

Harmony of the spheres

(The golden ratio and the Science of harmonics)



1st part: Introduction

If you love music, take a few minutes of your time and read attentively what is following and discover one of the greatest inventions based on a new musical perception. Everything started when the inventor associate together the diatonic scale and the chromatic scale by using numbers to indicate each degree (see Fig.1). By looking at the numbers that correspond to the intervals that constitute a Perfect Major Chord, you can see appear the legendary Fibonacci sequence (the golden section - PHI - or the divine proportion). With the discovery of a “Musical Universal Key”, M. Sylvain Lalonde, the inventor, demonstrates where this revolutionary easy approach method begin.

Fig.1

With the research of the harmony



The golden section influences the Western vision of the harmony
The sequence of Fibonacci... 1-1-2-3-5-8-13... applies to this rule

Ex: Perfect Majeur Chords: C,E,G,C

Tonic	Tone		Second	Tone		Third	Semitone		Fourth	Tone		Fifth	Tone		Sixth	Tone		Seventh	Semitone		Octave
1			2			3			4			5			6			7			8
C	C [#]	D ^b	D	D [#]	E ^b	E	F	F [#]	G ^b	G	G [#]	A ^b	A	A [#]	B ^b	B	C				
1	2	3	4	5	6	7	8	9	10	11	12	13									
Semitone		Semitone	Semitone	Semitone	Semitone	Semitone	Semitone	Semitone	Semitone	Semitone	Semitone	Semitone	Semitone	Semitone	Semitone	Semitone	Semitone	Semitone	Semitone	Semitone	Semitone
A													B								

When $(A + B) / A = A / B$. The A / B ratio is then equal to the golden ratio.

$$8 / 5 = 1.6$$

Uclide d'Alexandrie

	C	Db	D	Eb	E	F	Gb	G	Ab	A	Bb	B	C	
Do3	264	281,6	293,333	312,889	330	352	375,467	391,111	417,185	440	469,333	495,000	528,000	Do4
	268	277	298	308	335	358	369	397	411	447	462	503	520	
	273	273	303	303	341	364	364	404	404	454	454	511	511	
	277	268	308	298	346	369	358	411	397	462	447	520	503	
	281,6	264	312,889	293,333	352	375,467	352	417,185	391,111	469,333	440	528,000	495,000	
	1,6	-1,6	1,6	-1,6	1,6	1,6	-1,6	1,6	-1,6	1,6	-1,6	1,6	-1,6	
	1,6	-1,6	1,6	-1,6	1,6	1,6	-1,6	1,6	-1,6	1,6	-1,6	1,6	-1,6	
	1,6	-1,6	1,6	-1,6	1,6	1,6	-1,6	1,6	-1,6	1,6	-1,6	1,6	-1,6	
	1,6	-1,6	1,6	-1,6	1,6	1,6	-1,6	1,6	-1,6	1,6	-1,6	1,6	-1,6	
	1,06	1,06	1,06	1,06	1,06	1,06	1,06	1,06	1,06	1,06	1,06	1,06	1,06	

2nd part: Diatonic scale vs Chromatic scale

12 Sounds = 12 Tones = 12 Tonalties

Let's look at the same graphics on a 360-degree platform.. The use of black and white is used to represent the notes of a piano, to help understand this new perception.

Select and print Fig. 3 (picture) cut the two discs, then cut down the 7 windows that correspond to the intervals on disc 1 (Item 32). Overlap disc 1 over the second disc (Item 30) and then position the 1st degree (“Tonic”) on C, you’ll get the “C-D-E-F-G-A-B” sequence.

