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# A Supply Chain Decision Making Model for Replenishment Problem with Back-Order and Lost Sale

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Abstract

In devising an appropriate supply chain decision making policy, a production manager often needs to consider issues such as multiple suppliers, quantity discounts, transportation batch and back-order. An integrated supply chain decision making model should cover the management of business activities and relationships internally within a firm, with upstream suppliers, and downstream customers along the supply chain. Studies of individual topics in supply chain management (SCM) have been done abundantly. Among them, inventory management has caught the most attention, and various inventory models and methodologies have been proposed. Replenishment with back-orders problem has been studied, and problems that consider both the ordering and the purchase aspects have also been found. This research first proposes a mixed integer programming (MIP) model to minimize the total cost, which includes replenishment cost, transportation cost, back-order cost and lost sale cost. In addition, multiple suppliers with different quantity discount schemes are considered, and vehicles with different loading limits are present. A particle swarm optimization (PSO) model is constructed next to deal with large-scale problems that are too complicated to solve by the MIP. A case study is presented to examine the practicality of the MIP and the PSO models.



*Keywords:* Supply chain management (SCM); Replenishment; Back-order; Lost sale; Mixed integer programming (MIP); Particle swarm optimization (PSO).

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## Introduction

A good supply chain management (SCM) is important for firms to provide low cost and high quality products with greater flexibility in today's competitive market, and as a result, to survive and attain a reasonable profit. In devising an appropriate supply chain decision making policy, a production manager needs to consider multiple suppliers, transportation batch and quantity discounts. A supply chain decision making model should cover the management of business activities and relationships internally within a firm, with immediate suppliers, with the first and second-tier suppliers and customers along the supply chain, and with the entire supply chain. Inventory management is one of the most important topics in supply chain management, and various inventory and replenishment models have been developed.

The rest of this paper is organized as follows. Related works are reviewed in next section. Then, a mixed integer programming (MIP) model and a PSO model are constructed to solve the problem. A case study is demonstrated, and the conclusions are presented in the last section.

## Review of Related Works

In this section, some recent works on the joint replenishment problem (JRP) are reviewed. Khouja and Goyal (2008) reviewed the literature on the JRP from 1989 to 2005, and discussed several new methods for solving the JRP, including genetic algorithms (GAs). Reviews of some of the past works can also be found in Moin and Salhi (2007) and Anderson, Christiansen, Hasle, and Lokketangen (2010). Due to the interrelationships between inventory replenishments and routing patterns, it can be difficult to find the exact solution for the inventory routing problem (Federgruen & Zipkin, 1984; Chien, Balakrishnan, & Wong, 1989). Scholars have tried to apply heuristics to obtain near-optimal solutions. Chiou (2005) aimed to minimize total inventory cost and transportation cost over a period of planning horizon, and developed heuristic solution procedure and iterative solution strategies to solve the complicated problem.

PSO is one of the popular metaheuristics methods applied in different fields. Some recent production management works using the PSO are reviewed here. Tsai and Yeh (2008) proposed a PSO approach for inventory classification problems and developed a flexible classification algorithm based on a specific objective or multiple objectives, such as minimizing costs and maximizing inventory turnover ratios, etc. Huang, Yang, and Hsu (2015) studied a two-stage parallel multiprocessor flow shop scheduling problem. An integer programming model, an original PSO and an improved PSO, called subgroup PSO, were applied, the effectiveness and robustness of the models were compared. Liu, Hu, Zhao, and Wang (2015) proposed a hybrid PSO-GA algorithm for job shop scheduling in machine tool production. The genetic operators, such as crossover and mutation operators in GA, are used to update the particles in the PSO algorithm, and better solution quality and convergence rate are achieved.

## Construction of the Models

In this section, various costs for determining the total cost in a system are introduced first, a MIP model is constructed next, and the PSO procedure is described last.

### Various Costs

Eq. (1) shows the ordering cost, where $o_{iv}$ is the ordering cost of part *i* from supplier *v* for each purchase and $Z_{ivt}$ indicates whether an order of part *i* from supplier *v* in period *t* is placed.

$$Ordering\ cost = \sum_{i=1}^{I}\sum_{v=1}^{V}\sum_{t=1}^{T} o_{iv} * Z_{ivt} \tag{1}$$

Eq. (2) is the purchase cost. Based on the all-units discount brackets from suppliers and the purchase quantity in each period, the total purchase cost of the parts over the horizon can be calculated.

$$Purchase\ cost = \sum_{i=1}^{I}\sum_{v=1}^{V}\sum_{t=1}^{T} C(Q_{ivt}) * Q_{ivt} * Z_{ivt} \tag{2}$$

where $C(Q_{ivt})$ is the unit purchase cost of part *i* from supplier *v* in period *t*, $Q_{ivt}$ is the quantity of part *i* purchased from supplier *v* in period *t*, and $\alpha_{tvr}$ indicates whether an order of part *i* from supplier *v* in period *t* is placed.

