Modern Physics 2019 cont.

Section B: Modern Physics

Problem B1: *Proxima b* is the closest known exoplanet orbiting in the habitable zone of the star *Proxima Centauri*, approximately 4.2 light-years from the Earth. NASA decides to send its fastest experimental spacecraft that will travel at 0.75c to study this planet.

a. According to an observer on Earth, how long will this trip take?
b. According to the crew of the spacecraft, how long will this trip take?
c. According to the crew of the spacecraft, how far will they travel?
d. As the spacecraft approaches *Proxima b*, the crew sends a radio signal back to Earth informing Mission Control of their arrival. If Mission Control is expecting a radio signal with frequency of 2.0 GHz, at what frequency should the spacecraft broadcast? Assume that the spacecraft is still traveling at 0.75c when the signal is broadcast.

B1 a) $t_E = \frac{x}{v} = \frac{4.2\ c\ year}{0.75\ c} = 5.6\ years$

b) Use formula for time dilation:

$$t_C = t_E\sqrt{1-\frac{v^2}{c^2}} = t_E\sqrt{1-0.75^2} = 3.7\ years$$

c) Zero. In their reference frame they are at rest.

d) The Doppler shift is

$$f_E = \sqrt{\frac{1+v/c}{1-v/c}}f_C$$

One way of getting this is thinking in terms of a Lorentz transformations. The energy of the signal according to Earth is:

$$E_E = \gamma(E_C + v\,p_c c)$$

However, since photons are massless $E_C = cp_C$. This,

$$E_E = \gamma\left(1+\frac{v}{c}\right)E_C$$

$$= \frac{\left(1+\frac{v}{c}\right)E_C}{\sqrt{1-\frac{v^2}{c^2}}} \qquad\qquad Use\ 1-\frac{v^2}{c^2}=\left(1-\frac{v}{c}\right)\left(1+\frac{v}{c}\right)$$

$$= \frac{\left(1+\frac{v}{c}\right)E_C}{\sqrt{\left(1-\frac{v}{c}\right)\left(1+\frac{v}{c}\right)}}$$

$$\rightarrow \ E_E = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}\ E_C$$

Substitute $E = \hbar f$ (technically, don't need to invoke quantum mechanics, but it's quicker and gives right answer):

$$\hbar f_E = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}\ \hbar f_C$$

$$\rightarrow \ f_E = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}\ f_C$$

Now, since the signal and ship are going in opposite directions take $v = -0.75c$

$$2.0\ GHz \ = \sqrt{\frac{1-0.75}{1+0.75}}\ f_C$$

$$2.0\ GHz \ = \frac{1}{\sqrt{7}}\ f_C$$

$$\rightarrow f_C = 5.3\ GHz$$

Note: when objects move towards you the signal gets blue shifted (i.e., frequency increases); when objects move away the signal get red shifted (i.e., frequency decreases). Thus, the signal sent to Earth is going to be red shifted and should be sent at a higher frequency. Thus, if you get confused by signs take the physics into account.