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A Goal Programming Approach for Vehicle Routing Problem with Time Windows

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Abstract

Supply chain management is very important in today's competitive business environment, and logistics is one of the main areas in the management. Vehicle routing problem (VRP) is a popular research problem in the logistics area because it a challenging combinatorial optimization problem in which a number of vehicles need to serve various demands of customers in different locations. This study aims to solve the problem under which a manufacturer needs to purchase materials from multiple suppliers with different time windows and to ship the materials back to the company in multiple periods, and different vehicles, with different assignment costs, loading capacities and unit travelling costs, are available. A mixed integer programming (MIP) model is constructed to minimize the total transportation cost, which includes vehicle assignment cost, travelling cost, tardiness cost, and earliness cost and for the manufacturer. Genetic algorithm (GA) is applied to solve the problem so that a near optimal solution can be obtained when the problem is too difficult to be solved using the MIP. A case of a food manufacturing company is used to examine the practicality of the proposed MIP model and the GA model.

Keywords: Vehicle routing; Time windows; Mixed integer programming (MIP); Genetic algorithm (GA).

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Introduction

Vehicle routing problem (VRP) is a core issue in logistics, and it refers to a class of combinatorial optimization problems in which customers are to be served by a number of vehicles (Adelzadeh, Asl, & Koosha, 2014). VRP was first introduced by Dantzig and Ramser (1959), and the main objectives of the problem are minimizing the total travelling cost, time, or distance with a fleet of vehicles, starting and ending their routes at the depot while satisfying various demands of customers. Some recent works of VRP, especially the adoption of metaheuristics, are reviewed here. Adelzadeh et al. (2014) proposed a mathematical model and devised a heuristic solving procedure for multi-depot vehicle routing problem with fuzzy time window and heterogeneous vehicles that have different capacities, velocities and costs. The fuzzy concept was applied to describe the service levels associated with time windows, and a multi-objective model was developed. A three-stage algorithm was first used to break down the problem to some common VRPs, and a heuristic approach was developed to obtain an initial solution. Many VRP are solved by heuristics these days to obtain a near-optimal solution. Since the introduction, different types of VRP have been tackled to incorporate real-world issues (Abdoli, MirHassani, & Hooshmand, 2017).

Genetic algorithm (GA) has been applied in solving VRP. Some works are reviewed here. Baker and Ayechew (2003) stated that unlike other heuristics which had considerable progress in solving VRP, GA had not made a great impact. The authors developed a conceptually straightforward GA to solve the basic VRP and a hybrid heuristic, which incorporated neighborhood search into the GA. A comparison with other heuristics showed that the GA was competitive in terms of computing time and solution quality. Ombuki, Ross, and Hanshar (2006) studied the VRP with hard time windows and treated it as a multi-objective problem with two objectives: number of vehicles and total distance traveled. A multi-objective genetic algorithm approach using the Pareto ranking technique was developed so that the weighted sum, which might cause solution bias, did not need to be applied. Alvarenga, Mateus, and de Tomi (2007) studied the VRP with time windows and proposed a two-phase approach that incorporated GA and a set partitioning formulation. The objective was to minimize the travel distance, and the authors stated that the approach outperformed past heuristic methods in terms of the minimal travel distance. Vidal, Crainic, Gendreau, and Prins (2013) further proposed a hybrid genetic search with advanced diversity control to tackle four types of VRP: VRP with time windows, periodic VRP with time windows, multi-depot VRP with time windows, site dependent VRP with time windows. Some evaluation techniques were introduced to the GA to efficiently evaluate and prune neighborhoods and to decompose large instances.

In this study, A VRP problem that considers a manufacturer purchasing materials from multiple suppliers and shipping the materials back to the company is studied. Different vehicles with different assignment costs, loading capacities and unit travelling costs are available. In addition, the tardiness cost and earliness cost to a supplier are considered. Both a mixed integer programming (MIP) model and a genetic algorithm (GA) model are constructed to solve the problem. The rest of this paper is organized as follows. In the next section, a MIP model and a GA model for solving the VRP are constructed. Then, a case study is performed to examine the MIP and the GA models. Some conclusion remarks are made in the last section.

Formulation of the Vehicle Routing Problem with Time Windows

Mixed Integer Programming (MIP)

In this section, a coordinated approach for the vehicle routing problem with time windows that considers soft time window and heterogeneous vehicles is proposed by the MIP and the GA.

