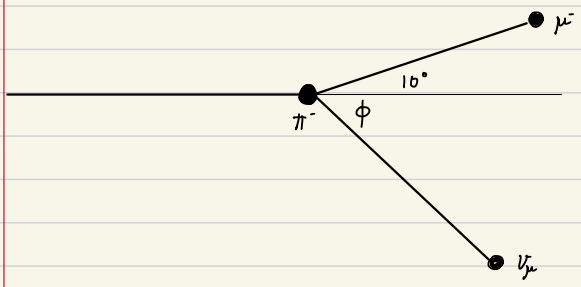
8. **Required** A pion, π^-, has a rest mass of 139.57 MeV/c and is moving with a speed $\beta = 0.5c$. It then decays into a muon/neutrino pair, $\mu^-\nu_\mu$, with the muon traveling off at an angle of 10^0 relative to the original direction of the pion. What is the energy and momentum of the muon and what is the angle, energy, and momentum of the neutrino? The muon has a rest mass of 105.65 MeV/c and you may assume the neutrino is massless



$P_{\mu^- x} + P_{\nu_\mu x} = P_{\pi^-}$ Conservation of P_x

$P_{\mu^- y} + P_{\nu_\mu y} = 0$ Conservation of P_y

$E_{\pi^-} = E_{\mu^-} + E_{\mu^-}$ Conservation of E

$\tan 10^\circ = \dfrac{P_{\mu^- y}}{P_{\mu^- x}} \rightarrow P_{\mu^- y} = P_{\mu^- x} \tan 10^\circ$ given

$$E_{\pi^-} = \sqrt{m_\pi^2 c^4 + p^2 c^2} = \sqrt{m_\mu^2 c^4 + (P_{\mu^- x}^2 + P_{\mu^- y}^2) c^2} + \sqrt{(P_{\nu_\mu x}^2 + P_{\nu_\mu y}^2)}\, c$$

First, solve for $P_{\mu^- x}$

$$139.57\ MeV = \sqrt{(105.65)^2 + P_{\mu^- x}^2 (1 + \tan^2 10^\circ)} + c\sqrt{(P_\pi - P_{\mu x})^2 + P_{\mu^- x}^2 \tan^2 10^\circ}$$

$P_{\nu_\mu y} = -P_{\mu^- y} = -P_{\mu^- x} \tan 10^\circ$

From $P_{\mu^- x}$ we can find $P_{\mu^- y}$ since $P_{\mu^- y} = P_{\mu^- x} \tan 10^\circ$

Then we can find $E_\mu = \sqrt{(105.65)^2 + P_{\mu^- x}^2 (1 + \tan^2 10^\circ)}$

Then we can find $E_\nu = E_\pi - E_\mu$

And we can find $P_{\nu_y} = -P_{\mu^- y}$

and $P_{\nu_x} = P_\pi - P_{\mu^- x}$ where $P_\pi = \dfrac{1}{\sqrt{1-\beta^2}} m_\pi \beta c$