# Classical Physics
## Mock Qualifier Exam 2022
## Department of Physics at FIU

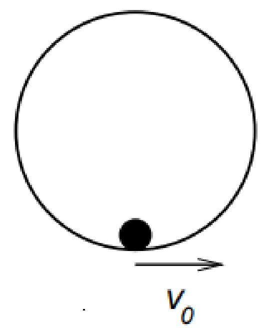
**Instructions:** There are nine problems on this exam. Three on Mechanics (**Section CM**), four on Electricity and Magnetism (**Section EM**), and two on Statistical Physics and Thermodynamics (**Section SP**). You must solve a total of six problems with at least **two** from **Section CM**, **two** from **Section EM**, and **one** from **Section SP**.

Do each problem on its own sheet (or sheets) of paper and write only on one side of the page. Do not forget to **write the problem identifier (letters and number)** on each page you turn in. Also turn in only those problems you want graded (**Do NOT submit for grading more than 6 problems all together**). Finally, write your panther ID on each page at top left-hand corner and the question identifier on each page. **DO NOT WRITE your name** anywhere on anything you turn in.

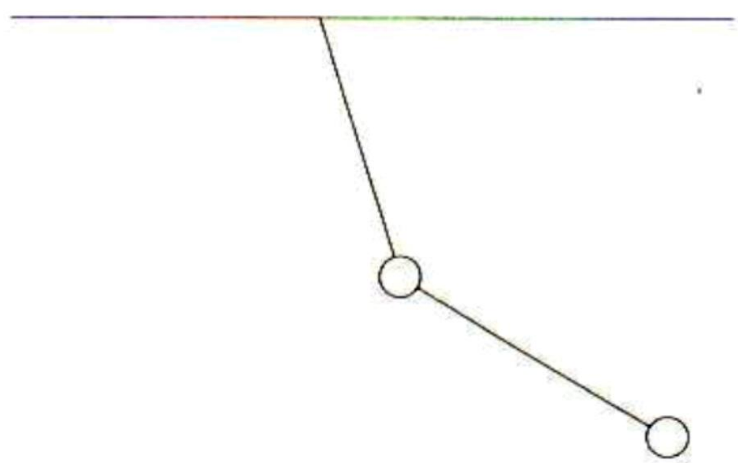
You may use a calculator and a math handbook as needed.

# Section: Classical Mechanics

Problem **CM1:** A particle of mass $m$ is moving without friction inside of a vertical circular track of radius $R$. When it is at its lowest position, its speed is $v_0$. Use the Lagrange method to find the minimum value of $v_0$ for which the particle will go completely around the circle without losing contact with the track?




**CM2:** Two identical simple pendulums (length $l$ and mass $m$) are coupled together as shown in the figure. Consider small oscillations. calculate the characteristic frequencies and derive the normal modes.



Problem **CM3:** A particle of mass $m$ moves on the surface of a circular cone of half angle $\alpha$ with its axis in the vertical direction. Gravity acts downward.

a) Write the equations of motion and find the conditions for motion of the particle to remain at a constant height $h$ above the cone's vertex.

b) Find the frequency of small oscillations about this horizontal trajectory.

# Section: Electricity and Magnetism

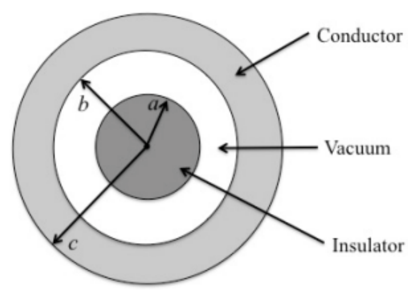
**EM1:** Two large parallel conducting plates are a distance $d$ apart. The region between them is filled with two linear isotropic and homogeneous (l.i.h.) layers. The first of thickness $x$ has conductivity $\sigma_1$ and permittivity $\varepsilon_1$. The second of thickness $d - x$ has conductivity $\sigma_2$ and permittivity $\varepsilon_2$. The plates are maintained at potentials $V_1$ and $V_2$, and there is a steady current flowing from one plate to the other.

a. Find the potential at the interface between the two layer.
b. Find the free surface charge density also at the interface between the two layers.