An MIP model is constructed to solve the vehicle routing problem with time windows which considers vehicle assigning and travelling cost, tardiness cost and earliness cost.

$$\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{j=0}^{I}\tau_v \times X_{t,0,j,v} + \sum_{t=1}^{T}\sum_{i=0}^{I}\sum_{j=0}^{I}\sum_{v=1}^{V}\pi_{i,j} \times \rho_v \times X_{t,i,j,v} = Z_1 \quad (1)$$

$$\sum_{t=1}^{T}\sum_{i=0}^{I}\sum_{v=1}^{V}p_i \times L_{t,i,v} = Z_2 \quad (2)$$

$$\sum_{t=1}^{T}\sum_{i=0}^{I}\sum_{v=1}^{V}q_i \times E_{t,i,v} = Z_3 \quad (3)$$

$$\text{Min} \quad Z = Z_1 + Z_2 + Z_3 \quad (4)$$

s.t.

Constraints.

Eq. (1) calculates the vehicle assigning and travelling cost, where τ_v is the fixed cost for assigning vehicle *v*, $X_{t,0,j,v}$ is a binary variable, indicating whether vehicle *v* departs from the depot (*i*=0) to supplier *j* in period *t*, $\pi_{i,j}$ is the travelling time from supplier *i* to supplier *j*, ρ_v is the travelling cost per unit of time, $X_{t,i,j,v}$ is a binary variable, indicating whether vehicle *v* travels from supplier *i* to supplier *j* in period *t*. Eq. (2) calculates the tardiness cost, where ρ_i is the tardiness cost per unit of time, and $L_{t,i,v}$ is the tardiness time of vehicle *v* when arriving supplier *i* in period *t*. Eq. (3) calculates the earliness cost, where q_i is the earliness cost per unit of time, and $E_{t,i,v}$ is the earliness time of vehicle *v* when arriving supplier *i* in period *t*. Objective function (4) is to minimize the total transportation cost in a planning horizon, including the three kinds of costs: vehicle assigning and travelling cost, tardiness cost and earliness cost.

Genetic Algorithm (GA) Model

In this research, the GA is used to solve the multi-period vehicle routing problem with soft time window and heterogeneous vehicles so that near-optimal solutions can be obtained in a short period of computational time for large-scale problems. The proposed procedures are as follows (Lee, Kang, Lai, & Hong, 2013; Lu, Liu, Niu, & Zhu, 2014; Kang, Pearn, Chung, & Lee, 2016):

Step 1. Code scheme.

In the travelling salesman problem, there are basically five different vector schemes to represent a tour among cities: a path representation, an adjacency representation, an ordinal representation, a position listing representation, and an adjacency listing representation. Due to its simplicity, the path representation has been frequently adopted in the GA to solve the travelling salesman problem.

Step 2. Initial population of chromosomes.

The initial population is generated randomly. Two types of chromosomes are used, partially matched crossover (PMX) and tie break crossover (TBX), and the chromosome type is determined randomly.

Step 3. Fitness function

The fitness function for each chromosome is defined as Min *TC*, where *TC* is the summation of vehicle assigning cost, travelling cost and tardiness cost. Min *TC* is the minimum cost among all the chromosomes across the population.

Step 4. Crossover operation

The standard two-cut-point crossover operator is applied to the selected pair of parent-individuals by recombining their genetic codes and producing two offspring.

Step 5. Mutation operator.

A mutation operator is used to counteract premature convergence and to maintain enough diversity in the population. This is done by changing a randomly selected gene in the genetic code (0-1, 1-0).

Step 6. Selection of subsequent population.

Parent selection is the selection of the subsequent population after mutation and crossover operations in a generation. Individuals with higher fitness values are more likely to be selected for the mating pool, and vice versa. Individuals are sorted by their fitness values, and the number of reproductive trails of each individual is then allocated according to its rank.

Step 7. Elitism selection.

In the GA applications, the concept of elitism is widely used. Elitism selection guarantees the survival of best individuals created during all generations. Under a non-elitist GA, some individuals might not survive due to crossover of mutation.

Step 8. Termination.

Repeat the processes of crossover, selection and replacement until the objective function is optimized or the stop criterion is met.