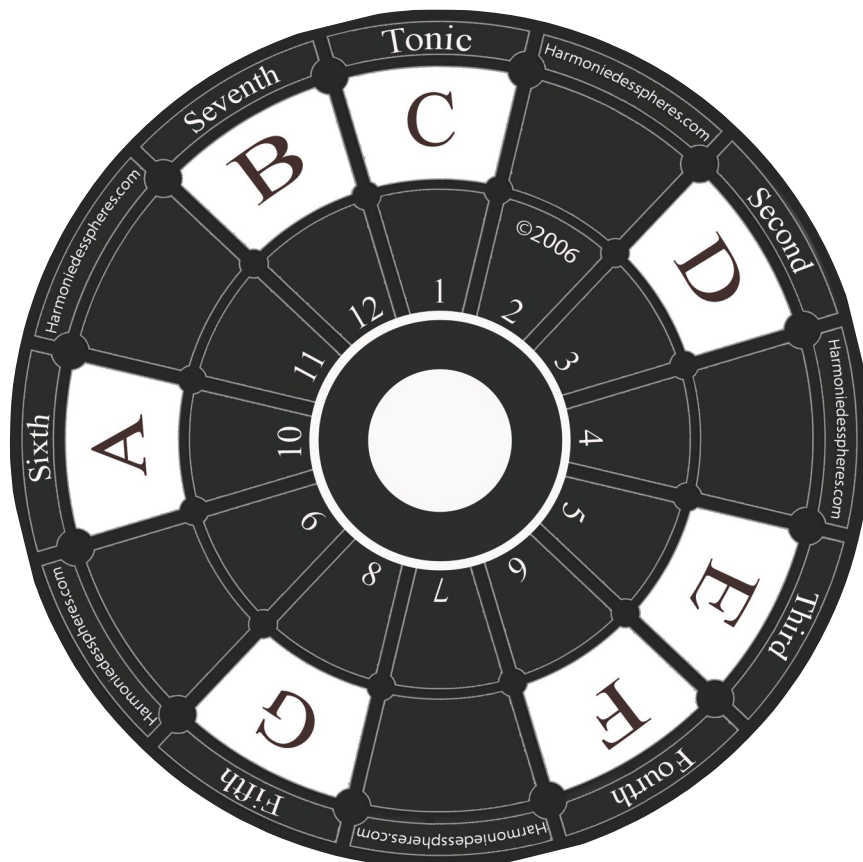
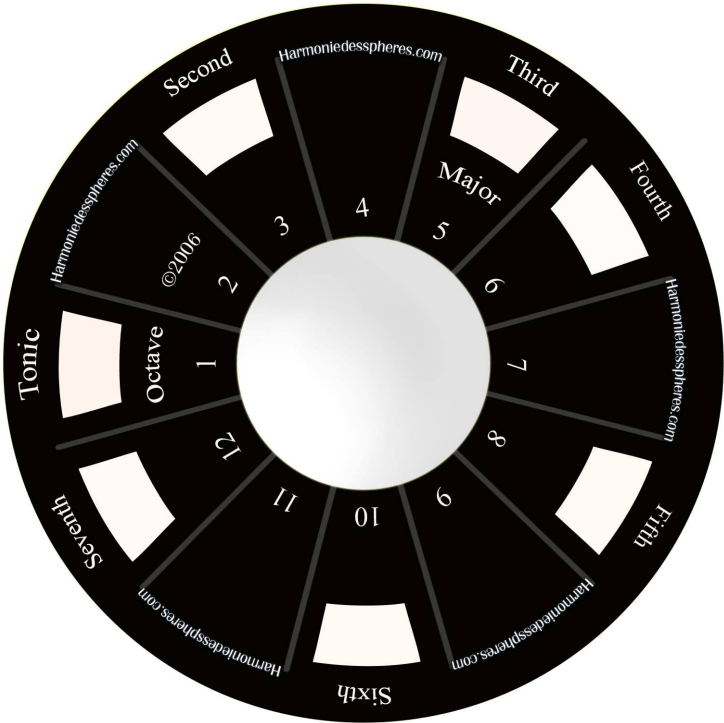


Fig.3



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« The music it's of space-times »

Sylvain Lalonde founding president : Harmonie des sphères inc.

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Now move the “Tonic” on the “Fifth”, the G, this way you obtain the alterations that constitutes the key frame of our tonal system and so on.
We call this:

« The Cycle of Fifths »

Ascending = Sharp order = F, C, G, D, A, E, B

Descending = Flat order = B, E, A, D, G, C, F

Fig.3B

Note that the order of flats is the opposite of sharps

order of #		F	C	G	D	A	E	B	order of b	
order of #		F#	C#	G#	D#	A#	E#	B#	order of b	
order of #		F#	C#	G#	D#	A#	E#	B	order of b	
order of #		F#	C#	G#	D#	A	E	B	order of b	
order of #		F#	C#	G#	D	A	E	B	order of b	
order of #		F#	C#	G	D	A	E	B	order of b	
order of #		F#	C	G	D	A	E	B	order of b	
order of #		F	C	G	D	A	E	B	order of b	
order of #		F	C	G	D	A	E	B ^b	order of b	
order of #		F	C	G	D	A	E ^b	B ^b	order of b	
order of #		F	C	G	D ^b	A ^b	E ^b	B ^b	order of b	
order of #		F	C	G ^b	D ^b	A ^b	E ^b	B ^b	order of b	
order of #		F	C ^b	G ^b	D ^b	A ^b	E ^b	B ^b	order of b	
order of #		F ^b	C ^b	G ^b	D ^b	A ^b	E ^b	B ^b	order of b	

Major Range

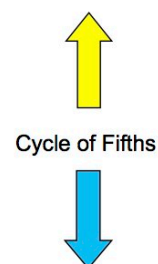


Fig.3c



HDS natural semitone Hz	
Proportion per semitone	1,0666666666667

B 247,50	/	C 264	x	Db 281,600
	1,066667		1,066667	
C# 275	/	D 293,333	x	Eb 312,889
	1,066667		1,066667	
D# 309,375	/	E 330	x	F 352
	1,066667		1,066667	
E 330	/	F 352	x	Gb 375,467
	1,066667		1,066667	
F# 366,667	/	G 391,111	x	Ab 417,185
	1,066667		1,066667	
G# 412,50	/	A 440	x	Bb 469,333
	1,066667		1,066667	
A# 458,333	/	B 488,889 495,000 1,0125	x	C 528
	1,066667		1,066667	

3rd part: the natural harmonics vs temperate

Each note is made of natural multiples called harmonics

At each degree of the scale, there is a three fundamental sound chord

The chords of three sounds consists of a fundamental
a third party (major or minor) and a fifth just.

