2015
ern - 9

Mod
(SR)

9. (a) A laser emits a pulse of light which travels at a speed of c (in vacuum) relative to the laser. Does this mean that the speed of the laser relative to the light pulse is also c? Give your reasoning for your answer.

(b) A laser emits a pulse of light in the positive *y* direction at the same time that a space-ship passes the laser at a speed of 0.9c in the positive *x* direction. What is the speed and direction of the light pulse relative to the space-ship?

a) No, the light pulse is not a valid inertial reference frame.

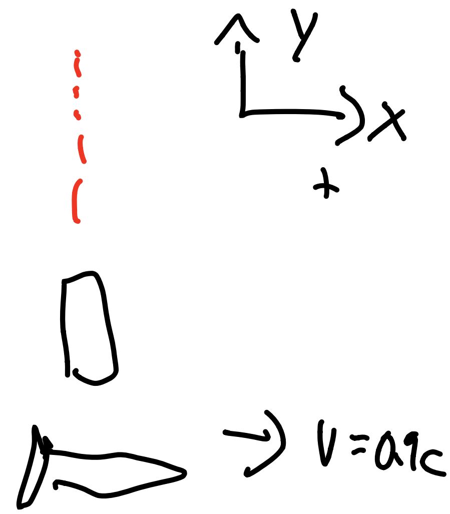
Also $P_{laser}^2 = M_{laser}^2 c^4 \neq 0$

If $v = c \Rightarrow P_{laser}^2 = 0$.

b) The speed is c since light travels at that speed for all inertial reference frames.

Let S be the reference frame of the laser and S' the

Space-ship. Then,



$$c\,dt' = \gamma\left(c\,dt - \frac{v\,dx}{c}\right)$$

$$dx' = \gamma(-v\,dt + dx)$$

$$dy' = dy$$

$$dz' = dz$$

Let $u_x = \dfrac{dx}{dt}$ $\quad u_x' = \dfrac{dx'}{dt'}$

$u_y = \dfrac{dy}{dt}$ $\quad u_y' = \dfrac{dy'}{dt'}$

Then

$$u_x' = \frac{dx'}{dt'} = \frac{\cancel{\gamma}(-v\,dt + dx)}{\frac{1}{c}\cancel{\gamma}\left(c\,dt - \frac{v\,dx}{c}\right)} \qquad \frac{1/dt}{1/dt}$$

$$= \frac{-v + dx/dt}{1 - v\,dx/dt}$$

$$\boxed{u_x' = \frac{-v + u_x}{1 - u_x v/c^2}} \qquad (1)$$

$$u_y' = \frac{dy'}{dt'} = \frac{dy}{\frac{1}{c}\gamma\left(c\,dt - \frac{v}{c}dx\right)}$$

$$= \frac{1}{\gamma}\frac{dy}{dt - \frac{v}{c^2}dx} \qquad \frac{1/dt}{1/dt}$$

$$\boxed{u_y' = \frac{1}{\gamma}\frac{u_y}{1 - uv/c^2}} \qquad (2)$$

Plugging in $v = 0.9c$ and

$u_x = 0 \quad u_y = c$ into (1) & (2)

$$u_x' = \frac{-0.9c + 0}{1 - 0}$$

$$u_x' = -0.9c$$

$$u_y' = \sqrt{1-0.9^2}\ \frac{c}{1+0}$$

$$u_y' = \sqrt{1-0.9^2}\ c$$

Therefore, the direction of the velocity of the light pulse is

$$\boxed{\begin{aligned}(u_x', u_y', u_z') &= (-0.9c, \sqrt{1-0.9^2}\,c, 0)\\ &\doteq (-0.9c, 0.437c, 0)\end{aligned}}$$

And $|\vec{u}'|^2 = c^2$