time, he should cross perpendicular to the river

Time to cross the river $= \dfrac{1km}{4km/hr} = 0.25hr$

Time taken round a trip $= 0.5\,hr = 30\,min.$

6. (a)



For shortest distance $v_B \sin\theta = V_w$

$$\Rightarrow \sin\theta = \frac{V_w}{v_B} = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

Time taken to cross the river

$$t = \frac{D}{V_B\cos\theta} = \frac{D}{V_B\sqrt{3}/2} = \frac{2D}{\sqrt{3}V_B}.$$

7. (d) $v_1 = \dfrac{s}{90}, v_2 = \dfrac{s}{60}$

Now, $t = \dfrac{s}{v_1+v_2} = \dfrac{s}{\frac{s}{90}+\frac{s}{60}} = \dfrac{90\times 60}{90+60} = 36$ s

8. (a) At 2s

$$x_A = 10\times 2 - \frac{1}{2}\times 4\times (2)^2 = 12\text{m}$$

$$x_B = 20\times 2 - \frac{1}{2}\times 2\times (2)^2$$

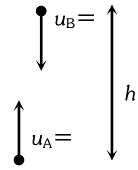
$$= 36 \text{ m}$$

$\therefore$ Distance between A and B at that instant is, 24 m.

9. (a) At any time

$v_A = u - gt$ (upward)

$v_B = gt$ (downward)



Relative velocity $A w.r.t.\ B$

$v_{AB} = u - gt - (-gt) = u.$

10. (d) Separation between two bodies

$$= \frac{1}{2}\times 9.8\,(3^2 - 2^2) = 24.5m.$$

11. (b) $v^2 = u^2 + 2gh$

$$\Rightarrow (3u)^2 = (-u)^2 + 2gh \Rightarrow h = \frac{4u^2}{g}.$$

12. (a) All particles will fall under the action of gravity only so their speeds will remain same.

13. (c) $h = ut + \frac{1}{2}gt^2 \Rightarrow 81 = -12t + \frac{1}{2}\times 10t^2$

$$\Rightarrow 5t^2 - 12t - 81 = 0 \Rightarrow t = 5.4sec.$$

14. (d) Time of flight $= \dfrac{2u}{g}$

$$\Rightarrow 4 = \frac{2u}{10} \Rightarrow u = 20\,m/s.$$

15. (b) Time of ascent = Time of decent = 5 s upward = 5 s downward.

16. (b) $h_1 = \frac{1}{2}gt^2, h_2 = 50t - \frac{1}{2}gt^2$

$$h_1 + h_2 = 100 \Rightarrow 50t = 100 \Rightarrow t = 2sec.$$

17. (a) $t \propto \sqrt{h} \Rightarrow \dfrac{t_1}{t_2} = \sqrt{\dfrac{h}{2h}} = \dfrac{1}{\sqrt{2}}.$

18. (d) Relative to lift

$$u_r = 0,$$

$$a_r = (98 + 1.2) = 11\text{m/s}^2$$

$$s_r = \frac{1}{2}a_r t^2$$

$$\therefore\ 2.7 = \frac{1}{2}\times 11\times t^2$$

or $$t = \sqrt{\frac{5.4}{11}}\text{s}$$

19. (b) Distance travelled in n^{th} second

$$S_n = u + \frac{a}{2}(2n-1)$$

$$S_5 = \frac{10}{2}(5\times 2 - 1) = 45m.$$

20. (d) Relative velocity of stone w.r.t. ground

$v_r = 10 + 5 = 15\,m/s$

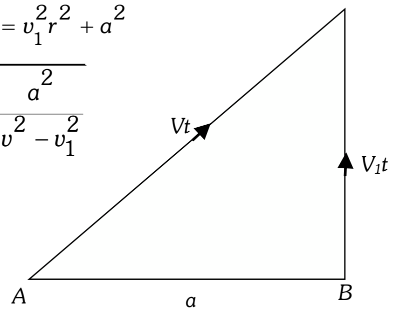
Velocity after $t = 2$ sec

$v = u_r - gt \Rightarrow v = 15 - 10\times 2 = -5m/s$

i.e. 5*m/sec* (downward).

21. (b) $v^2t^2 = v_1^2r^2 + a^2$

$\therefore \quad t = \sqrt{\dfrac{a^2}{v^2 - v_1^2}}$



22. (b) Height of minaret

$$H = \frac{1}{2}gt^2$$

In last 2 *sec* body travels distance of 40 *m*

$$(H - 40) = \frac{1}{2}g(t-2)^2$$

$$\Rightarrow H - 40 = \frac{1}{2}gt^2 - 2gt + 2g$$

$$\Rightarrow H - 40 = H - 2gt + 2g$$

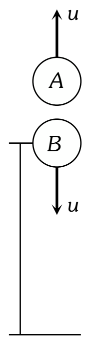
$$\Rightarrow t = \frac{60}{20} = 3\,sec$$

$$H = \frac{1}{2} \times 10 \times (3)^2 = 45\,m.$$

23. (c) $V_A^2 = u^2 - 2g(-h) = u^2 + 2gh$

$V_B^2 = u^2 + 2gh = u^2 + 2gh$

$V_A = V_B$ (Attain the same final velocity).



