Problem C2: Given equation of state as $P(v-b) = RT$, show that the relation between P and v during an adiabatic process is $P(v-b)^\gamma = constant$, where γ is the ratio of C_p and C_v.

$$dU = \delta Q - P dV$$

- at $dV = 0$

$$dU = \delta Q = C_v\, dT$$

(1)

- At $P = \text{const}$

$$C_v\, dT = \delta Q - \frac{RT}{V-b}\, dV \quad (2)$$

$$P\, dV = R\, dT$$

$$\Rightarrow \frac{RT}{V-b} dV = R\, dT \quad (3)$$

Inserting Eq. (3) into Eq. (2)

$$C_v\, dT = \delta Q - R\, dT$$

$$\delta Q = \underbrace{(C_v + R)}_{= C_p}\, dT$$

- Adiabatic $\delta Q = 0$

$$dU = -P\, dV$$

$$C_v\, dT = -\frac{RT}{V-b}\, dV$$

$$\frac{dT}{T} = -\frac{R}{C_v}\, \frac{dV}{V-b}$$

$$\ln\left(\frac{T}{T_0}\right) = -\frac{R}{C_v}\, \ln\left(\frac{V-b}{V_0-b}\right)$$

$$\frac{T}{T_0} = \left[\frac{V-b}{V_0-b}\right]^{-\frac{R}{C_v}}$$

$$T(V-b)^{\frac{R}{c_v}} = T_0(V_0-b)^{\frac{R}{c_v}} = \text{const.}$$

$$\left(\frac{P(V-b)}{R}\right) V^{\frac{R}{c_v}} = \text{const.}$$

$$P(V-b)^{\frac{R}{c_v}+1} = \text{const.}$$

$$P(V-b)^{\underbrace{\frac{R+c_v}{c_v}}_{c_p/c_v}} = \text{const.}$$

$$\boxed{P(V-b)^{\gamma} = \text{const.}}$$