« THE THIRD CYCLE »

« C, E, G, B, D, F, A »

Harmonie des sphères



Clé universelle musicale
Musique Instrument Digital Interface
Codex

I	II	III	IV	V	VI	VII	VIII
Major	minor	minor	Major	Major	minor	Diminished	Major
Sol	La	Si	Do	Ré	Mi	H Fa	Sol
Mi	Fa	Sol	La	Si	Do	Ré	Mi
Do	Ré	Mi	Fa	Sol	La	Si	Do
Time	Time	Interval	Time	Time	Time	Interval	

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Harmony of the spheres



Universal musical key
Music Instrument Digital Interface
Codex

I	II	III	IV	V	VI	VII	VIII
Major	minor	minor	Major	Major	minor	Diminished	Major
G	A	B	C	D	E	H F	G
E	F	G	A	B	C	D	E
C	D	E	F	G	A	B	C
Time	Time	Interval	Time	Time	Time	Interval	

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The notes of a chord can be duplicated at the octave or arpeggiated without
altering the identity of the chord.

You can select and print Fig.3c (Codex) in legal format.

Cut the two parts (A and B) according to the language you want.

Also cut the twenty-four (24) windows (Part A) that correspond to the

« Music intervals »

I degree	C, E, G
II degree	D, F, A
III degree	E, G, B
IV degree	F, A, C
V degree	G, B, D
VI degree	A, C, E
VII degree	B, D, F
VIII degree	C, E, G

Then superimpose part A on part B and position the 1st degree on the C, you will obtain at each degree the notes of all the chords.

You can change the tone as you wish,

The results will be the same.

P.S. You can also use an ABS tube with a diameter of 1.5 inch or 3.81 cm to make it circular shape.

Fig.3c (Codex)

P
A
R
T
I
E

A

Harmonie des sphères



Clé universelle musicale
Musique Instrument Digital Interface
Codex

I	II	III	IV	V	VI	VII	VIII
Majeur	mineur	mineur	Majeur	Majeur	mineur	Diminuer	Majeur
Sol	La	Si	Do	Ré	Mi	H Fa	Sol
Mi			La	Si			Mi
	Fa	Sol		Do	Ré		
Do	Ré	Mi	Fa	Sol	La	Si	Do
	Ton	Ton	Demi-ton	Ton	Ton	Ton	Demi-ton

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Harmony of the spheres



Universal musical key
Music Instrument Digital Interface
Codex

I	II	III	IV	V	VI	VII	VIII
Majeur	mineur	mineur	Majeur	Majeur	mineur	Diminish	Majeur
G	A	B	C	D	E	H F	G
E			A	B			E
	F	G			C	D	
C	D	E	F	G	A	B	C
	Tone	Tone	Semi-tone	Tone	Tone	Tone	Semi-tone

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P
A
R
T

A

P
A
R
T
I
E

B

Do	Ré	Mi	Fa	Sol	La	Si	Do
Si	Ré ^b	Mi ^b	Mi	Sol ^b	La ^b	Si ^b	Si
La	Do	Ré	Ré	Fa	Sol	La	La
Sol	Si	Do	Do	Mi	Fa	Sol	Sol
Fa	La	Si	Si	Ré ^b	Mi ^b	Fa	Sol ^b
Mi	Sol	La	La	Do	Ré	Mi	Fa
Ré	Mi	Sol	Sol	La	Si	Ré	Mi
Do	Ré	Mi	Fa	Sol	La	Si	Do
Si	Do	Ré	Mi	Fa	Sol	La	Si
La	Si	Ré	Ré	Mi	Sol	La	La
Sol	La	Si	Do	Ré	Mi	Sol	Sol
Fa	Sol	La	Si	Do	Ré	Mi	Fa
Mi	Sol	La	La	Si	Ré ^b	Mi ^b	Mi
Ré	Mi	Sol	Sol	La	Si	Ré	Ré
Do	Ré	Mi	Fa	Sol	La	Si	Do

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C	D	E	F	G	A	B	C
B	D ^b	E ^b	D	G ^b	A ^b	B ^b	B
A	C	D	D	F	G	A	A
G	A	B	C	D	E	F	G
F	G	A	B	C	D	E	F
E	G ^b	A ^b	A	B	D ^b	E ^b	E
D	E	F	G	A	C	D	D
C	D	E	F	G	A	B	C
B	D ^b	E ^b	D	F	G	A	B
A	C	D	D	E	G ^b	A ^b	A
G	A	B	C	D	E	F	G
F	G	A	B	C	D	E	F
E	G ^b	A ^b	A	B	D ^b	E ^b	E
D	E	F	G	A	B	D ^b	D
C	D	E	F	G	A	B	C