24. (c) We take downward as the positive direction with $y = 0$ and $t = 0$ at the top of the cliff. The freely falling pebble then has $v_0 = 0$

and $a = g = +9.8 m/s^2$.

The displacement of the pebble at $t = 1.0$ s is given by $y_1 = 4.9$ m. The displacement of the pebble at $t = 3.0$ s is found from

$$y_3 = v_0t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(9.80 \text{ m/s}^2)(3.0\text{s})^2 = 44 \text{ m}.$$

The distance fallen in the 2.0 s interval from $t = 1.0$ s to $t = 3.0$ s is then

$$\Delta y = y_3 - y_1 = 44\text{m} - 4.9\text{ m} = 39\text{ m}$$

25. (c) $\because h = ut + \frac{1}{2}gt^2 \Rightarrow h = \frac{1}{2}gT^2$

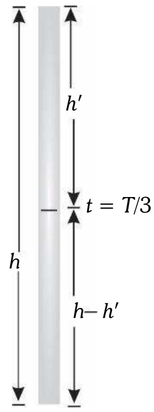
After $\frac{T}{3}$ seconds, the position of ball,

$$h' = 0 + \frac{1}{2}g\left(\frac{T}{3}\right)^2 = \frac{1}{2} \times \frac{g}{9} \times T^2$$

$$h' = \frac{1}{2} \times \frac{g}{9} \times T^2 = \frac{h}{9} \text{ m from top}$$

$\therefore$ Position of ball from ground

$$= h - \frac{h}{9} = \frac{8h}{9}\text{m.}$$



26. (7) Relative velocity of bird w.r.t. train $= 30 m/s$

$$\text{time} = \frac{210}{30} = 7\text{ sec}.$$

27. (0) Velocity of passenger w.r.t. combine system (train + passenger) is zero.

28. (3) Let speed of train $= x$

$$\frac{x-u}{x+u} = \frac{1}{2} \Rightarrow x + u = 2x - 2u$$

$$\Rightarrow \quad x = 3u.$$

29. (5) Distance covered in first 3 *sec*.

$$S = \frac{1}{2}gt^2 = \frac{1}{2} \times g \times (3)^2$$

distance covered in last second

$$S = \frac{9}{2}(2n-1)$$

$$\frac{1}{2} \times g \times 9 = \frac{9}{2}(2n-1)$$

$$\Rightarrow \quad n = 5\,sec.$$

30. (60) $H_{max} = 5$

$$H_{max} = \frac{u^2}{2g} \Rightarrow u = \sqrt{2 \times 10 \times 5} = 10 m/s$$

Time interval between two balls

$$t = \frac{u}{g} = \frac{10}{10} = 1 sec = \frac{1}{60} min.$$

Number of ball thrown per *min*. = 60.

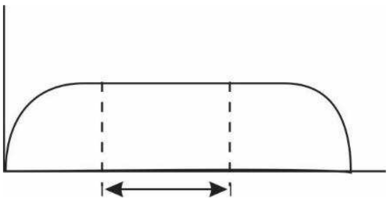
1. (c) Since, $v = r\omega \Rightarrow \omega = \frac{v}{r}$ = constant

[as v and r are constant].

2. (a) $\omega = \frac{2\pi}{T} = \frac{2\pi}{86400} rad/sec$

$\therefore T = 24\,hr = 24 \times 60 \times 60 = 86400\,sec.$

3. (d) For 5 s weight of the body is balanced by the given force. Hence, it will move in a straight line as shown.



$R = \frac{u^2 \sin 2\theta}{g} + (u\cos\theta)(5)$

$= \frac{(50)^2 . \sin 60^o}{10} + (50 \times \cos 30^o)(5) = 250\sqrt{3}$ m

4. (d) Since, $a \propto v^2$

$\frac{a_2}{a_1} = \frac{(2v)^2}{v^2} = \frac{4}{1}.$

5. (d) Because kinetic energy is a scalar quantity.

6. (b) Due to centrifugal force

7. (c) Maximum force of friction = centripetal force

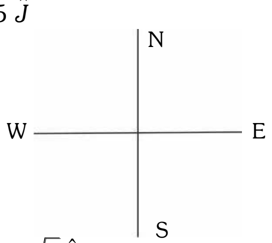
$\frac{mv^2}{r} = \frac{100 \times (9)^2}{30} = 270\,N.$

8. (a) Banking angle

$\tan\theta = \frac{v^2}{Rg}$ and $\tan\theta = \frac{h}{b}$

$\frac{h}{b} = \frac{v^2}{Rg} \Rightarrow h = \frac{v^2 b}{Rg}.$

9. (c) $\vec{v}_c = -5\,\hat{J}$



$v_{a,b,c} = 2\sqrt{6}\,\hat{i}$

$\vec{v}_b = \vec{v}_{bc} + \vec{v}_c = 2\sqrt{6}\,\hat{i} - 5\hat{j}$

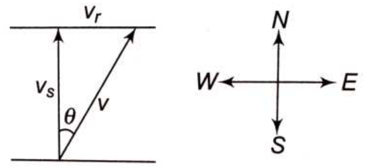
$|\vec{v}_b| = \sqrt{4 \times 6 + 25} = 7$m/s

