MP2: Two events occur at locations x_1 and x_2 and at times t_1 and t_2 in reference frame S.

a. What is the time difference, $\Delta t' = t_2' - t_1'$ between these two events in a reference frame S' that is moving with speed βc in the positive x direction relative to S? Your answer should be in terms of Δt, Δx, β, and γ, where $\Delta t = t_2 - t_1$ and $\Delta x = x_2 - x_1$

b. Describe the special case of $x_1 = x_2$.

c. Find β such that the two events occur simultaneously in S' and describe any limiting conditions.

a) A Lorentz transformation gives

$$\Delta t' = \frac{1}{\sqrt{1-\beta^2}}\left(\Delta t - \frac{\Delta x}{c}\beta\right) \qquad (1)$$

b) In this case

$$\Delta t' = \frac{\Delta t}{\sqrt{1-\beta^2}}$$

Since $\beta^2 < 1$, $\Delta t' > \Delta t$. This is known as a time dilation.

c) Looking at Eq. (1),

$$0 = \frac{1}{\sqrt{1-\beta^2}}\left(\Delta t - \frac{\Delta x}{c}\beta\right)$$

$$\Rightarrow \qquad \Delta t = \frac{\Delta x}{c} \beta$$

$$\boxed{\beta = c\,\frac{\Delta t}{\Delta x}}$$

Since $\beta < 1 \Rightarrow \boxed{c\Delta t < \Delta x}$

This means the invariant length:

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 < 0$$

This holds for all inertial reference frames since Δs^2 is invariant. Thus, it's required that

$$\boxed{(\Delta s)^2 < 0}$$

or: the two events must be space-like seperated.