Vectors & Scalars

SCALAR

The physical quantity which has a magnitude but no specific direction. eg. - distance, mass, speed.

VECTOR

The physical quantity which has a magnitude as well as direction and follows the vector law of addition. eg.- Force, velocity, displacement, momentum.

NOTE -1 Current is not a vector quantity though it has direction and magnitude as it does not follow vector law of addition.

REPRESENTATION OF A VECTOR QUANTITY

A vector is represented by drawing an arrow propotional in length to the physical quantity being represented.

- A vector variable is represented by an arrow over english or greek alphabet.
- The same alphabet without the vector sign represents the magnitude of the vector and the same alphabet with a cap sign represents the direction of the vector.

Practice Time

(a) $\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

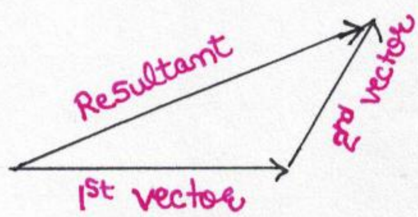
(b) $\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

(c) $\tan\left(\frac{3\pi}{4}\right) = -1$

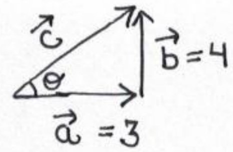
(d) $\tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$

TRIANGLE LAW OF ADDITION

To add two vectors, using triangle law, place the tail of the second vector on the head of the first vector. Now complete the triangle and the resultant vector is given by the arrow starting from tail of the first vector to the head of the second vector.



Eg -



$\vec{c} = \vec{a} + \vec{b}$

$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

For example -

If $\vec{a}$ = 1Km North

then, a = 1km and $\hat{a}$ = North

INVERSE TRIGONOMETRIC NOTATION FOR ANGLES

$\sin^{-1}(x)$ represents an angle whose Sin is x

Eg - $\tan^{-1}(x) = \tan^{-1}(1) = 45^\circ$

$\sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$

RADIAN MEASURE OF AN ANGLE

$360^\circ = 2\pi$ radian

If the angle (θ) of an arc is expressed in radians then the arc length is simply

$$\boxed{l = r\theta}$$

Trigonometric ratios for angles $> 90^\circ$

$\sin(\pi - \theta) = \sin\theta$

$\cos(\pi - \theta) = -\cos\theta$

$\tan(\pi - \theta) = -\tan\theta$

$\sin(\pi + \theta) = -\sin\theta$

$\cos(\pi + \theta) = -\cos\theta$

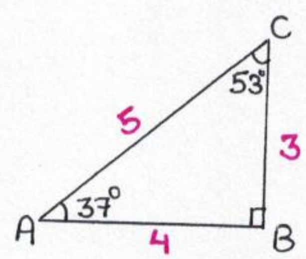
$\tan(\pi + \theta) = \tan\theta$

NOTE - 2 $\sin 37^\circ = \frac{3}{5}$

$\cos 37^\circ = \frac{4}{5}$

$\sin 53^\circ = \frac{4}{5}$

$\cos 53^\circ = \frac{3}{5}$



NOTE - 3 Vector sum is Commutative i.e.

$\vec{a} + \vec{b} = \vec{b} + \vec{a}$

ANGLE BETWEEN TWO VECTORS

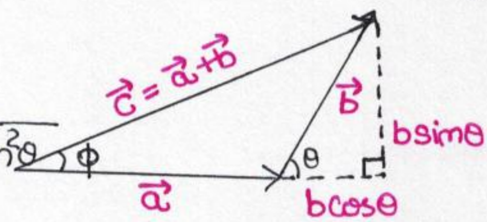
The angle between two vectors is defined as the non-reflex angle, between the vectors when they are joined tail to tail.

Eg - The angle between the vectors is $60°$.

ANALYTICAL ADDITION OF VECTORS

When we consider two vectors $\vec{a}$ and $\vec{b}$ represented for finding the resultant $(\vec{a}+\vec{b})$ using the triangle law as shown in fig.