The transportation cost of the system is obtained by equation (3), which calculates the total costs of transporting parts from the suppliers to the manufacturer over the periods.

$$Transportation\ \ cost = R = \sum_{v=1}^{V}\sum_{t=1}^{T} r_v \times \left\lceil \sum_{i=1}^{I} Q_{ivt} \Big/ \varphi \right\rceil = \sum_{v=1}^{V}\sum_{t=1}^{T} r_v \times Y_{vt} \tag{3}$$

where $r_v$ is the transportation cost per time from supplier *v* per period, $\varphi$ is the maximum transportation batch size from supplier *v* , $\lceil \sum_{i=1}^{I} Q_{ivt} / \varphi \rceil$ is the smallest integer greater than or equal to $\sum_{i=1}^{I} Q_{ivt} / \varphi$, $Y_{vt}$ is number of transportations from supplier *v* in period *t*.

Eq. (4) calculates the inventory holding cost of parts. Ending inventory of part *i* in a period is the sum of the beginning inventory of part *i* in the period and the purchase quantity of part *i* in the period and minus the quantity of part *r* used in production in the period. The inventory holding cost is as follows:

$$Holding\ cost = \sum_{i=1}^{I}\sum_{t=1}^{T} h_i * I_{it}^{+} \tag{4}$$

where $I_{it}^{+}$ is the ending inventory of part *i* in period *t*, $h_i$ is the unit holding cost of part *i* per period.

Eq. (5) calculates the back-order cost of parts.

$$\textit{Back-order cost} = \sum_{i=1}^{I}\sum_{t=1}^{T} b_i * k_{1i} * I_{it}^{-} \tag{5}$$

where $I_{it}^{-}$ is the shortage of part *i* in period *t*, $k_{1i}$ is the backorder ratio of part *i* , $b_i$ is the unit back-order cost of part *i* per period.

Eq. (6) calculates the lost sale cost of parts.

$$\textit{Lost sale cost} = \sum_{i=1}^{I}\sum_{t=1}^{T} l_i * k_{2i} * I_{it}^{-} \tag{6}$$

where $I_{it}^{-}$ is the shortage of part *i* in period *t*, $k_{2i}$ is the lost sale ratio of part *i* , $l_i$ is the unit lost-sale loss of part *i* per period.

**Mixed Integer Programming (MIP)**

In this sub-section, a mixed integer programming (MIP) model is developed to solve the multiple-supplier, multiple-product replenishment problem, and to devise the replenishment plan and production mode in each period in the planning horizon. The MIP model is as follows:

*Minimize*

$$TC = \sum_{i=1}^{I}\sum_{v=1}^{V}\sum_{t=1}^{T} o_{iv} * Z_{ivt} + \sum_{i=1}^{I}\sum_{v=1}^{V}\sum_{t=1}^{T} C(Q_{ivt}) * Q_{ivt} * Z_{ivt} + \sum_{v=1}^{V}\sum_{t=1}^{T} r_v * Y_{vt} + \sum_{i=1}^{I}\sum_{t=1}^{T} h_i * I_{it}^{+} + \sum_{i=1}^{I}\sum_{t=1}^{T} b_i * k_{1i} * I_{it}^{-} + \sum_{i=1}^{I}\sum_{t=1}^{T} l_i * k_{2i} * I_{it}^{-} \tag{7}$$

*s.t.*

Constraints.

Objective function (7) is to minimize the total cost in the planning horizon. The costs include six kinds of costs: ordering cost, purchase cost, transportation cost, holding cost, back-order cost and order sale cost.

**Particle Swarm Optimization (PSO)**

In this research, the PSO procedure is developed based on the constriction factor proposed by Kennedy and Eberhart (1995).