10. (b) From energy conservation

$\frac{1}{2}mv^2 = mgh$

$\Rightarrow v = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.2} = 2\,m/s.$

11. (b)



Here, Velocity of water flowing in river, $n_r = 3ms^{-1}$ Velocity of swimmer in still water, $n_s = 4ms^{-1}$ From figure, the resultant velocity of the swimmer is

$v = \sqrt{V_s^2 + V_r^2} = \sqrt{(4)^2 + (3)^2} = \sqrt{25} = 5ms^{-1}$

12. (b) Minimum velocity at B

$v_B = \sqrt{gl}$

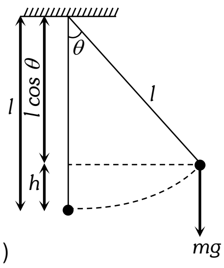
Range $(R) = v_B\sqrt{\frac{2h}{g}} = \sqrt{gl}\sqrt{\frac{2(2l)}{g}} \Rightarrow R = 2l.$

13. (d) Tension at lowest point

$T_{max} = \frac{mv^2}{r} + mg = m\omega^2 r + mg = m[g + (2\pi f)^2 r]$

$= m\left[g + 4\pi^2 \frac{n^2}{(60)^2} r\right] = m\left[g + \frac{\pi^2 n^2 r}{900}\right].$

14. (d) Maximum tension



$T = \frac{mv_B^2}{r} + mg$

Since, $mgh = \frac{1}{2}mv_B^2$

$\Rightarrow v_B^2 = 2gh$

$= 2gl(1 - \cos\theta_0)$

$\therefore T = mg + \frac{2mgl}{l}(1 - \cos\theta_0)$

$= mg + 2mg \times 2\sin^2\frac{\theta_0}{2}$

For small angle $\sin\theta \approx \theta$

$T = mg + 4mg \times \left(\frac{\theta_0}{2}\right)^2 = mg(1 + \theta_0^2).$

15. (c) Because horizontal velocity is same for coin and the observer. So relative horizontal displacement will be zero.

16. (d) H $\propto \sin\alpha$

$\therefore \frac{H_1}{H_2} = \frac{\sin^2\alpha}{\sin^2(90^o - \alpha)} = \tan^2\alpha$

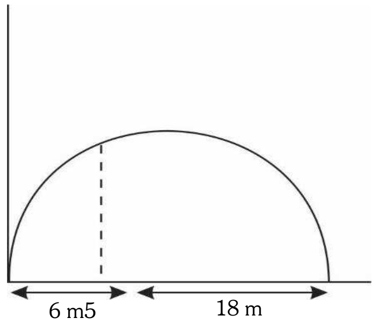
17. (c) Time taken by the bullet to hit target

$t = \frac{200}{2000} = \frac{1}{10} sec$

Vertical-height

$y = h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times \left(\frac{1}{10}\right)^2 = \frac{1}{20}m = 5\,cm\ [u_y = 0]$

18. (b) $R = \dfrac{u^2 \sin 2\theta}{R} = 24$



In $y = x\,tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

$3 = 6\tan\theta - \dfrac{36g}{2u^2\cos^2\theta}$

From Eq.(i) $\dfrac{g}{u^2} = \dfrac{\sin 2\theta}{24} = \dfrac{\sin\theta\cos\theta}{12}$

Substituting in Eq. (ii), we have

$3 = 6\tan\theta - \dfrac{3}{2}\tan\theta = \dfrac{9}{2}\tan\theta$

$\theta = \tan^{-1}\left(\dfrac{2}{3}\right)$

19. (c) Since, horizontal component of velocity is constant.

20. (b) The relative acceleration of one particle w.r.t. to the other is zero, so relative velocity is constant in magnitude and direction.

21. (d) $\vec{v}_B = \vec{v}_{BA} + \vec{v}_A$

$= (5\hat{i} + 12\hat{j}) + (3\hat{i} - 4\hat{j})$

$\vec{v}_B = 8\hat{i} + 8\hat{j}$

22. (a) $T/2 = 2+1 = 3s$ or $T = 6s$

$\therefore \dfrac{2u_y}{g} = 6$

$\therefore u_y = 30\text{m/s}$

Further, $\tan 30^\circ = \dfrac{v_y}{v_s} = \dfrac{u_y - gt}{u_x} = \dfrac{30 - 20}{u_x}$

or $u_s = 10\sqrt{3}$ m/s

or $u = \sqrt{u_x^2 + u_y^2}$

$= 20\sqrt{3}$ m/s

$\tan\theta = \dfrac{u_y}{u_x} = \dfrac{30}{10\sqrt{3}} = \sqrt{3}$

or $\theta = 60^\circ$

23. (d) $R = \dfrac{u^2\sin 2\theta}{g} = 200\,m \Rightarrow \dfrac{2u^2\sin\theta\cos\theta}{g} = 200$