Using pythagoras theorem



$$|\vec{c}| = \sqrt{(a+b\cos\theta)^2 + (b\sin\theta)^2}$$

$$|\vec{c}| = \sqrt{a^2+b^2\cos^2\theta + 2ab\cos\theta + b^2\sin^2\theta}$$

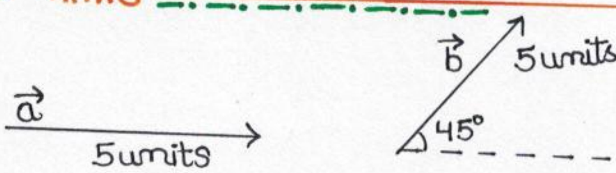
$$|\vec{c}| = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

and direction is given by

$$\phi = \tan^{-1}\left(\frac{b\sin\theta}{a+b\cos\theta}\right)$$

Practice Time

■



$$\text{resultant} = \sqrt{25 + 25 + 50 \times \frac{1}{\sqrt{2}}}$$

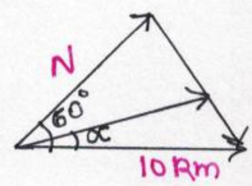
$$= \sqrt{50 + \frac{50}{\sqrt{2}}}$$

$$= \sqrt{\frac{50\sqrt{2} + 50}{\sqrt{2}}}$$

$$\phi = \tan^{-1}\left(\frac{5/\sqrt{2}}{5 + 5/\sqrt{2}}\right) = \tan^{-1}\left(\frac{5}{5\sqrt{2}+5}\right)$$

$$\phi = \tan^{-1}\left(\frac{1}{\sqrt{2}+1}\right)$$

■ A man walks 10 Km towards east & then another 10 km towards $60°$ North east. Find its resultant & direction.



$$\vec{c} = \sqrt{200 + 200 \times \frac{1}{2}}$$

$$= \sqrt{300}$$

$$= 10\sqrt{3}$$

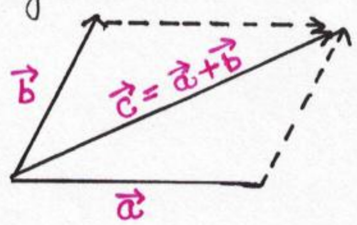
$$\alpha = \tan^{-1}\left(\frac{5\sqrt{3}}{10+15}\right)$$

$$\alpha = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$\alpha = 30^\circ$$

PARALLELOGRAM LAW OF ADDITION

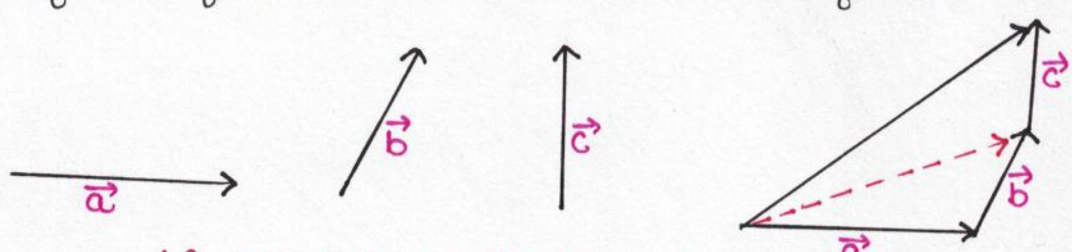
Here we place the vector's tail to tail and complete the parallelogram. Now the resultant is given by diagonal of the parallelogram passing through the common tail.



NOTE-4 For any given vector $\vec{a}$, the symbols a and $|\vec{a}|$ mean exactly the same thing.

POLYGON LAW OF ADDITION

Here we join all the vectors to be added in a head to tail configuration and now the resultant is given by the vector joining tail of the first vector to the head of the last vector.



(A repeated application of Triangle law)

NEGATIVE OF A VECTOR

Negative of a vector is a vector with the same magnitude but with reversed direction.

Eg - $\vec{a}$ = 1 unit $\hat{N}$

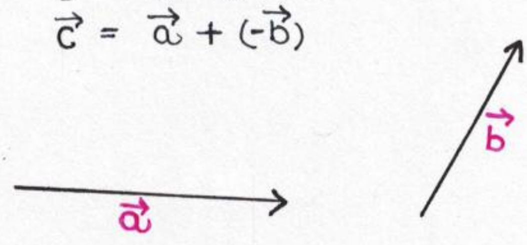
$-\vec{a}$ = 1 unit $\hat{S}$

SUBTRACTION OF VECTORS

Subtraction is same as the addition of the negatives.

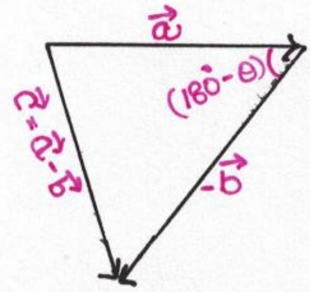
$$\vec{c} = \vec{a} - \vec{b}$$

$$\vec{c} = \vec{a} + (-\vec{b})$$



$$|\vec{c}| = \sqrt{a^2+b^2+2ab\cos(180^\circ-\theta)}$$

$$|\vec{c}| = \sqrt{a^2+b^2-2ab\cos\theta}$$



UNIT VECTOR

A vector having magnitude and some specific direction is called Unit Vector. It is **unitless**.