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P
A
R
T

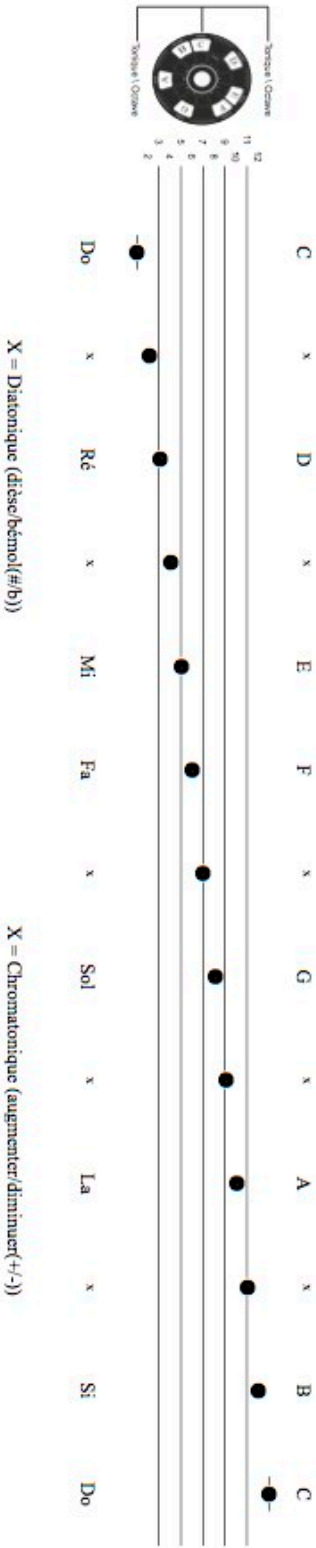
B

Harmony of the spheres



Fig.4

Musical universal key
Musical Instrument Digital Interface

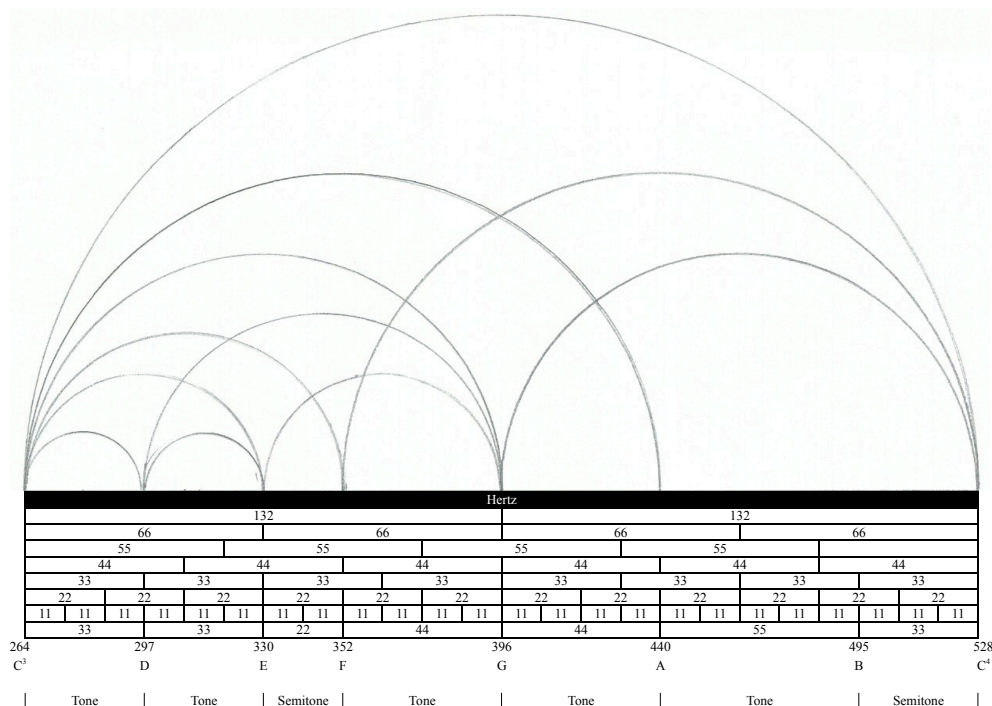


Pythagore and the harmony of the spheres

Fig.7

HARMONY OF THE SPHERES

MUSICAL UNIVERSAL KEY



HDS Universal Proportion

1,125	*	1,111	1,06666667	1,125	*	1,111	1,125	1,06666667
1,250				1,250			1,250	
1,500				1,33333333				
1,66666667				1,2				
1,33333333				1,500				
1,250				1,6				
1,875				1,06666667				

HDS Fraction Pythagorean

1 1/8								
1 1/4								
1 1/3								
1 1/2								
1 2/3								
1 7/8								
2								

For more details on natural harmonics see the section R/D

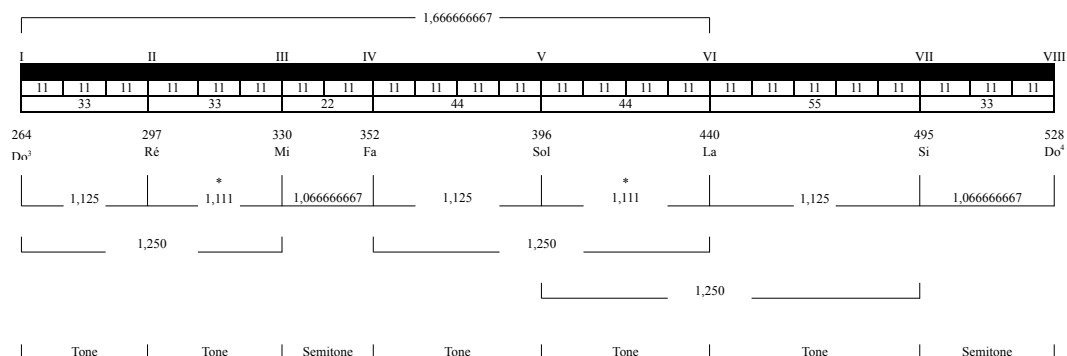
« Natural Harmonics »

Fig.7a



NATURAL HARMONICS

Starting from the fact that the natural resonance (harmonics) of each note are relatively all multiples of each of them and the frequency (Hz) is given by the simple arithmetic operation of fundamental sound F^0 multiplication by the rank number of it



