Problem C2: The equation of state for radiant energy in equilibrium with the temperature of the walls of a cavity of volume V is $P = aT^4/3$, where a is a constant. The internal energy is $U = aT^4V$.

a) Calculate the amount of heat supplied in an isothermal doubling of the volume of the cavity.

b) Show that an adiabatic process is characterized by $VT^3 = const$. Hint: use the first law and $U = U(V,T)$.

a)

$$dU = \delta Q - PdV$$

$$aT^4 dV = \delta Q - \frac{aT^4}{3} dV$$

$$\delta Q = \frac{4}{3} T^4 dV$$

$$Q = \frac{4}{3} T^4 V$$

Doubling

$$Q_2 - Q_1 = \frac{4}{3} T^4 (2V - V)$$

$$\boxed{\Delta Q = \frac{4}{3} T^4 V}$$

b) Adiabatic : $\delta Q = 0$

$$\partial U = -P\partial V$$

$$\partial(\cancel{a}T^4V) = -\frac{aT^4}{3}\partial V$$

$$4VT^3\partial T + T^4\partial V = -\frac{1}{3}T^4\partial V$$

$$\cancel{4}VT^{\cancel{3}}\partial T = -\frac{\cancel{4}}{3}T^{\cancel{4}}\partial V$$

$$\frac{\partial T}{T} = -\frac{1}{3}\frac{\partial V}{V}$$

$$3\frac{dT}{T} = -dV$$ Integrate

$$3\ln\left(\frac{T}{T_0}\right) = -\ln\left(\frac{V}{V_0}\right)$$

$$\frac{T^3}{T_0^3} = \frac{V_0}{V}$$

$$\boxed{V T^3 = V_0 T^3 = \text{const.}}$$