$T = \dfrac{2u\sin\theta}{g} = 5 \Rightarrow \left(\dfrac{2u\sin\theta}{g}\right)u\cos\theta = 200$

$\Rightarrow u\cos\theta = \dfrac{200}{5} = 40\,m/s.$

24. (c) $v\cos\theta = u\cos 2\theta$

$v = \dfrac{u(2\cos^2\theta - 1)}{\cos\theta} = u(2\cos\theta - \sec\theta).$

25. (d) $R = \dfrac{u^2\sin\theta}{g}$ at angles θ and $(90^\circ - \theta)$

Now, $h_1 = \dfrac{u^2\sin^2\theta}{2g}$ (given)

and $h_2 = \dfrac{u^2\sin^2(90^\circ - \theta)}{2g} = \dfrac{u^2\cos^2\theta}{2g}$

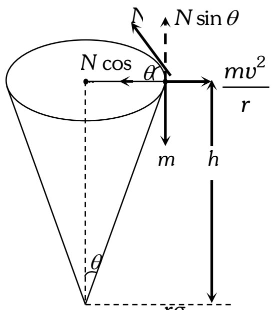
$h_1h_2 = \left(\dfrac{u^2\sin\theta}{g}\right)^2 \dfrac{1}{16} = \dfrac{R^2}{16}$

$R = 4\sqrt{h_1h_2}$

26. (25) $mg = N\sin\theta$ …(i)

Centripetal force is provided by $N\cos\theta$

$N\cos\theta = \dfrac{mv^2}{r}$ …(ii)

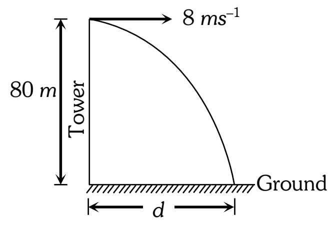


By (i) / (ii) – $\tan\theta = \dfrac{rg}{v^2}$ and $\tan\theta = \dfrac{r}{h}$

$\therefore\ h = \dfrac{v^2}{g} = \dfrac{(0.5)^2}{10} = 0.025\,m = 2.5\,cm.$

27. (32) Here, $u = 8\,ms^{-1}$, $h = 80\,m$

The time taken by the stone to reach the ground is



$t = \sqrt{\dfrac{2h}{g}}$

$= \sqrt{\dfrac{2\times 80\,m}{10\,ms^{-2}}} = 4\,s$

Distance, $d = u\times t = 8\,ms^{-1}\times 4\,s = 32\,m.$

28. (25) $H = \dfrac{u^2 \sin^2\theta}{2g} = \dfrac{(10)^2 \times \sin^2 30}{2\times 10} = 1.25\,m$

$t = \dfrac{u\sin\theta}{g}$ [time to reach top point]

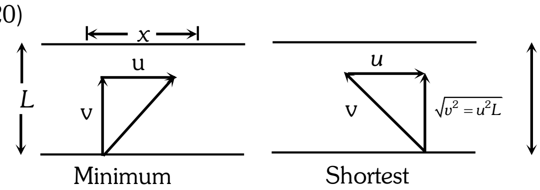
$= \dfrac{10\times\frac{1}{2}}{10} = 0.5\,\text{sec}$

Distance of vertical fall in 0.5 *sec*

$S = \dfrac{1}{2}gt^2 = \dfrac{1}{2}\times 10\times(0.5)^2 = 1.25\,m$

Height of second ball $= 1.25 + 1.25 = 2.5\,m$.

29. (20)



$10 = \dfrac{L}{V}$ ...(i)

$12.5 = \dfrac{L}{\sqrt{v^2 - u^2}} = \dfrac{L}{v\sqrt{1 - u^2/v^2}}$ ...(ii)

From (i) and (ii) $\dfrac{1}{12.5} = \dfrac{L}{v}\times\dfrac{v\sqrt{1-u^2/v^2}}{L}$

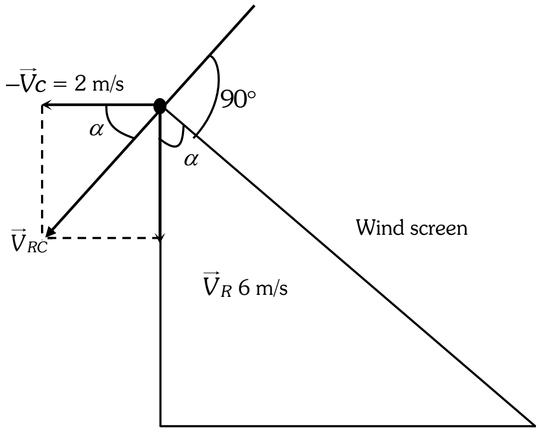
$\dfrac{4}{5} = \sqrt{1 - \dfrac{12^2}{v^2}}$