II	X	=
11	1	11
11	2	22
11	3	33
11	4	44
11	5	55
11	6	66
11	7	77
11	8	88
11	9	99
11	10	110
11	11	121
11	12	132
11	13	143
11	14	154
11	15	165
11	16	176
11	17	187
11	18	198
11	19	209
11	20	220
11	21	231
11	22	242
11	23	253

Octave	11	24	264	C	1
	11	25	275		2
	11	26	286		3
Second	11	27	297	D	4
	11	28	308		5
	11	29	319		6
Third	11	30	330	E	7
	11	31	341		8
Fourth	11	32	352	F	9
	11	33	363		10
	11	34	374		11
	11	35	385		12
Fifth	11	36	396	G	13
	11	37	407		14
	11	38	418		15
	11	39	429		16
Sixth	11	40	440	A	17
	11	41	451		18
	11	42	462		19
	11	43	473		20
	11	44	484		21
Seventh	11	45	495	B	22
	11	46	506		23
	11	47	517		24
Octave	11	48	528	C	25

For more details on natural harmonics see figures 6 and 7 section R/D

Note that the ratio between A 440 Hz and C 264 Hz has a proportion of

$$1.66666666666667$$

$$\text{either: } 440/264 = 1.66666666666667$$

this proportion will never change ... whatever the frequency chosen:

$$\dots 328 - 426,7 - 432 - 440 - 442 - 444 \dots$$

it's a good way to find your C

then divide it by 24 to get what I call the unit of measure.

P: S. The tuning frequency of A at 426.7 Hz also called Scientific range
or Range of physicists, with a Do at 256 Hz

$$\text{either : } 426.666666666667 / 256 = 1.66666666666667.$$

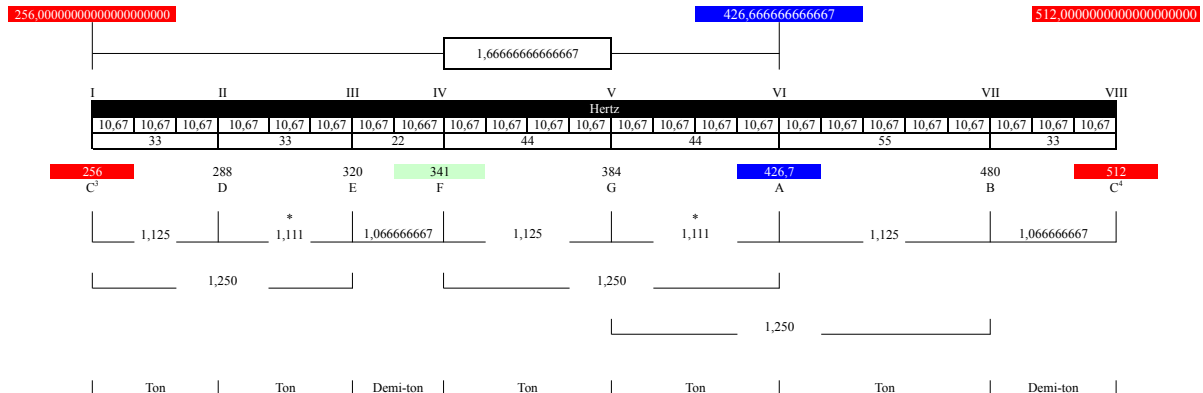
Fig.7f



NATURAL HARMONICS

426,7 Hz

Starting from the fact that the natural resonance (harmonics) of each note are relatively all multiples of each of them and the frequency (Hz) is given by the simple arithmetic operation of fundamental sound F0 multiplication by the rank number of it



10,666666666667					
	10,67	X	=		
	10,67	1	10,67		
	10,67	2	21,33		
	10,67	3	32		
	10,67	4	42,67		
	10,67	5	53,33		
	10,67	6	64		
	10,67	7	74,67		
	10,67	8	85,33		
	10,67	9	96		
	10,67	10	106,7		
	10,67	11	117,3		
	10,67	12	128		
	10,67	13	138,7		
	10,67	14	149,3		
	10,67	15	160		
	10,67	16	170,7		
	10,67	17	181,3		
	10,67	18	192		
	10,67	19	202,7		
	10,67	20	213,3		
	10,67	21	224		
	10,67	22	234,7		
	10,67	23	245,3		
Octave	10,67	24	256	C	1
	10,67	25	266,7		2
	10,67	26	277,3		3
	10,67	27	288	D	4
Seconde	10,67	28	298,7		5
	10,67	29	309,3		6
	10,67	30	320	E	7
Tierce	10,67	31	330,7		8
	10,67	32	341,3	F	9
Quarte	10,67	33	352		10
	10,67	34	362,7		11
	10,67	35	373,3		12
Quinte	10,67	36	384	G	13
	10,67	37	394,7		14
	10,67	38	405,3		15
Sixième	10,67	39	416		16
	10,67	40	426,7	A	17
	10,67	41	437,3		18
Septième	10,67	42	448		19
	10,67	43	458,7		20
	10,67	44	469,3		21
	10,67	45	480	B	22
	10,67	46	490,7		23
Octave	10,67	47	501,3		24
	10,67	48	512	C	25

