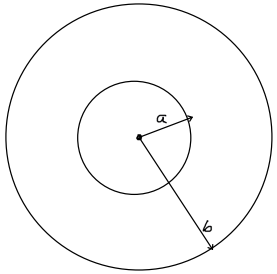
Consider the configuration of three concentric spherical capacitors as shown in the figures below. Please explain your reasoning when answering the following questions

a) In the figure to the left, in a) the concentric conducting spheres have inner radii "a" and outer radii "b" with the region between the spheres a vacuum. Fint the capacitance of the configuration

b) In the figure on the left a dielectric material with constant k fills up the volume between the spheres up to radiu (a+b)/2. Find the total capacitance of the configuration of spheres

c) In the figure on the left, find the total capacitance if now the dielectric material fills up the upper hemisphere of the space between the spheres as shown. The capcitance of the dielectriç material is k.

a) First lets find the capacitance of a spherical shell of radius a to b. Capacitance $C = \frac{Q}{V}$

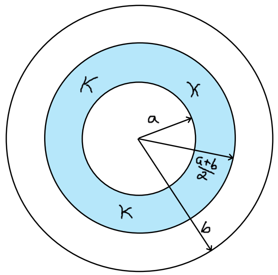


$$V_{a\to b} = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \quad = -\int_a^b \frac{Q}{4\pi\epsilon_0 r^2}\,dr = -\frac{Q}{4\pi\epsilon_0}r\Big|_a^b = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$C = \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)} = \boxed{C = \frac{4\pi\epsilon_0 ab}{(b-a)}}$$

b) Now find the total capacitance of the dielectric sphere within the metal sphere first we note the configuration of the sphere within the sphere is that of two capacitors in series because the charge that accumulates on all surface does so with the same magnitude



$$C_T = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} \quad \text{where } C_1 = \frac{4\pi\epsilon_0\left(\frac{a+b}{2}\right)b}{b-\left(\frac{a+b}{2}\right)} \;;\; C_2 = \frac{4\pi\epsilon_0\left(\frac{a+b}{2}\right)aK}{\frac{a+b}{2}-b}$$

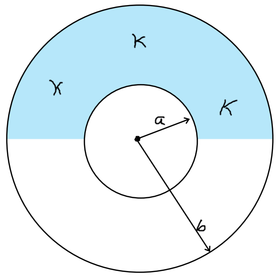
the rest is algebra:

$$\frac{1}{\frac{\frac{a+b}{2}-a}{4\pi\epsilon_0\left(\frac{a+b}{2}\right)aK} + \frac{b-\left(\frac{a+b}{2}\right)}{4\pi\epsilon_0\left(\frac{a+b}{2}\right)b}} = \frac{4\pi\epsilon_0\left(\frac{a+b}{\cancel{2}}\right)abK}{\left[\frac{a+b}{\cancel{2}}-2a\right]b+\left[2b-\left(\frac{a+b}{\cancel{2}}\right)\right]aK}$$

$$= \frac{4\pi\epsilon_0(a+b)abK}{[b-a]b+[b-a]aK}$$

$$\boxed{C_T = \frac{4\pi\epsilon_0(a+b)abK}{(b-a)(b+aK)}}$$

c) Now find the total capacitance of the two half spherical shells one filled with a dielectric material the other empty
Now we note the configuration of the two half spheres is that of two capacitors in parallel because the potential btw a and b are the same.



$$C_T = C_1 + C_2 \quad \text{where } C_1 = \frac{4\pi\epsilon_0 ab}{2(b-a)} \;;\; C_2 = \frac{4\pi\epsilon_0 abK}{2(b-a)}$$

$$C_T = \frac{2\pi\epsilon_0 ab}{(b-a)} + \frac{2\pi\epsilon_0 abK}{(b-a)} \qquad \boxed{C_T = \frac{2\pi\epsilon_0 ab(K+1)}{(b-a)}}$$