2017 Modern - 8 (SR)

8) Given the stated measurements below each with independent, uncorrelated and random uncertainties, calculate the value and uncertainty for the results q for each of the scenarios. note: quote properly the units in your final answer

a. When q depends on x and y as $q = x^2 y$, $x = 3.0 \pm 0.3$ cm, $y = 2.0 \pm 0.1$ m.

b. The charge to mass ratio e/m of an electron is determined from accelerating voltages V, and currents I and I_e,

$$q = e/m = 1.6 \times 10^{10} \frac{V}{(I - I_e)^2}$$

where $V = 22.2 \pm 1.1$ [V], $I = 1.7 \pm 0.2$ [A] and $I_e = 0.2 \pm 0.1$ [A]

a) $$\Delta q = \sqrt{\left[\frac{\partial q}{\partial x}\Delta x\right]^2 + \left[\frac{\partial q}{\partial y}\Delta y\right]^2}$$

$$= \sqrt{(2xy\,\Delta x)^2 + [x^2\,\Delta y]}$$

$$= \sqrt{\Big(\underbrace{2x^2y}_{q}\,\frac{\Delta x}{x}\Big)^2 + \Big(\underbrace{x^2y}_{q}\,\frac{\Delta y}{y}\Big)^2}$$

$$\Delta q = q\sqrt{4\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

$$= (0.03)^2\, 2^2 \sqrt{4(0.1)^2 + (0.05)^2} \qquad m^2$$

b) $$\Delta q = \sqrt{\left(\frac{\partial q}{\partial V}\Delta V\right)^2 + \left(\frac{\partial q}{\partial I}\Delta I\right)^2 + \left(\frac{\partial q}{\partial I_e}\Delta I_e\right)^2}$$

$$= \sqrt{\left(\frac{1}{(I-I_e)^2}\Delta V\right)^2 + \left(-2\frac{V}{(I-I_e)^3}\Delta I\right)^2 + \left(2\frac{V}{(I-I_e)^3}\Delta I_e\right)^2}$$

$$= \sqrt{\left(\underbrace{\frac{V}{(I-I_e)^2}}_{q}\frac{\Delta V}{V}\right)^2 + 4\left(\underbrace{\frac{V}{(I-I_e)^2}}_{q}\frac{\Delta I}{(I-I_e)}\right)^2 + 4\left(\underbrace{\frac{V}{(I-I_e)^2}}_{q}\frac{\Delta I_e}{I-I_e}\right)^2}$$

$$= q\sqrt{\left(\frac{\Delta V}{V}\right)^2 + 4\left(\frac{\Delta V}{I-I_e}\right)^2 + 4\left(\frac{\Delta I_e}{I-I_e}\right)^2}$$