341,33333333334

426,66666666667

For more details on natural harmonics see figures 6 and 7 section R/D

Fig.7H



LES HARMONIQUES NATURELS - NATURAL HARMONICS

F Fa 302	C C 264	G Gb 390,313	D D 293,333																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
<table> <tr><td>1</td><td>14.666667</td><td>1</td><td>14.262222</td></tr> <tr><td>2</td><td>29.333333</td><td>2</td><td>28.444444</td></tr> <tr><td>3</td><td>44</td><td>3</td><td>36.666667</td></tr> <tr><td>4</td><td>58.666667</td><td>4</td><td>48.888889</td></tr> <tr><td>5</td><td>73.333333</td><td>5</td><td>61.111111</td></tr> <tr><td>6</td><td>88</td><td>6</td><td>73.333333</td></tr> <tr><td>7</td><td>102.666667</td><td>7</td><td>85.555556</td></tr> <tr><td>8</td><td>117.333333</td><td>8</td><td>97.777778</td></tr> <tr><td>9</td><td>132</td><td>9</td><td>102.222222</td></tr> <tr><td>10</td><td>146.666667</td><td>10</td><td>114.444444</td></tr> <tr><td>11</td><td>161.333333</td><td>11</td><td>126.666667</td></tr> <tr><td>12</td><td>176</td><td>12</td><td>138.888889</td></tr> <tr><td>13</td><td>190.666667</td><td>13</td><td>151.111111</td></tr> <tr><td>14</td><td>205.333333</td><td>14</td><td>163.333333</td></tr> <tr><td>15</td><td>220</td><td>15</td><td>175.555556</td></tr> <tr><td>16</td><td>234.666667</td><td>16</td><td>187.777778</td></tr> <tr><td>17</td><td>249.333333</td><td>17</td><td>200.000000</td></tr> <tr><td>18</td><td>264</td><td>18</td><td>212.222222</td></tr> <tr><td>19</td><td>279.666667</td><td>19</td><td>224.444444</td></tr> <tr><td>20</td><td>293.333333</td><td>20</td><td>236.666667</td></tr> <tr><td>21</td><td>308</td><td>21</td><td>248.888889</td></tr> <tr><td>22</td><td>322.666667</td><td>22</td><td>261.111111</td></tr> <tr><td>23</td><td>337.333333</td><td>23</td><td>273.333333</td></tr> <tr><td>24</td><td>352</td><td>24</td><td>285.555556</td></tr> <tr><td>25</td><td>366.666667</td><td>25</td><td>297.777778</td></tr> <tr><td>26</td><td>381.333333</td><td>26</td><td>310.000000</td></tr> <tr><td>27</td><td>396</td><td>27</td><td>322.222222</td></tr> <tr><td>28</td><td>410.666667</td><td>28</td><td>334.444444</td></tr> <tr><td>29</td><td>425.333333</td><td>29</td><td>346.666667</td></tr> <tr><td>30</td><td>440</td><td>30</td><td>358.888889</td></tr> <tr><td>31</td><td>454.666667</td><td>31</td><td>371.111111</td></tr> <tr><td>32</td><td>469.333333</td><td>32</td><td>383.333333</td></tr> <tr><td>33</td><td>484</td><td>33</td><td>395.555556</td></tr> <tr><td>34</td><td>498.666667</td><td>34</td><td>407.777778</td></tr> <tr><td>35</td><td>513.333333</td><td>35</td><td>420.000000</td></tr> <tr><td>36</td><td>528</td><td>36</td><td>432.222222</td></tr> <tr><td>37</td><td>542.666667</td><td>37</td><td>444.444444</td></tr> <tr><td>38</td><td>557.333333</td><td>38</td><td>456.666667</td></tr> <tr><td>39</td><td>572</td><td>39</td><td>468.888889</td></tr> <tr><td>40</td><td>586.666667</td><td>40</td><td>481.111111</td></tr> <tr><td>41</td><td>601.333333</td><td>41</td><td>493.333333</td></tr> <tr><td>42</td><td>616</td><td>42</td><td>505.555556</td></tr> <tr><td>43</td><td>630.666667</td><td>43</td><td>517.777778</td></tr> <tr><td>44</td><td>645.333333</td><td>44</td><td>530.000000</td></tr> <tr><td>45</td><td>660</td><td>45</td><td>542.222222</td></tr> <tr><td>46</td><td>674.666667</td><td>46</td><td>554.444444</td></tr> <tr><td>47</td><td>689.333333</td><td>47</td><td>566.666667</td></tr> <tr><td>48</td><td>704</td><td>48</td><td>578.888889</td></tr> </table>	1	14.666667	1	14.262222	2	29.333333	2	28.444444	3	44	3	36.666667	4	58.666667	4	48.888889	5	73.333333	5	61.111111	6	88	6	73.333333	7	102.666667	7	85.555556	8	117.333333	8	97.777778	9	132	9	102.222222	10	146.666667	10	114.444444	11	161.333333	11	126.666667	12	176	12	138.888889	13	190.666667	13	151.111111	14	205.333333	14	163.333333	15	220	15	175.555556	16	234.666667	16	187.777778	17	249.333333	17	200.000000	18	264	18	212.222222	19	279.666667	19	224.444444	20	293.333333	20	236.666667	21	308	21	248.888889	22	322.666667	22	261.111111	23	337.333333	23	273.333333	24	352	24	285.555556	25	366.666667	25	297.777778	26	381.333333	26	310.000000	27	396	27	322.222222	28	410.666667	28	334.444444	29	425.333333	29	346.666667	30	440	30	358.888889	31	454.666667	31	371.111111	32	469.333333	32	383.333333	33	484	33	395.555556	34	498.666667	34	407.777778	35	513.333333	35	420.000000	36	528	36	432.222222	37	542.666667	37	444.444444	38	557.333333	38	456.666667	39	572	39	468.888889	40	