*Step 1. Initialize particles with random positions and velocities.* With a search space of *d*-dimensions, a set of random particles (solutions) is first initialized. Let the lower and the upper bounds on the variables' values be $\lambda_{min}$ and $\lambda_{max}$. We can randomly generate the positions, $\lambda_n^e$ (the superscript denotes the $e^{th}$ particle, and the subscript denotes the $n^{th}$ iteration), and the exploration velocities, $v_n^e$, of the initial swarm of particles:

$$\lambda_0^e = \lambda_{min} + rand(\lambda_{max} - \lambda_{min}) \tag{8}$$

$$v_0^e = \frac{\lambda_{min} + rand(\lambda_{max} - \lambda_{min})}{\Delta} = \frac{Position}{\Delta} \tag{9}$$

*Step 2*. *Evaluate the fitness of all particles.* The performance of each solution is evaluated with the fitness function.

*Step 3*. *Keep track of the locations where each individual has its highest fitness.*

*Step 4*. *Keep track of the position with the global best fitness.*

*Step 5*. *Update the velocity of each particle*:

$$v_{n+1}^e = \omega_n \times v_n^e + \varphi_1 \times rand_1 \times (pbest_n^e - \lambda_n^e) + \varphi_2 \times rand_2 \times (gbest_n - \lambda_n^e) \tag{10}$$

*Step 6*. *Update the position of each particle:*

$$\lambda_{n+1}^e = \lambda_n^e + v_{n+1}^e \cdot \Delta \tag{11}$$

*Step 7*. *Terminate the process*. If the standard deviation of fitness is less than an error, a predetermined (CPU) time is reached, or a maximum number of iterations is attained. Otherwise, go to *Step 2*.

**Case Study**

The proposed MIP and PSO models for joint replenishment problem with back-order and lost sale are applied in a case study here. The MIP model is implemented using the software LINGO 10, and the PSO is implemented using the software MATLAB (2015).

A case is presented here. The planning horizon contains three periods, and the demand of the finished good in each period is shown in Table 1. Table 2 also shows each kind of cost and the total cost of the firm in the horizon. Since both the MIP and the PSO lead to the same results, the total cost is the same, i.e. \$115,140. In addition, Figure 1 shows the PSO result generated from MATLAB. The computational time for the MIP and the PSO is 2 seconds and 156 seconds, respectively. For PSO, the initialized particle swarm size is selected as 50, and the maximum evolution number of the particle swarm is 1000. It can be seen that the PSO converges after the $38^{th}$ generation.

Table 1

*Demand of Finished Goods* ($d_{it}$)

| Period ($t$) | 1 | 2 | 3 |
|---|---|---|---|
| Demand | $d_{11}$=90, $d_{21}$=100 | $d_{12}$=20 | $d_{13}$=90, $d_{23}$=150 |

Table 2

*Relevant Results in Each Period under the Case Using the MIP and the PSO*

| Decision variables | $t$=1 | $t$ =2 | $t$ =3 |
|---|---|---|---|
| $Z_{ivt}$ | $Z_{111}=1$, $Z_{221}=1$ | | $Z_{113}=1$, $Z_{223}=1$ |
| $Q_{ivt}$ | $Q_{111}=100$, $Q_{221}=100$ | | $Q_{111}=96$, $Q_{221}=150$ |
| $C(Q_{ivt})$ | $C(Q_{111})=180$, $C(Q_{221})=150$ | | $C(Q_{111})=180$, $C(Q_{221})=140$ |
| $Y_{vt}$ | $Y_{11}$ =1, $Y_{21}$ =2 | | $Y_{13}$ =1, $Y_{23}$ =3 |
| $I_{it}^{+}$ | $I_{11}^{+}=10$ | | |
| $I_{it}^{-}$ | | $I_{12}^{-}$ =10 | |

| Ordering cost | Purchase cost | Transportation cost | Holding cost | Back-order cost | Lost sale cost | Total cost |
|---|---|---|---|---|---|---|
| $360 | $71,280 | $42,000 | $100 | 600 | 800 | $115,140 |

## Conclusions

Replenishment problem with back-order and lost sale has been studied in this research. The objective is to order the optimal quantities of different parts from right suppliers in different periods to reduce the total cost for the firm. Both a mixed integer programming (MIP) model and a particle swarm optimization (PSO) model are constructed to solve this joint replenishment problem with back-order and lost sale to minimize the total cost in the system during a planning horizon. The case study shows that the PSO model can generate optimal solutions under a short duration of time. When the problem scale is small, both the MIP and the PSO can lead to optimal solutions within a reasonable time. However, when the problem becomes complicated and reflects real application more, the MIP model may require a long computational time and may still not obtain the optimal solution. Therefore, the PSO model is an effective and efficient algorithm to search for solutions.

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