$\dfrac{16}{25} = 1 - \dfrac{12^2}{v^2} \Rightarrow \dfrac{12^2}{v^2} = 1 - \dfrac{16}{25} = \dfrac{9}{25}$

$\dfrac{12}{v} = \dfrac{3}{5} \Rightarrow v = \dfrac{12\times 5}{3} = 20\text{ m/s}$

30. (3) Velocity of rain with respect to car $\vec{v}_{RC} = \vec{v}_{RC} - \vec{v}_C$ should be perpendicular to the windscreen.

i.e. components of $\vec{v}_R$ and $\vec{v}_C$ parallel to wind – screen should be zero.



or $6\cos\alpha = 2\sin\alpha$

or $\tan\theta = 3$

or $\alpha = \tan^{-1}(3)$

1. (b) Because of inertia of motion upper part of body continues to be in motion in forward direction while feet come to rest as soon as man touches the road.

2. (b) Since, force needed to overcome frictional force.

3. (a) Velocity of train increases.

4. (d) Suppose F is reading of spring balance. Then,
$F - 2g = 2g$ or $F = 4g$
So, it is 4 kg or 4 kg- N
Note That 1 kg = g-N

5. (d) $p = a + bt^2 \Rightarrow F = \frac{dp}{dt} = 2bt$ *i.e.* $F \propto t$.

6. (a) $\vec{p} = p_x\hat{i} + p_y\hat{j} = 2\cos t\,\hat{i} + 2\sin t\,\hat{j}$

$\vec{F} = \frac{d\vec{p}}{dt} = -2\sin t\,\hat{i} + 2\cos t\,\hat{j} \Rightarrow \vec{F}.\vec{p} = 0$ *i.e.* $\theta = 90°$.

7. (b) Spring balance $= mg\sin 30 = 5\times 10\times\frac{1}{2} = 25N$.

8. (d) $F = m\frac{dv}{dt} = \frac{0.25\times[10-(-10)]}{0.01} = 25\times 20 = 500N$.

9. (c) $a = \frac{1000-500}{1000} = \frac{1}{2} = 0.5\,m/s^2$

$u = 10\,m/s \quad t = 10\sec$

$v = u + at = 10 + 0.5\times 10$

$v = 15\,m/s$.

10. (a) F = Rate of change of momentum of ball

$= \frac{mv-(-mv)}{t} = \frac{2mv}{t}$.

11. (a) $m = 15\,kg$; $u = 1.5\,m/s$ (horizontal)

$a = \frac{5}{5} = 1\,m/s^2$ (Perpendicular to velocity)

$S_H = ut = 1.5\times 4 = 6\,m$

$S_V = \frac{1}{2}at^2 = \frac{1}{2}\times 1\times(4)^2 = 8\,m$

$S_{net} = \sqrt{6^2+8^2} = 10\,m$.

12. (a) Weight of the disc = force applied by bullet

$\Rightarrow Mg = nmv \Rightarrow m = \frac{40\times 0.05\times 6}{10} = 1.2\,kg$.

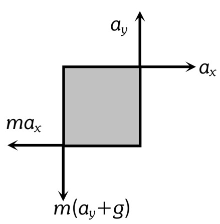
13. (c) As $\vec{v} = 5t\,\hat{i} + 2t\,\hat{j}$: $a = \frac{dv}{dt}$

$\Rightarrow \vec{a} = a_x\hat{i} + a_y\hat{j} = 5\hat{i} + 2\hat{j}$

$\vec{F} = ma_x\hat{i} + m(g+a_y)\hat{j}$

$= m\sqrt{5^2+(12)^2}$

$= 26N$.



14. (c) At 11th second $a = \frac{0-3.6}{2} = -1.8\,m/s^2$

Tension in rope

$T = m(g+a) = 1500\,(9.8-1.8) = 12000N$.

15. (b) $F_u = m(g+a) : F_d = m(g-a)$.

16. (b) $T = 2\pi\sqrt{l/g}$

$\Rightarrow T \propto \frac{1}{\sqrt{g}}$

$\frac{T_1}{T_2} = \sqrt{\frac{g+\frac{g}{3}}{g}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}} \Rightarrow T_2 = \frac{\sqrt{3}}{2}T$.

17. (a) $m = 0.05\,kg$; $a = 9.5\,m/s^2$

$mg - f_{air} = ma \Rightarrow f_{air} = m(g-a)$

$= 0.05\times(9.8-9.5)$

$= 0.015\,N$.

18. (d) As by an internal force momentum of the system can not be changed.

19. (b) Due to Newton's third law. The force experienced by the bird as compared to that of the aeroplane is equal.

20. (a) Due to 3rd law of motion.

21. (c) Reading = weight of cage + Reaction of bird

$= 20 + 0.5(10+2) = 26N$.

22. (d) Since action and reaction acts in opposite direction on same line, hence angle between them is 180°.