**EM2:** An insulating sphere of radius $a$ carries a charge density of

$$\rho = \frac{k}{r}$$

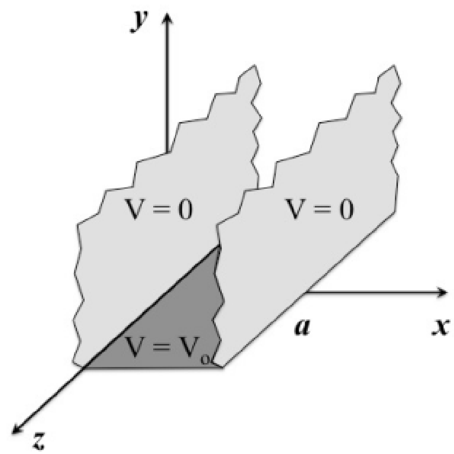
in the region $r \leq a$, where $k$ is a positive constant and $r$ is the distance from the center of the sphere. The sphere is surrounded by a thick, concentric conducting metal shell with an inner radius $b$ and an outer radius $c$. The conducting shell carries no net charge.




a. Determine the surface charge density at the inner ($r = b$) and outer surfaces ($r = c$) of the conductor.
b. Determine the electric field $\vec{E}$ in all four regions (i) $r < a$, (ii) $a < r < b$, (iii) $b < r < c$, and (iv) $r > c$.
c. Determine the electric potential V at the center of the sphere using infinity ($r = \infty$) as a reference point.
d. If the outer shell is grounded, what would the potential at the center of the sphere be using the same reference point as in part (c).

**EM 3:** A sphere of radius $a$, has a volume charge density $\rho = Ar^2$ where $A$ is a constant. It rotates about a diameter coinciding with the z-axis with constant angular velocity, $\omega$. What is its magnetic dipole moment?

**EM4:** Two infinitely large metal plates lie parallel to the $y$-$z$ plane, one at $x = 0$ and one at $x = a$, as shown in the figure to the right. These two plates are maintained at zero potential ($V = 0$) and both extend from $y = 0$ to $y = +\infty$ and from $z = -\infty$ to $z = +\infty$. A third plate, this one maintained at a constant potential $V_0$, lies in the $x$-$z$ plane and forms the bottom of a "slot". Determine an expression for the potential $V(x,y)$ for any point within the "slot". Notice that due to symmetry, the potential is independent of $z$.



# Section: Statistical Physics & Thermodynamics

**SP1:** Two state paramagnet: Consider a solid made up of a collection of magnetic dipoles. Each dipole can be in one of two states: antiparallel to the applied magnetic field and parallel to the applied magnetic field. Let $\epsilon$ and $-\epsilon$ be their respective energies ($\epsilon$ can be positive or negative). Let the solid be made up of $N$ dipoles.

(a) Derive an expression for the number of microstates in the macrostate with $N_\uparrow$ dipoles aligned parallel to the applied magnetic field.

(b) Write down the expression for the total energy $U$ in terms of $N$ and $N_\uparrow$.

(c) Assume $N, N_\uparrow, (N - N_\uparrow)$ are all large compared to unity. Prove that

$$U = -N\epsilon \tanh \frac{\epsilon}{kT}.$$

**SP2:** The complete thermodynamic cycle of a heat engine using an ideal gas with constant specific heat capacities consists of four steps. Step A is an *adiabatic* compression from pressure $P_1$ and volume $V_1$ to pressure $P_2$ and volume $V_2$. Step B is an *isobaric* expansion at pressure $P_2$ from volume $V_2$ to volume $V_3$. Step C is an *adiabatic* expansion from pressure $P_2$ and volume $V_3$ to pressure $P_1$ and volume $V_4$. Step D is an *isobaric* compression at pressure $P_1$ from volume $V_4$ back to the original volume $V_1$.

a) Make a *PV* diagram for the complete cycle.

b) Show that the ratio of the heat flow out of the engine during step D to the heat flow into the engine during step B is given by
$Q_{out}/Q_{in} = (T_4\text{-}T_1)/(T_3\text{-}T_2)$

c) The efficiency of the engine is defined as the ratio of the work done by the engine to the input heat transfer. Use the ideal gas equation together with the fact that $PV^\gamma$ is constant for an adiabatic process, where $\gamma$ is the adiabatic index of the gas, to show that the efficiency of the engine is given by
$e = 1 - \left(\frac{P_1}{P_2}\right)^\alpha$,
where $\alpha = \frac{(\gamma-1)}{\gamma}$.