Case Study

The case study is based on a manufacturer in the food industry. Assume that there are one manufacturer, several suppliers and several vehicles. The objective is to minimize the total transportation cost, which includes the vehicle assigning and travelling cost, tardiness cost and earliness cost. Software packages, LINGO 10 and MATLAB 2015, are used to solve the problem and compare the results from the MIP and the GA. In the case, there are three suppliers. Table 1 shows the travelling time matrix among suppliers. There are two vehicles, one small vehicle and one medium vehicle. The loading capacities for the small and medium vehicles are 50 and 100 units, respectively. The assignment costs for the small and medium vehicles each time are $500 and $1000, respectively. Vehicle 1 is a small truck, while vehicle 2 is a medium truck. The planning horizon contains three periods. The travelling costs per minute for the small and medium vehicles are $9 and $12, respectively. The tardiness cost and earliness cost per minute to a supplier are $30 and $10, respectively. The earliest time and latest time arriving to supplier 1 without penalty are 200 and 260 minutes, respectively, in the day. The earliest time and latest time arriving to supplier 2 without penalty are 180 and 240 minutes, respectively, in the day. The earliest time and latest time arriving to supplier 3 without penalty are 250 and 310 minutes, respectively, in the day. Table 2 shows the demand from each supplier in each period.

Table 1

Travelling Time Matrix among Suppliers Unit of measure: min

w_{ij}	j=0	j=1	j=2	j=3
i=0		90	60	72
i=1	90		96	186
i=2	60	96		90

i=3	72	186	90	

Table 2

Demand from Suppliers for a Case

Period	Supplier 1	Supplier 2	Supplier 3
1	21	40	26
2	35	63	30
3	22		23

Table 3

Relevant Results in Each Period Using the MIP and the GA

Decision variables	t=1	t =2	t =3
$X_{t,0,j,v}$	$X_{1,0,1,2} = 1$	$X_{2,0,3,1} = 1, X_{2,0,1,2} = 1$	$X_{3,0,1,1} = 1$
$X_{t,i,j,v}$	$X_{1,1,2,2} = 1, X_{1,2,3,2} = 1,$ $X_{1,3,0,2} = 1$	$X_{2,3,0,1} = 1, X_{2,1,2,2} = 1,$ $X_{2,2,0,2} = 1$	$X_{3,1,3,1} = 1, X_{3,3,0,1} = 1$
$L_{t,i,v}$	$L_{1,3,2} = 38$		
$E_{t,i,v}$	$E_{1,1,2} = 110$	$E_{2,1,2} = 110, E_{2,3,1} = 178$	$E_{3,1,1} = 110$

Assignment cost	Travelling cost	Earliness cost	Tardiness cost	Total cost
\$3,000	\$10,314	\$5,080	\$1,140	\$19,534

The solutions of Case obtained by the MIP model and by the GA model are the same, as shown in Table 3. In period 1, vehicle 2 travels from the depot to supplier 1, to supplier 2, to supplier 3, and then back to the depot. The tardiness time of the vehicle when arriving supplier 3 is 38 minutes, and the earliness time of the vehicle when arriving supplier 1 is 110 minutes. In period 2, vehicle 1 travels from the depot to supplier 3, and then back to the depot, while vehicle 2 travels from the depot to supplier 1, to supplier 2, and then back to the depot. The earliness time of vehicle 2 when arriving supplier 1 is 110 minutes, and the earliness time of vehicle 1 when arriving supplier 3 is 178 minutes. In period 3, vehicle 1 travels from the depot to supplier 1, to supplier 3, and then back to the depot. The earliness time of vehicle 1 when arriving supplier 1 is 110 minutes. The assigning cost is \$3,000, the travelling cost is \$10,314, the earliness cost is \$5,080, the tardiness cost is \$1,140, and the total transportation cost is \$19,534. In this case, the computational time using the MIP is 0.9 seconds, while the computational time using the GA is 203 seconds. The results show that the MIP is effective for this case.

Conclusions

This paper studies a vehicle routing problem with time windows. In this problem, a manufacturer needs to outsource different amount of materials from different suppliers using different vehicles with limited loading sizes and limited travelling distances in each period. A mixed integer programming (MIP) model is proposed first to minimize the total transportation cost for the manufacturer, and the optimal vehicle routing and loading size of each vehicle in each period can be generated. When a problem becomes too complicated to solve by the MIP, the proposed genetic algorithm (GA) could be used to obtain near optimal solution in a short computational time.

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