2025-26 II PUC PREPARATORY EXAMINATION

Subject: 35-Mathematics **Max. Marks: 80**
Time: 3.00 hours **No. of Questions: 47**

Instructions:
1) *The question paper has five parts namely A, B, C, D and E. Answer all the parts.*
2) *PART A has 15 MCQ's ,5 Fill in the blanks of 1 mark each.*
3) *Use the graph sheet for question on linear programming in PART E.*
4) *For questions having figure/graph, alternate questions are given at the end of question paper in separate section for visually challenged students.*

PART A

I. Answer ALL the Multiple Choice Questions **15 × 1 = 15**

1. A function $f: R \to R$ defined by $f(x) = 3x + 6$ is a bijective mapping then $f^{-1}(x)$ is given by

(A) $\frac{x}{3} - 2$ (B) $2x + 6$ (C) $x - 2$ (D) $6x + 3$.

*2.*Match Column I with Column II

Column I	Column II
a) Range of $\sin^{-1} x$	i) R
b) Range of $\cos^{-1} x$	ii) $[0, \pi]$
c) Domain of $tan^{-1} x$	iii) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Choose the correct answer from the options given below:
(A) a-i, b-ii, c-iii (B) a-iii, b-ii, c-i (C) a-i, b-iii, c-ii (D) a-iii, b-i, c-ii

*3.*The product of matrices A and B is equal to a square matrix. If the order of the matrix B is 2x3, then order of the matrix A is
(A) 3x3 (B) 2x2 (C) 3x2 (D) 2x3

4. If A is a square matrix of order 2 and $|A| = 3$, then $|A^{-1}| =$

(A) 3 (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) 12.

5. If $y = \cos(\sqrt{x})$, then $\frac{dy}{dx} =$

(A) $sin(\sqrt{x})$ (B) $-sin(\sqrt{x})$ (C) $\frac{sin(\sqrt{x})}{2\sqrt{x}}$ (D) $\frac{-sin(\sqrt{x})}{2\sqrt{x}}$.

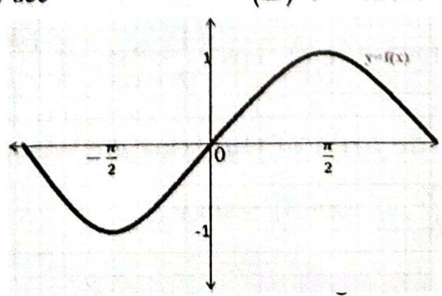
6. If $y = x^{20}$, then $\frac{d^2y}{dx^2} =$

(A) $20x^{19}$ (B) $20x^{18}$ (C)$380\,x^{18}$ (D)$360x^{18}$.

7. The edge of a cube is increasing at a constant rate. At the moment when the edge is 2 cm long, the rate at which the volume of the cube is increasing is:

(A) $32cm^3$ / sec (B) $60cm^3$ / sec (C) $150cm^3$ / sec (D) $5^3 \times 12cm^3$ / sec

8. The point of inflection for the following graph is



(A) $-\frac{\pi}{2}$
(B) $\frac{\pi}{2}$
(C) 0
(D) point of inflection does not exist

9. $\int \frac{2-3\sin x}{\cos^2 x}dx =$

(A) $2\tan x - 3\sec x + c$ (B) $2\tan x + 3\sec x + c$

(C) $2\tan x - 2\sec x + c$ (D) $2\sec x - 3\tan x + c$

10. $\int_0^1 xe^x\, dx =$

(A) $e-1$ (B) 1 (C) -1 (D) $2e-1$

11. A unit vector in the direction of vector $\vec{a} = 2\,\hat{\imath} + 3\hat{\jmath} + \hat{k}$ is

(A) $\frac{2\,\hat{\imath}+3\hat{\jmath}+\hat{k}}{\sqrt{14}}$ (B) $\frac{2\,\hat{\imath}+3\hat{\jmath}+\hat{k}}{14}$

(C) $\frac{2\,\hat{\imath}+3\hat{\jmath}+\hat{k}}{\sqrt{6}}$ (D) $\frac{-(2\,\hat{\imath}+3\hat{\jmath}+\hat{k})}{\sqrt{14}}$

12. **Statement 1:** If either $|\vec{a}| = 0$ or $|\vec{b}| = 0$ then $\vec{a}.\vec{b} = 0$

Statement 2: If $\vec{a} \times \vec{b} = \vec{0}$, *then* $\vec{a} \perp$ *to* $\vec{b}$.

