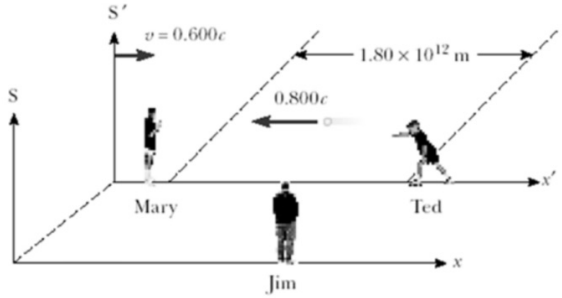
Problem MP3: Ted and Mary are playing a friendly game of relativistic catch in frame S', which is moving at $0.600c$ with respect to frame S. Jim, at rest in frame S, watches the action. Ted throws the ball to Mary at $0.800c$ (according to Ted) and their separation (measured in S') is 1.80×10^{12} m.

a) According to Mary, how fast is the ball moving?
b) According to Mary, how long does it take the ball to reach her?
c) According to Jim, how far apart are Ted and Mary,
d) According to Jim, how fast is the ball moving?
e) According to Jim, how long does it take the ball to reach Mary?



a) $V = -0.8c$

b) $\Delta t = \frac{\Delta x'}{|V|} = \frac{1.8 \times 10^{12}\ m}{0.8(3 \times 10^8 m/s)}$

$= \frac{1.8}{2.4} \times 10^4\ s = 7.5 \times 10^3 s$

c) $\Delta x = \sqrt{1 - \frac{v^2}{c^2}}\ \Delta x'$

$$\Delta X = \sqrt{1 - 0.6^2}\ (1.8 \times 10^{12}\ m)$$

$$\Delta X = 1.44 \times 10^{12}\ m$$

d)

$$\Delta X = \gamma(\Delta X' + V\Delta t')$$

$$\Delta t = \gamma\left(\Delta t' + \frac{V\Delta X'}{c^2}\right)$$

$$\frac{\Delta X}{\Delta t} = \frac{\Delta X' + V\Delta t'}{\Delta t' + \frac{V\Delta X'}{c^2}}$$

$$\Rightarrow u_x = \frac{u_x' + V}{1 + \frac{V u_x'}{c^2}}$$

$$u_x = \frac{-0.8c + 0.6c}{1 - \frac{0.8(0.6)c^2}{c^2}}$$

$$= \frac{-0.2c}{1 - 0.48} = -0.385c$$

e)

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}} = \frac{7.5 \times 10^{3}\,s}{0.8}$$

$$= 9.38 \times 10^{9}\,s$$