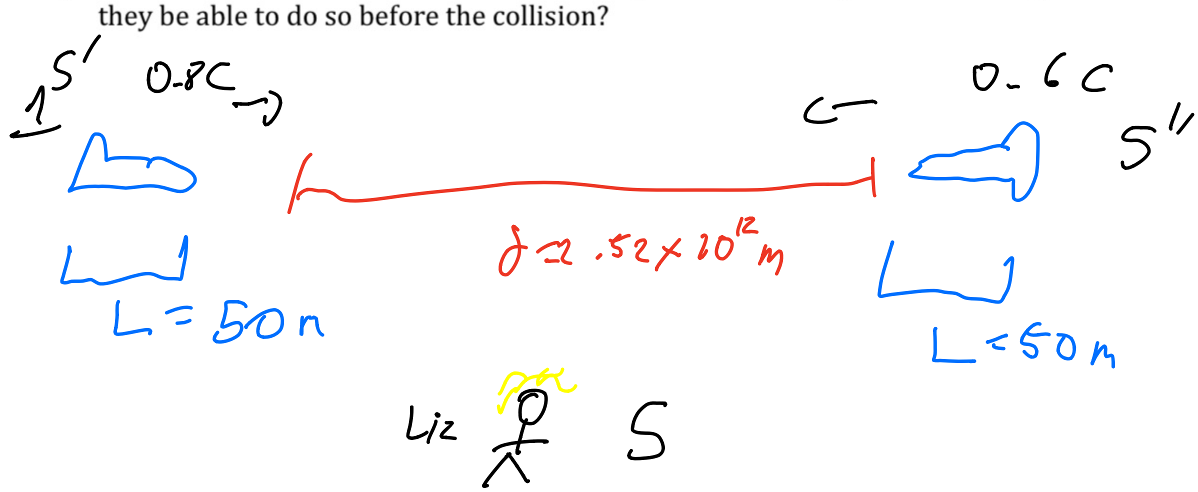
2021 Modern - MP1

MP1: Two rockets that are coasting (engines off) are heading towards each other on a collision course. As measured by Liz, a stationary Earth observer, Rocket 1 has a speed of 0.800c, Rocket 2 has a speed of 0.600c, both rockets are 50.0 m in length, and are initially 2.52 billion kilometers apart.

a) What are their respective proper lengths?
b) What is the length of each rocket as observed by a stationary observer in the other rocket?
c) According to Liz, how long before the rockets collide?
d) According to Rocket 1, how long before they collide?
e) According to Rocket 2, how long before they collide?
f) If the crews can evacuate their rockets safely within 50 min (their own time), will they be able to do so before the collision?



a) Use the formula for length contraction.

$$L_1 = L_1' \sqrt{1 - v_1^2/c^2}$$

$$50\ m = L_1' \sqrt{1 - 0.8^2}$$

$$L_1' = \frac{50\ m}{\sqrt{0.36}} = \frac{50\ m}{0.6}$$

$$L_1' = 83.3\ m$$

$$L_2 = L_2'' \sqrt{1 - V_2^2/c^2}$$

$$50\ m = L_2'' \sqrt{1 - 0.6^2}$$

$$L_2'' = \frac{50\ m}{0.8}$$

$$\boxed{L_2'' = 62.5\ m}$$

b) Use relativistic velocity sum formula

$$u_x' = \frac{u_x - V}{1 - \frac{u_x V}{c^2}}$$

Speed of 2 in RF of S (u_x)

Speed of S' in RF of S (V)

Speed of 2 in RF of S' (u_x')

$$= \frac{-0.6c - 0.8c}{1 - \frac{(-0.6)(0.8)c^2}{c^2}}$$

$$u_x' = \frac{-1.4c}{1 + 0.48} = -0.946c$$

$$L_2' = L_2'' \sqrt{1 - u_x'^2/c^2}$$

$$L_2' = 62.5\,m \sqrt{1 - 0.946^2}$$

$$62.5\,m\,(0.324)$$

$$\boxed{L_2' = 20.3\ m}$$

According to 2, 1 is moving at $v = 0.946c$.

So,

$$L_1'' = L_1' \sqrt{1 - 0.946^2}$$

$$= 83.3\,(0.324)$$

$$\boxed{L_1'' = 27.0\ m}$$

c)

$$v_1 t - d/2 = v_2 t + d/2$$

$$(v_1 - v_2)\,t = d$$

$$t = \frac{d}{v_1 - v_2}$$

$$= \frac{2.52 \times 10^{12}\,m}{0.8c - 0.6c}$$

$$= \frac{2.52 \times 10^{12} m}{1.4 c}$$

$$= \frac{2.52 \times 10^{12} m}{1.4 \times 3 \times 10^{8} m/s}$$

$$t = 0.6 \times 10^{4} s$$

$$\boxed{t = 6 \times 10^{3} s}$$

Or 100 minutes = 1 hr 40 min

d) $t_1' = t \sqrt{1 - \frac{v_1^2}{c^2}}$

proper time of 1

$$= t \sqrt{1 - 0.8^2}$$

$$= 6 \times 10^{3} \times 0.6 \ s$$

$$t' = 3.6 \times 10^{3} s$$

$$= 60 min = 1 hr$$

e) $t_2 = t\sqrt{1-0.6^2}$

$t_2 = 0.8 \times 6 \times 10^3 s$

$t_2 = 4.8 \times 10^3 s$

$= 80$ min $= 1$ hr 20 m

f) Rocket 1 → No

Rocket 2 → No