586.666667	40	481.111111	41	601.333333	41	493.333333	42	616	42	505.555556	43	630.666667	43	517.777778	44	645.333333	44	530.000000	45	660	45	542.222222	46	674.666667	46	554.444444	47	689.333333	47	566.666667	48	704	48	578.888889	<table> 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<tr><td>19</td><td>209</td><td>19</td><td>209</td></tr> <tr><td>20</td><td>220</td><td>20</td><td>220</td></tr> <tr><td>21</td><td>231</td><td>21</td><td>231</td></tr> <tr><td>22</td><td>242</td><td>22</td><td>242</td></tr> <tr><td>23</td><td>253</td><td>23</td><td>253</td></tr> <tr><td>24</td><td>264</td><td>24</td><td>264</td></tr> <tr><td>25</td><td>275</td><td>25</td><td>275</td></tr> <tr><td>26</td><td>286</td><td>26</td><td>286</td></tr> <tr><td>27</td><td>297</td><td>27</td><td>297</td></tr> <tr><td>28</td><td>308</td><td>28</td><td>308</td></tr> <tr><td>29</td><td>319</td><td>29</td><td>319</td></tr> <tr><td>30</td><td>330</td><td>30</td><td>330</td></tr> <tr><td>31</td><td>341</td><td>31</td><td>341</td></tr> <tr><td>32</td><td>352</td><td>32</td><td>352</td></tr> <tr><td>33</td><td>363</td><td>33</td><td>363</td></tr> <tr><td>34</td><td>374</td><td>34</td><td>374</td></tr> <tr><td>35</td><td>385</td><td>35</td><td>385</td></tr> <tr><td>36</td><td>396</td><td>36</td><td>396</td></tr> <tr><td>37</td><td>407</td><td>37</td><td>407</td></tr> <tr><td>38</td><td>418</td><td>38</td><td>418</td></tr> <tr><td>39</td><td>429</td><td>39</td><td>429</td></tr> <tr><td>40</td><td>440</td><td>40</td><td>440</td></tr> <tr><td>41</td><td>451</td><td>41</td><td>451</td></tr> <tr><td>42</td><td>462</td><td>42</td><td>462</td></tr> <tr><td>43</td><td>473</td><td>43</td><td>473</td></tr> <tr><td>44</td><td>484</td><td>44</td><td>484</td></tr> <tr><td>45</td><td>495</td><td>45</td><td>495</td></tr> <tr><td>46</td><td>506</td><td>46</td><td>506</td></tr> <tr><td>47</td><td>517</td><td>47</td><td>517</td></tr> <tr><td>48</td><td>528</td><td>48</td><td>528</td></tr> 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<tr><td>6</td><td>97.777778</td><td>6</td><td>97.777778</td></tr> <tr><td>7</td><td>114.074107</td><td>7</td><td>114.074107</td></tr> <tr><td>8</td><td>130.370370</td><td>8</td><td>130.370370</td></tr> <tr><td>9</td><td>146.666667</td><td>9</td><td>146.666667</td></tr> <tr><td>10</td><td>162.962963</td><td>10</td><td>162.962963</td></tr> <tr><td>11</td><td>179.259259</td><td>11</td><td>179.259259</td></tr> <tr><td>12</td><td>195.555556</td><td>12</td><td>195.555556</td></tr> <tr><td>13</td><td>211.851852</td><td>13</td><td>211.851852</td></tr> <tr><td>14</td><td>228.148148</td><td>14</td><td>228.148148</td></tr> <tr><td>15</td><td>244.444444</td><td>15</td><td>244.444444</td></tr> <tr><td>16</td><td>260.740741</td><td>16</td><td>260.740741</td></tr> <tr><td>17</td><td>277.037037</td><td>17</td><td>277.037037</td></tr> <tr><td>18</td><td>293.333333</td><td>18</td><td>293.333333</td></tr> <tr><td>19</td><td>309.629629</td><td>19</td><td>309.629629</td></tr> <tr><td>20</td><td>325.925926</td><td>20</td><td>325.925926</td></tr> <tr><td>21</td><td>342.222222</td><td>21</td><td>342.222222</td></tr> <tr><td>22</td><td>358.518519</td><td>22</td><td>358.518519</td></tr> <tr><td>23</td><td>374.814815</td><td>23</td><td>374.814815</td></tr> <tr><td>24</td><td>391.111111</td><td>24</td><td>391.111111</td></tr> <tr><td>25</td><td>407.407407</td><td>25</td><td>407.407407</td></tr> <tr><td>26</td><td>423.703704</td><td>26</td><td>423.703704</td></tr> <tr><td>27</td><td>440</td><td>27</td><td>440</td></tr> <tr><td>28</td><td>456.296297</td><td>28</td><td>456.296297</td></tr> <tr><td>29</td><td>472.592592</td><td>29</td><td>472.592592</td></tr> <tr><td>30</td><td>488.888889</td><td>30</td><td>488.888889</td></tr> <tr><td>31</td><td>505.185185</td><td>31</td><td>505.185185</td></tr> <tr><td>32</td><td>521.481481</td><td>32</td><td>521.481481</td></tr> <tr><td>33</td><td>537.777778</td><td>33</td><td>537.777778</td></tr> 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<tr><td>48</td><td>782.222222</td><td>48</td><td>782.222222</td></tr> 