23. (d) Rate of flow of water $\frac{V}{t} = \frac{10cm^2}{\text{sec}} = 10\times 10^{-6}\,\frac{\text{m}^3}{\text{sec}}$

Density of water $\rho = \frac{10^3\,\text{kg}}{\text{m}^3}$

Cross-sectional area of pipe A = $\pi(0.5\times 10^{-3})^2$

Force = $m\frac{dv}{dt} = \frac{mv}{t} = \frac{v\rho v}{t} = \frac{\rho v}{t}\times\frac{V}{At} = \left(\frac{V}{t}\right)^2\frac{\rho}{A}$

$\left(\because v = \frac{V}{At}\right)$

By substituting the value in the above formula,
We get F = 0.127 N

24. (b) The acceleration of a rocker is given by

$a = \frac{v}{m}\left(\frac{\Delta m}{\Delta t}\right) - g = \frac{400}{100}\left(\frac{5}{1}\right) - 10$

$= (20-10) = 10\,\text{m/s}^2$

25. (a) Velocity by which the ball hits the bat

$v_1 = \sqrt{2gh} = \sqrt{2\times 10\times 5}$ or $\vec{v}_1 = +10\,\text{m/s} = 10\,\text{m/s}$

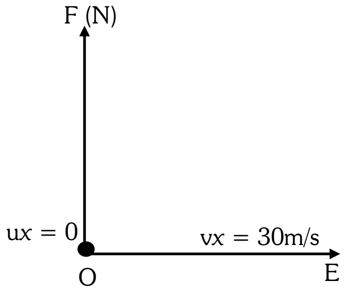
Velocity of rebound

$v_2 = \sqrt{2gh_2} = \sqrt{2\times 10\times 20} = 20$ m/s or $\vec{v}_2 = 20\,\text{m/s}$

$F = m\frac{dv}{dt} = \frac{m(\overrightarrow{v_2}-\overrightarrow{v_1})}{dt} = \frac{4(-20-10)}{dt} = 10N$

By solving, dt = 0.12 sec

26. (20)



Displacement of body in 4 sec along *OE*

$$S_x = v_x t = 3 \times 4 = 12\,m$$

Force along *OF* (perpendicular to *OE*) = 4 N

$\therefore \quad a_y = \dfrac{F}{m} = \dfrac{4}{2} = 2 \text{ m/s}^2$

Displacement of body in 4 sec along *OF*

$\Rightarrow \; S_y = u_y t + \dfrac{1}{2} a_y t^2 = \dfrac{1}{2} \times 2 \times (4)^2 = 16\text{ m} \qquad [As\ u_y = 0]$

$\therefore$ Net displacement s = $\sqrt{s_x^2 + s_y^2} = \sqrt{(12)^2 + (16)^2}$

= 20 m

27. (1) $m = 10\,kg$; breaking force $= 30\,kg\,wt = 300N = T_{\max}$

$T - mg = ma$

$\Rightarrow \quad a = \dfrac{300 - 10 \times 10}{10} = 20\,m/s^2$

$\Rightarrow \; S = \dfrac{1}{2} \times a \times t^2 \Rightarrow 10 = \dfrac{1}{2} \times 20 \times t^2$

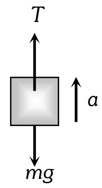
$\Rightarrow \quad t = 1\,\text{sec}.$

28. (3) $T = m(g + a)$

Since, $\quad T = 4\,mg$

$\Rightarrow \quad 4\,mg = m(g + a)$

$\Rightarrow \quad a = 3g.$



29. (10) The acceleration of a rocket is given by

$$a = \frac{v}{m}\left(\frac{\Delta m}{\Delta t}\right) - g = \frac{400}{100}\left(\frac{5}{1}\right) - 10$$

= (20 – 10) = 10 m/s^2

30. (25) When all are pulling

$\vec{F}_{net} - 100 \times 3\hat{i}$

when A stops

$\vec{F}_{net} - \vec{F}_A = 100 \times 1(-\hat{i})$

When B stops

$\vec{F}_{net} - \vec{F}_B = 100 \times 24\hat{j}$

from these three get $\vec{F}_A + \vec{F}_B = (700\hat{i} - 2400\hat{j})N$

hence acceleration of the cart

$$\vec{a} = \frac{\vec{F}_A + \vec{F}_b}{m} = \frac{(700\hat{i} - 2400\,\hat{j})}{100} m/s^2$$

$\vec{a} = (7\hat{i} - 24\,\hat{j}) m/s^2 \Rightarrow |\vec{a}| = \sqrt{7^2 + 24^2} = 25 \text{ m/s}^2$

1. (c) $MV = m_1v_1 + m_2v_2$

$\Rightarrow 10\times 10 = 0 + 1\times v_2 \Rightarrow v_2 = 100\, m/s.$

2. (a) $m_1v_1 = m_2v_2 \Rightarrow 1000\times 50 = (1000+250)v_2$

$v_2 = \frac{50000}{1250} = 40\, m/s.$

3. (a) Newton's second and third laws lead to the conservation of linear momentum.