- Any vector divided by its magnitude gives the unit vector in the same direction.

For eg- If $\vec{a} = 5km\ \hat{N}$

$a = 5km$

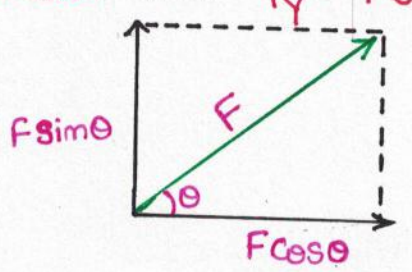
$\hat{a} = \dfrac{\vec{a}}{a} \quad 1\hat{N}$

RECTANGULAR COMPONENTS OF A VECTOR

Whenever a vector is expressed as a sum of two mutually perpendicular vectors, then we called the vectors as **rectangular components of the vector.**

There are infinite ways of doing this. To resolve a vector into mutually perpendicular components, imagine a rectangle whose diagonal is the given vector. The sides of such rectangle are the rectangular components.

For instance let there be a force F inclined at an angle θ with the horizontal, then its horizontal and vertical components are given by $F_x = F\cos\theta$ and $F_Y = F\sin\theta$

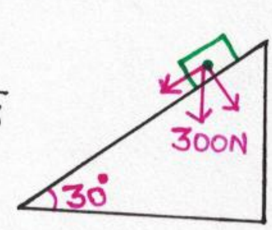


Practice Time

(1) Resolve the wt. vector of block into Components

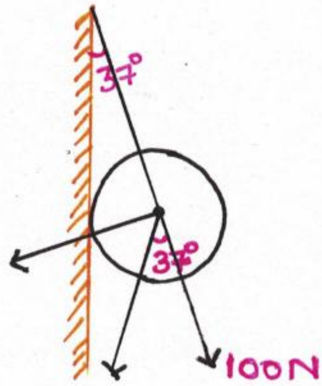
$W_{\parallel} = mg\sin\theta = 300 \times \frac{1}{2} = 150$

$W_{\perp} = mg\cos\theta = 300 \times \frac{\sqrt{3}}{2} = 150\sqrt{3}$



(2) $\parallel$ to thread $= 100 \times \frac{4}{5} = 80N$

$\perp$ to thread $= 100 \times \frac{3}{5} = 60N$

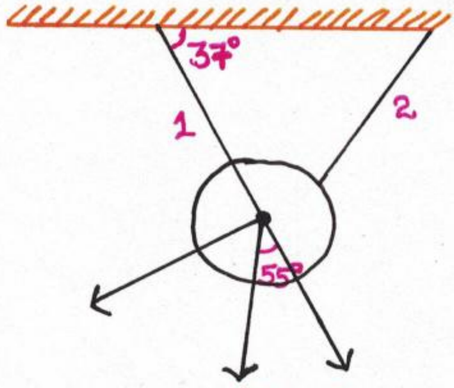


(3) $\perp = 1000 \times \frac{4}{5}$

$= 800$

$\parallel = 1000 \times \frac{3}{5}$

$= 600$

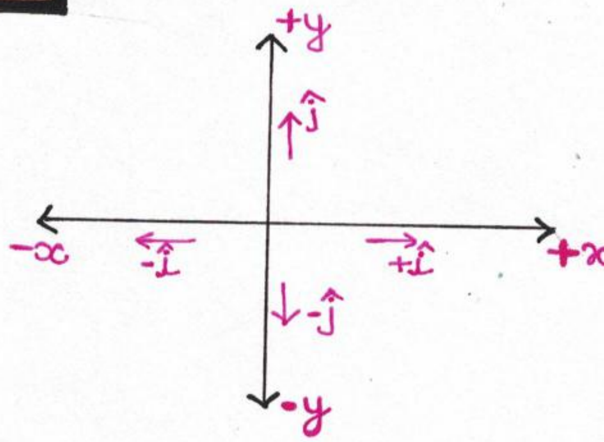


UNIT VECTORS ALONG COORDINATE AXIS

Any vector, say $\vec{a}$ drawn in the x-y plane can always be written in terms of its components $\parallel$ to the x and y-axis respectively.

