Modern Physics

Mock Qualifier Exam 2022

Department of Physics at FIU

Instructions: There are nine problems on this exam. Five on Quantum Mechanics (**Section QM**), Four on general Modern Physics (**Section MP**), You must solve a total of six problems with at least **two** from **each section**.

Do each problem on its own sheet (or sheets) of paper and write only on one side of the page. Do not forget to **write the problem identifier (letters and number)** on each page you turn in. Also turn in only those problems you want graded (**Do NOT submit for grading more than 6 problems all together**). Finally, write your panther ID on each page at top left-hand corner and the question identifier on each page. **DO NOT WRITE your name** anywhere on anything you turn in.

You may use a calculator and a math handbook as needed.

Section: Quantum Mechanics

QM1: An electron spin is in a uniform magnetic field $\vec{B} = B_0\,\hat{\jmath}$ (where $\hat{\jmath}$ is the unit vector in the $+y$ direction). At $t = 0$, the electron spin is aligned in the $+z$ direction. Find the spin wavefunction at late time t. Express your answer in terms of the $-\rangle$ and $+\rangle$ eigenstates in the z direction.

QM2: A particle of mass m is stuck in a 2-D rectangular box of length a and width b. Derive the normalized wavefunctions and allowed energies for the ground and first excited states using the Schrodinger equation.

QM3: A particle of mass m and charge q is in a one-dimensional harmonic oscillator potential moving with frequency ω. In addition, it is subject to a *weak* electric field $\mathcal{E}$.

a) Find the exact expression for the energy.

b) Calculate the energy up to the first non-zero correction in non-degenerate perturbation theory. Compare your result to what you found in part a.

QM4: A particle with positive energy ($E > 0$) encounters a Dirac delta-function potential barrier

$$V(x) = \alpha\delta(x)$$

where α is positive real constant with the units of energy×length.

a) Solve the Schrödinger equation for the regions $x < 0$ and $x > 0$. If you make any assumptions, you must verify your answer to receive full credit.
b) State and apply the appropriate boundary conditions for the potential interface at $x = 0$. Obtain expressions relating the different coefficients.
c) Using the results from part b), determine the transmission coefficient T for a particle incident and transmitted through this potential barrier. Discuss the limiting cases for T as the energy E approaches zero ($E \rightarrow 0$) and E approaches infinity ($E \rightarrow \infty$).

QM5: The Hamiltonian of a quantum system is given by $H = \begin{pmatrix} \alpha & 0 \\ 0 & \alpha \end{pmatrix} + \begin{pmatrix} 0 & -i\beta \\ i\beta & 0 \end{pmatrix}$ where α and β are real constants.

a. Find the energy eigenvalues and eigenfunctions.
b. With $\beta \ll \alpha$ treat the 2^{nd} term as a small perturbation, find the first-order correction to the energy eigenvalues and the corresponding eigenfunctions. Compare with your results in (a).

Section: Modern Physics

MP1: Let $\mathbf{u} = d\mathbf{r}/dt$ be the velocity of a particle observed in an inertial frame K. The same quantity observed in an inertial frame K' moving with velocity $\mathbf{v}$ with respect to K is $\mathbf{u}' = d\mathbf{r}'/dt$.

a) Use the transformation properties of dt, $\mathbf{r}_\parallel$ and $\mathbf{r}_\perp$(the direction along the velocity $\mathbf{v}$ or perpendicular to the moving K' frame, respectively), to directly derive the velocity addition rule,

$$\mathbf{u}_\parallel = \frac{d\mathbf{r}_\parallel}{dt} = \frac{\mathbf{u}'_\parallel + \mathbf{v}}{1 + \frac{\mathbf{v}\cdot\mathbf{u}'_\parallel}{c^2}} \quad \text{and} \quad \mathbf{u}_\perp = \frac{d\mathbf{r}_\perp}{dt} = \frac{\mathbf{u}'_\perp}{\gamma(v)\left[1 + \frac{\mathbf{v}\cdot\mathbf{u}'_\parallel}{c^2}\right]}.$$

b) Let $\mathbf{v}$ define a polar axis with polar coordinates $\mathbf{u} = (u, \theta)$, and $\mathbf{u}' = (u', \theta')$ for the particle velocities as measured in K and K'. Write the transformation laws in part (a) in the form of $u = u(u', \theta')$ and $\theta = \theta(u', \theta')$

c) Use the results of (b) to show that $u \to c$ when $v \to c$.

MP2: Two events occur at locations x_1 and x_2 and at times t_1 and t_2 in reference frame S.

a. What is the time difference, $\Delta t' = t_2' - t_1'$ between these two events in a reference frame S' that is moving with speed βc in the positive x direction relative to S? Your answer should be in terms of Δt, Δx, β, and γ, where $\Delta t = t_2 - t_1$ and $\Delta x = x_2 - x_1$

b. Describe the special case of $x_1 = x_2$.

c. Find β such that the two events occur simultaneously in S' and describe any limiting conditions.

MP3: A pion, π^-, has a rest mass of 139.57 MeV/c and is moving with a speed $\beta = 0.5c$. It then decays into a muon/neutrino pair, $\mu^-\nu_\mu$, with the muon traveling off at an angle of 10^0 relative to the original direction of the pion. What is the energy and momentum of the muon and what is the angle, energy, and momentum of the neutrino? The muon has a rest mass of 105.65 MeV/c and you may assume the neutrino is massless

MP4: Consider the process of Compton scattering. A photon of wavelength λ is scattered off a free electron initially at rest. Let λ' be the wavelength of the photon scattered in a direction θ relative to the photon incident direction.

(a) Find λ' in terms of λ and θ and universal constants.

(b) Find the kinetic energy of the recoiled electron.