4. (b) $m_b = 0.01\, kg$; $M = 5\, kg$

$m\times 300 + M\times 0 = m\times 0 + Mv$

$v = \frac{0.01\times 300}{5} = 0.6\, m/s = 60\, cm/\sec.$

5. (a) Impulse = Change in momentum

$= m[10-(-10)] = 0.15\times 20 = 3\, Ns.$

6. (c) Change in momentum = Impulse = $F\times t$

$\Rightarrow F = \frac{0.5\times 10}{1/50} = 250 N.$

7. (d) Law of conservation of momentum

$m_3v_3 = -1\times(3\hat{i} + 4\hat{j})\, kg\, ms^{-1}$

Impulse = Average force × time

$\Rightarrow$ Average force $= \frac{\text{Impulse}}{\text{time}} = \frac{\text{Change in momentum}}{\text{time}}$

$= \frac{-(3\hat{i}+4\hat{j})}{10^{-4}} = -(3\hat{i}+4\hat{j})\times 10^4 N.$

8. (b) $\left|(m_1v_1)\hat{i} + (m_2v_2)\hat{j}\right| = m_3v_3$

$\Rightarrow \sqrt{(1\times 5)^2 + (2\times 6)^2} = m_3\times 6.5$

$m_3 = \frac{13}{6.5} = 2\, kg$

Total mass = 1 + 2 + 2 = 5 kg.

9. (c) $|(m_1v_1)\hat{i} + (m_2v_2)\hat{j}| = m_3v_3$

$\Rightarrow v_3 = \frac{\sqrt{\left(\frac{M}{4}\times 3\right)^2 + \left(\frac{M}{4}\times 4\right)^2}}{M/2} = \frac{5}{2} m/s = 2.5\, m/s.$

10. (d) $|(m_1v_1)\hat{i} + (m_2v_2)\hat{j}| = m_3v_3$

$\Rightarrow v_3 = \frac{\sqrt{(2\times 8)^2 + (1\times 12)^2}}{0.5} = 40\, m/s.$

11. (c) $[(m_1v_1)\hat{i} - (m_2v_2)\hat{i}] = m_3v_3$

$(M\times 2v)\hat{i} - (2M\times v)\hat{i} = m_3v_3 \Rightarrow m_3v_3 = 0 \Rightarrow v_3 = 0.$

12. (d) $\frac{dm}{dt} = 0.1\, kg/\sec$; $v = 1\, km/\sec = 1000\, m/s$

$mv =$ constant

$\Rightarrow m\frac{dv}{dt} - v\frac{dm}{dt} = 0$ [as $m\downarrow$]

$\Rightarrow \frac{dv}{dt} = \frac{1000\times 0.1}{100} = 1\, m/s^2.$

13. (c) $F = v\frac{dm}{dt} = 5\times 10^4\times 40 = 2\times 10^6 N.$

14. (a) $F = v\frac{dm}{dt} = 20\times\frac{50}{60} = 16.66 N.$

15. (d) Rate of flow of water $= \frac{\text{Volume}}{t} = 10\times 10^{-6} m^3/\sec$

Density of water $(\rho_w) = 1000\, kg/m^3$

Area $(A) = \pi r^2 = \pi(0.5\times 10^{-3})^2$

Force $(F) = m\frac{dv}{dt} = \frac{mv}{t} = \frac{V\rho v}{t}$

$= \left(\frac{V}{t}\right)\rho v = \left(\frac{V}{t}\right)\rho\left(\frac{V}{At}\right)$

$\left[\text{Since, } v = \frac{\text{distance}}{\text{time}} = \frac{V}{At} = \frac{Al}{At}\right]$

$= \left(\frac{V}{t}\right)^2\frac{\rho}{A} = (10\times 10^{-6})^2\times\frac{1000}{\pi\times(0.5\times 10^{-3})^2}$

$= \frac{100\times 1000\times 10^{-12}}{\pi\times 25\times 10^{-8}}$

$= \frac{10^{-7}\times 10^8}{25\pi} = \frac{10}{25\pi} = 0.127 N.$

16. (c) Impulse = Area under $F-t$ graph is maximum in case III & IV.

17. (c) Impulse (I) = area under $F-t$ graph

$= \frac{1}{2}\times 2\times 4 + 2\times 4 + \frac{1}{2}(0.5)(4+2.5) + 2\times 2.5$

$= 4 + 8 + 1.625 + 5 = 18.625$

Now, Impulse (I) = Change in moment

$18.625 = m(v_f - v_i)$

$\Rightarrow v_f = v_i + \frac{18.625}{2} = 5 + \frac{18.625}{2}$

$v_f = 14.31\, m/s.$

18. (d) Momentum acquired= Impulse = area under F–t graph

$= \frac{1}{2}\times 2\times 10 + 4\times 10$

$= 50$ N-s.