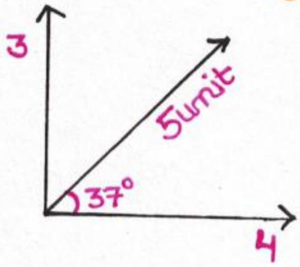
For eg - In the shown fig., $\vec{a}$ can be written as

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$



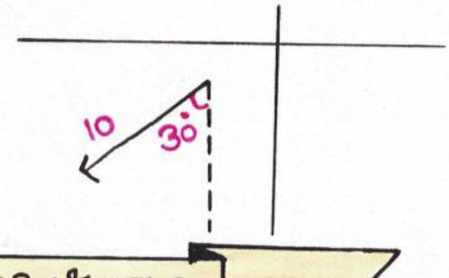
Such representation is called Cartesian representation.

(1) $\vec{5} = 4\hat{i} + 3\hat{j}$



(2) $\vec{10} = -10\cos 30\hat{i} - 10\sin 30\hat{j}$

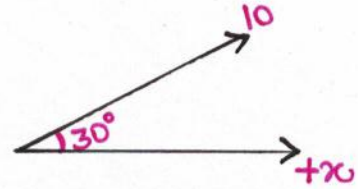
$= -5\hat{i} - 5\sqrt{3}\hat{j}$



POLAR REPRESENTATION OF VECTORS

When a vector is specified in terms of its magnitude and angle which it makes with the x-axis, we call it polar representation.

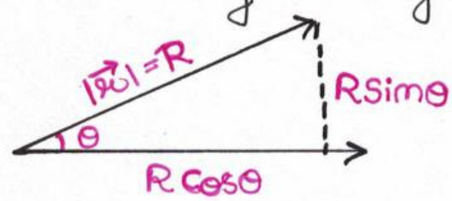
For eg.- 10 @ 30°



CONVERTING FROM POLAR FORM TO CARTESIAN FORM

Consider a vector $\vec{R}$ having a magnitude $\vec{R}$ making an angle θ with the x-axis as shown.

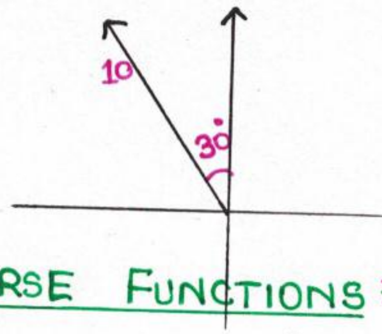
$$\vec{r} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$$



- Represent the shown vector in cartesian form:

$$\vec{v} = -5\hat{i} + 5\sqrt{3}\hat{j}$$

$$= 10\cos 120\,\hat{i} + 10\sin 120\,\hat{j}$$



PRINCIPAL RANGES FOR INVERSE FUNCTIONS

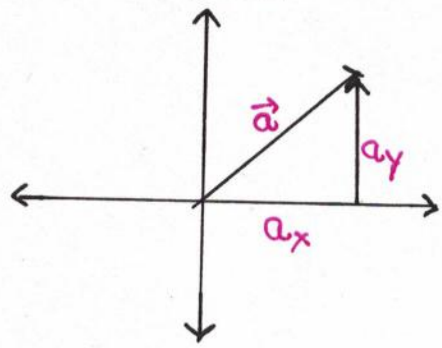
$\sin^{-1} = -90°$ to $90°$

$\tan^{-1} = -90°$ to $90°$

$\cos^{-1} = 0°$ to $180°$

CONVERTING FROM CARTESIAN FORM TO POLAR FORM

Consider a vector $\vec{a}$, as represented in cartesian form $\vec{a} = a_x\hat{i} + a_y\hat{j}$ as shown in fig.



$$a = \sqrt{a_x^2 + a_y^2}$$

$$\tan\theta = \frac{a_y}{a_x}$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) \text{ if } a_x \text{ is positive}$$

$$\theta = 180° + \tan^{-1}\left(\frac{a_y}{a_x}\right) \text{ if } a_x \text{ is negative}$$

Convert the following from cartesian to Polar forms:

(a) $3\hat{i} + 4\hat{j} = 5 @ 53°$

(b) $-4\hat{i} + 3\hat{j} = 5 @ 143°$

(c) $-3\hat{i} - 4\hat{j} = 5 @ 233°$

(d) $4\hat{i} - 3\hat{j} = 5 @ \tan^{-1}\left(-\frac{5}{4}\right) = 5 @ -37°$