(A) Statement 1 is true and Statement 2 is false

(B) Statement 1 is false and Statement 2 is true

(C) Statement 1 is true and Statement 2 is true

(D) Statement 1 is false and Statement 2 is false

13. If a line makes $\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}$ with x, y, z axes respectively, then its direction cosines are

(A) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (B) $0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$ (C) $1, \frac{1}{2}, \frac{1}{\sqrt{2}}$ (D) $0, \frac{1}{2}, \frac{1}{\sqrt{2}}$

14. Two cards are drawn at random without replacement from a pack of 52 playing cards,then the probability that both the cards are black is

(A) $\frac{1}{26}$ (B) $\frac{1}{4}$ (C) $\frac{25}{102}$ (D) $\frac{25}{104}$

15. If A is a subset of B and P(A)≠0, then P(B | A)=

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) do not exist

II. Fill in the blanks by choosing the appropriate answer from those given in the bracket (−2, 8, 0, 3, 2, 4) **5 × 1 = 5**

16. If $f(x) = |x-2|$, then f is continuous but not differentiable at x=_______.

17. The absolute maximum value of the function f given by $f(x) = x^3, x\epsilon[0,2]$ is _______.

18. Find the number of arbitrary constants in the particular solution of a differential equation of third order is ________.

19. If $\hat{\imath}$, $\hat{\jmath}$ *and* $\hat{k}$ are unit vectors, then the value of $i.\hat{i} + j.j + k.k$ is _______.

20. If $P(B) = \frac{5}{12}$, $P(A \cap B) = \frac{4}{12}$ *and* $P(A|B) = \frac{k}{5}$, then k = ______.

PART B

III. Answer Any SIX Questions: **6 × 2 = 12**

21. Find the value of $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\left(\frac{1}{2}\right)$.

22. Find the area of the triangle whose, vertices are $(3,8), (-4,2) \& (5,1)$ using determinants.

23. Find $\frac{dy}{dx}$, if $2x + 3y = \sin y$.

24. Find the intervals in which the function f given by $f(x)=x^2-4x+6$ is strictly decreasing.

25. Evaluate $\int xe^{(x^2+1)}dx$.

26. Find the general solution of the differential equation $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$

27. Find the area of the parallelogram whose adjacent sides are given by the vectors $\vec{a}=3\hat{i}+j+4k$ & $\vec{b}=\hat{i}-j+k$.

28. Find the vector and Cartesian equation of the line passing through the point (5,2,-4) and to the vector $2\hat{\imath}+6\hat{\jmath}+3\hat{k}$.

29. If A and B are two independent events such that $P(A)=\frac{1}{4}$ and $P(B)=\frac{1}{2}$. then find P(not A and not B) .

PART C

IV. Answer Any SIX Questions: **6 × 3 = 18**

30. Let T be the set of triangles and let R be a relation on T defined as $R=\{(T_1, T_2) : T_1$ is congruent to $T_2\}$. Show that R is an equivalence relation.

31. Prove that $\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$.

32. For the matrix A = $\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}$, verify that

(i) (A + A′) is a symmetric matrix (ii) (A - A′) is a skew symmetric matrix.

33. Find $\frac{dy}{dx}$, if $x=a(\theta+\sin\theta)$, $y=a(1-\cos\theta)$.

34. Find two positive numbers whose sum is 15 and the sum of whose squares is minimum.

35. Find $\int\frac{x}{(x+1)(x+2)}dx$.

36. Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Evaluate the quantity $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$, if $|\vec{a}|=1, |\vec{b}|=4$ & $|\vec{c}|=2$.

37. Find distance between the lines l_1 and l_2 given by $\vec{r}=\hat{\imath}+2\hat{\jmath}-4\hat{k}+\lambda(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$ and $\vec{r}=3\hat{\imath}+3\hat{\jmath}-5\hat{k}+(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$.

38. A bag contains 3 red and 4 black balls another bag contains 5 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

PART D

V. Answer Any FOUR Questions: **5 × 4 = 20**

39. State whether the function $f: R \rightarrow R$ defined by $f(x) = 3 - 4x$ is one-one, onto or bijective. Justify your answer.

40. For the matrices, $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & -6 \end{bmatrix}$, verify that $(AB)' = B'A'$.

41. Solve the system of equations: $x + y + z = 6,\ y + 3z = 11$ and $x - 2y + z = 0$ by matrix method.

42. If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2 y_2 + xy_1 + y = 0$.

43. Find the integral of $\frac{1}{\sqrt{a^2 - x^2}}$ with respect to x and evaluate $\int \frac{1}{\sqrt{9 - 25x^2}} dx$.

44. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by the method of integration.

45. Find the general solution of the differential equation $x\frac{dy}{dx} + 2y = x^2 (x \neq 0)$.

PART E

VI. Answer the Following Questions:

46. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$.

6M

OR

Solve the following graphically, Minimize Z = $200x + 500y$, subject to the constraints $x + 2y \geq 10,\ 3x + 4y \leq 24,\ x \geq 0,\ y \geq 0$.

47. Show that $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 5A + 7I = 0$ where I is 2 x 2 identity matrix and O is 2 x 2 zero matrix and using this equation find A^{-1}.

4M

OR

Find the value of k so that the function $f(x) = \begin{cases} kx^2, & if\ x \leq 2 \\ 3, & if\ x > 2 \end{cases}$ is continuous at x=2.

PART F

(For Visually Challenged Students only)

8. The point of inflection of the function $y = x^3$ is
A) $(2, 8)$ B) $(1, 1)$ C) $(0, 0)$ D) $(-3, -27)$.