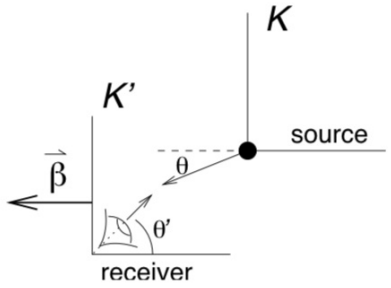
2008 Modern - A1 (SR)

Problem A1: The energy of a photon is given by $E = h\nu$ and its momentum is given by $p = h\nu/c$ (or $p = E$ setting $c = 1$), where $\nu$ is the frequency of the emitted light.

a) Use the Lorentz transformation of the energy to find the frequency, $\nu'$, of the light observed by a receiver moving *directly toward* the source. What is the frequency when the receiver is moving *directly away* from the source.

b) Now consider the case where the light is emitted at an angle of $\theta$ in the frame of the source and observed at an angle of $\theta'$ at the receiver.




a) Find the Doppler shifted frequency.

b) Show that

$$\tan\theta' = \frac{\sin\theta}{\gamma\,(\cos\theta - \beta)}.$$

a) Since photon is massless

$\therefore p = E = h\nu$

By Lorentz transformation

$E' = \gamma(E - pv)$

$= \frac{1}{\sqrt{1-v^2/c^2}}\left(E - \frac{Ev}{c}\right)$

$$E' = E\frac{(1-V/c)}{\sqrt{1-v^2/c^2}}$$

$$E' = E\sqrt{\frac{c(1-V/c)}{(1-V/c)(c+V/c)}}$$

$$E' = E\sqrt{\frac{1-V/c}{1+V/c}}$$

$$h\nu' = h\nu\sqrt{\frac{1-V/c}{1+V/c}}$$

$$\boxed{\nu' = \nu\sqrt{\frac{1-V/c}{1+V/c}}}$$

Assume $x_0 > 0$.

When $V = |V|$ (away from source)

$\nu'$ decreases. Red shift.

$V = -|V|$ (toward source). $\nu'$ increases

Blue shift.

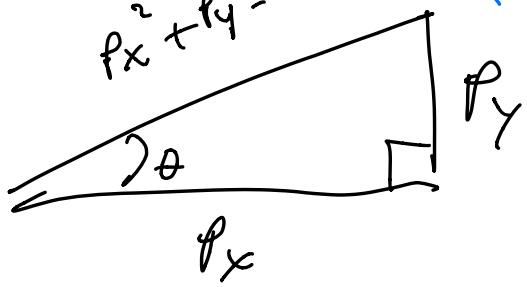
b) $$p_x' = \gamma\left(p_x - \frac{EV}{c^2}\right)$$

$$p_y' = p_y$$

$$\tan\theta' = \frac{p_y'}{p_x'} = \frac{p_y}{\gamma\left(p_x - \frac{EV}{c^2}\right)} \quad \frac{1/p_x}{1/p_x}$$

$$\Rightarrow \frac{\tan\theta}{\gamma\left(1 - \frac{EV}{p_x c^2}\right)}$$

Now $p_x^2 + p_y^2 = E^2/c^2$ ← $E^2 - c^2(p_x^2 + p_y^2) = 0^2$




$$\frac{E/c}{p_x} = \cos\theta$$

$$\tan\theta' = \frac{\tan\theta}{\gamma\left(1 - \frac{v}{c}\frac{1}{\cos\theta}\right)} \quad \frac{\cos\theta}{\cos\theta}$$

$$\Rightarrow \quad \tan\theta' = \frac{\sin\theta}{\gamma(\cos\theta - \beta)}$$