19. (d) Change in momentum = Impulse

= area under $F-t$ graph

$\Rightarrow mv = \frac{1}{2}\times 2\times 10 + 2\times 10 + \frac{1}{2}\times 2(10+20)$

$+ \frac{1}{2}\times 4\times 20$

$= 10 + 20 + 30 + 40$

$= 100$

$v = \frac{100}{2} = 50\, m/s.$

20. (c) Impulse (I) = area under $F-t$ graph $=\frac{\pi r^2}{2}$

Now, Impulse (I) = Change in momentum

$\frac{\pi r^2}{2}=(mu)\Rightarrow u=\frac{\pi rr}{2m}\Rightarrow u=\frac{\pi F_0(T/2)}{2m}=\frac{\pi F_0 T}{4m}$.

21. (c) $P_x=2\cos t$, $P_y=2\sin t\ \therefore \vec{P}=2\cos t\,\hat{i}+2\sin t\,\hat{j}$

$\vec{F}=\frac{d\vec{P}}{dt}=-2\sin t\,\hat{i}+2\cos t\,\hat{j}\Rightarrow \vec{F}.\vec{P}=0\ \therefore \theta=90°$.

22. (a) By conservation of linear momentum,

$0=3\times16-6\times\upsilon$, $\therefore \upsilon=8\,m/s$ for 6 kg

$\therefore$ Kinetic energy of 6 kg mass $=\frac{1}{2}\times6\times8^2=192J$.

23. (a) Here, Mass of the shell, $m=200g=200\times10^{-3}kg$

Mass of the gun, M = 100 kg

Muzzle speed of the shell, V = 80 ms^{-1}

Recoil speed of the gun, υ= ?

According to the principle of conservation of linear momentum, $mV + M\upsilon = 0$

$$\upsilon=-\frac{mV}{M}=-\frac{(200\times10^{-3}kg)(80ms^{-1})}{100\,kg}$$

-ve sign shows that the recoil speed of the gun will be in a direction opposite to that of the shell.

24. (b) Here,

Mass of the gun, M = 100 kg

Mass of the ball, m = 1kg

Height of the cliff, h = 500 m

and g = 10 ms^{-2}

Time taken by the ball to reach the ground is

$$t=\sqrt{\frac{2h}{g}}=\sqrt{\frac{2\times500m}{10ms^{-2}}}=10s$$

Horizontal distance covered =ut

$\therefore\ 400=u\times10$

Where u is the velocity of the ball.

U = 40 m s^{-1}

According to law of conservation of linear momentum, we get

$0 = M\upsilon + mu$

$$\upsilon=-\frac{mu}{M}=-\frac{(1kg)(40ms^{-1})}{100\ kg}=-0.4ms^{-1}$$

-ve sign shows that the direction

-ve sign shows that the direction of recoil of

25. (c) $F=600-2\times10^5 t=0\Rightarrow t=3\times10^{-3}$ sec

Impulse $I=\int_0^t F\,dt=\int_0^{3\times10^{-3}}(600-2\times10^3 t)dt$

$=[600t-10^5t^2]_0^{3\times10^{-3}}=0.9\text{N}\times\text{sec}$

26. (72) Here, m_2 = 10 kg ;

m_1 = 30 g = 30 × 10^{-3} kg

Number of bullets fired/sec, n = 6 ; F = ?

v_1 = 400 m/s

As force = rate of change of linear momentum

$\therefore\ F=\frac{\text{change in linear momentum}}{\text{time}}=\frac{nm_1\upsilon_1}{1}$

$=\frac{6\times30\times10^{-3}\times400}{1}=72\,N$

27. (80) Let n bullets be fired in one second. To stop the tiger in his track, in his track, rate of change of momentum of bullets = rate of change of momentum of tiger $n\ (50\times10^{-3})\times150=60\times10$

$$n=\frac{60\times10}{50\times10^{-3}\times150}=80$$

28. (9) Here m_1 = 1000 kg, $u_1=32$ m/s,

$m_2=8000\,kg$, $u_2=4$ m/s, $\upsilon_1=-8$ m/s, $\upsilon_2=?$

Using principle of conservation of linear momentum,

$m_2\upsilon_2+m_1\upsilon_1=m_1u_1+m_2\ u_1\,8000$

$\upsilon_2+1000\times(-8)=1000\times32+8000\times4=64000$

$$\upsilon_2=\frac{64000+8000}{8000}=9\text{ m/s}$$

29. (98) Here, M = 10 g = 10^{-2} kg, n = 10,

M = 5 g = 5 × 10^{-3} kg, v = ?

To keep the disc floating horizontally.

Weight of disc = upward force on the disc = rate of change of momentum of marbles

$10^{-2}\times9.8=n\times m\times2\upsilon=10\times5\times10^{-3}\times2\upsilon$

$=\frac{\upsilon}{10}$

$\upsilon=9.8\times10^{-2}\times10$ m/s = 0.98 m/s

30. (5) Here, $m=2\times10^4$ kg, $F=5\times10^5\,N, t=20\,s$

$\upsilon=?, u=0.$

As Impulse = change in momentum,

$\therefore\ F\times t=m(\upsilon-u)=m\upsilon$

$\upsilon=\frac{F\times t}{m}=\frac{5\times10^5\times20}{2\times10^4}$=500 m/s