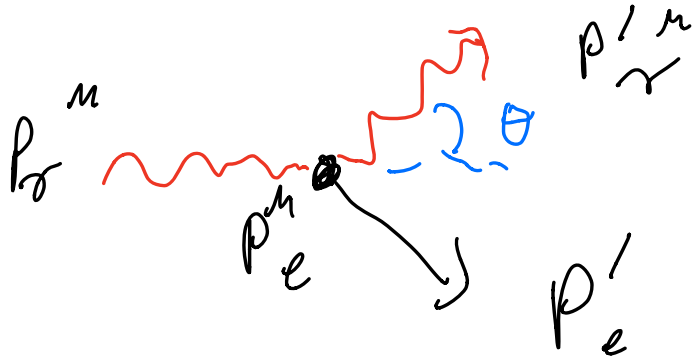
Problem A2: A photon of energy E scatters off of a free electron (mass m_e) through an angle of θ. Suppressing factors of c,
a) show that the energy of the scattered photon is given by

$$E' = \frac{Em_e}{m_e + E(1-\cos\theta)}.$$

b) Show that the kinetic energy of the electron after the collision is given by

$$T_e = \frac{E^2(1-\cos\theta)}{m_e + E(1-\cos\theta)}.$$

a)



By conservation of 4-momentum:

$$p_\gamma^\mu + p_e^\mu = p_\gamma'^\mu + p_e'^\mu$$

$$p_\gamma^\mu - p_\gamma'^\mu = p_e'^\mu - p_e^\mu$$

Squaring both sides:

$$(p_\gamma^\mu - p_\gamma'^\mu)^2 = (p_e'^\mu - p_e^\mu)^2$$

$$p_\gamma^2 - 2p_\gamma \cdot p_\gamma' + p_\gamma'^2 = p_e'^2 - 2p_e \cdot p_e' + p_e^2$$

$m_\gamma^2 = 0$ $\quad$ $m_\gamma^2 = 0$ $\quad$ m_e^2 $\quad$ m_e^2

$$-2p_\gamma \cdot p_\gamma' = 2m_e^2 - 2p_e \cdot p_e'$$

$$-p_\gamma \cdot p_\gamma' = m_e^2 - p_e \cdot p_e'$$

$$(EE' - |\vec{p}_\gamma||\vec{p}_\gamma'|\cos\theta) = m_e^2 - (E_e E_e' - |\vec{p}_e||\vec{p}_e'|\cos\theta_e)$$

Assuming initial e @ rest: $E_e = m_e$, $\vec{p}_e = \vec{0}$.

Also, since γ massless $\Rightarrow$ $|\vec{p}_\gamma| = E$, $|\vec{p}_\gamma'| = E'$

$$-(EE' - EE'\cos\theta) = m_e^2 - (m_e E_e' - 0)$$

$$-EE'(1-\cos\theta) = m_e^2 - m_e E_e' \qquad (1)$$

From conservation of energy,

$$E + m_e = E_e' + E'$$

$$E_e' = E + m_e - E' \qquad (2)$$

Inserting (2) into (1),

$$-EE'(1-\cos\theta) = m_e^2 - m_e(E + m_e - E')$$

$$-EE'(1-\cos\theta) = m_e^2 - m_e E - m_e^2 + m_e E'$$

$$-EE'(1-\cos\theta) = -m_e E + m_e E'$$

$$-EE'(1-\cos\theta) - m_e E' = -m_e E$$

$$E'(-E(1-\cos\theta) - m_e) = -m_e E$$

$$E' = \frac{-m_e E}{-E(1-\cos\theta) - m_e}$$

$$\Rightarrow \quad \boxed{E' = \frac{m_e E}{E(1-\cos\theta) + m_e}} \qquad (3)$$

b) $T_e = E_e' - m_e$

Using (2) and (3)

$$T_e = E + m_e - \frac{m_e E}{E(1-\cos\theta) + m_e} - m_e$$

$$T_e = E - \frac{m_e E}{E(1-\cos\theta) + m_e}$$

$$= \frac{E\left(E(1-\cos\theta) + m_e\right) - m_e E}{E(1-\cos\theta) + m_e}$$

$$= \frac{E^2(1-\cos\theta) + m_e E - m_e E}{E(1-\cos\theta) + m_e}$$

$$\Rightarrow \boxed{T_e = \frac{E^2(1-\cos\theta)}{E(1-\cos\theta) + m_e}}$$