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<tr><td>1</td><td>12.222222</td><td>1</td><td>12.222222</td></tr> <tr><td>2</td><td>24.444444</td><td>2</td><td>24.444444</td></tr> <tr><td>3</td><td>36.666667</td><td>3</td><td>36.666667</td></tr> <tr><td>4</td><td>48.888889</td><td>4</td><td>48.888889</td></tr> <tr><td>5</td><td>61.111111</td><td>5</td><td>61.111111</td></tr> <tr><td>6</td><td>73.333333</td><td>6</td><td>73.333333</td></tr> <tr><td>7</td><td>85.555556</td><td>7</td><td>85.555556</td></tr> <tr><td>8</td><td>97.777778</td><td>8</td><td>97.777778</td></tr> <tr><td>9</td><td>102.222222</td><td>9</td><td>102.222222</td></tr> <tr><td>10</td><td>114.444444</td><td>10</td><td>114.444444</td></tr> <tr><td>11</td><td>126.666667</td><td>11</td><td>126.666667</td></tr> <tr><td>12</td><td>138.888889</td><td>12</td><td>138.888889</td></tr> <tr><td>13</td><td>151.111111</td><td>13</td><td>151.111111</td></tr> <tr><td>14</td><td>163.333333</td><td>14</td><td>163.333333</td></tr> <tr><td>15</td><td>175.555556</td><td>15</td><td>175.555556</td></tr> <tr><td>16</td><td>187.777778</td><td>16</td><td>187.777778</td></tr> <tr><td>17</td><td>200.000000</td><td>17</td><td>200.000000</td></tr> <tr><td>18</td><td>212.222222</td><td>18</td><td>212.222222</td></tr> <tr><td>19</td><td>224.444444</td><td>19</td><td>224.444444</td></tr> <tr><td>20</td><td>236.666667</td><td>20</td><td>236.666667</td></tr> <tr><td>21</td><td>248.888889</td><td>21</td><td>248.888889</td></tr> <tr><td>22</td><td>261.111111</td><td>22</td><td>261.111111</td></tr> <tr><td>23</td><td>273.333333</td><td>23</td><td>273.333333</td></tr> <tr><td>24</td><td>285.555556</td><td>24</td><td>285.555556</td></tr> <tr><td>25</td><td>297.777778</td><td>25</td><td>297.777778</td></tr> <tr><td>26</td><td>310.000000</td><td>26</td><td>310.000000</td></tr> <tr><td>27</td><td>322.222222</td><td>27</td><td>322.222222</td></tr> <tr><td>28</td><td>334.444444</td><td>28</td><td>334.444444</td></tr> 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<tr><td>43</td><td>517.777778</td><td>43</td><td>517.777778</td></tr> <tr><td>44</td><td>530.000000</td><td>44</td><td>530.000000</td></tr> <tr><td>45</td><td>542.222222</td><td>45</td><td>542.222222</td></tr> <tr><td>46</td><td>554.444444</td><td>46</td><td>554.444444</td></tr> <tr><td>47</td><td>566.666667</td><td>47</td><td>566.666667</td></tr> <tr><td>48</td><td>578.888889</td><td>48</td><td>578.888889</td></tr> </table>	1	12.222222	1	12.222222	2	24.444444	2	24.444444	3	36.666667	3	36.666667	4	48.888889	4	48.888889	5	61.111111	5	61.111111	6	73.333333	6	73.333333	7	85.555556	7	85.555556	8	97.777778	8	97.777778	9	102.222222	9	102.222222	10	114.444444	10	114.444444	11	126.666667	11	126.666667	12	138.888889	12	138.888889	13	151.111111	13	151.111111	14	163.333333	14	163.333333	15	175.555556	15	175.555556	16	187.777778	16	187.777778	17	200.000000	17	200.000000	18	212.222222	18	212.222222	19	224.444444	19	224.444444	20	236.666667	20	236.666667	21	248.888889	21	248.888889	22	261.111111	22	261.111111	23	273.333333	23	273.333333	24	285.555556	24	285.555556	25	297.777778	25	297.777778	26	310.000000	26	310.000000	27	322.222222	27	322.222222	28	334.444444	28	334.444444	29	346.666667	29	346.666667	30	358.888889	30	358.888889	31	371.111111	31	371.111111	32	383.333333	32	383.333333	33	395.555556	33	395.555556	34	407.777778	34	407.777778	35	420.000000	35	420.000000	36	432.222222	36	432.222222	37	444.444444	37	444.444444	38	456.666667	38	456.666667	39	468.888889	39	468.888889	40	481.111111	40	481.111111	41	493.333333	41	493.333333	42	505.555556	42	505.555556	43	517.777778	43	517.777778	44	530.000000	44	530.000000	45	542.222222	45	542.222222	46	554.444444	46	554.444444	47	566.666667	47	566.666667	48	578.888889	48	578.888889
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5	68.75	5	68.75																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
6	82.5	6	82.5																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
7	96.25	7	96.25																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
8	110	8	110																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
9	123.75	9	123.75																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
10	137.5	10	137.5																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
11	151.25	11	151.25																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
12	165	12	165																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
13	178.75	13	178.75																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
14	192.5	14	192.5																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
15	206.25	15	206.25																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
16	220	16	220																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
17	233.75	17	233.75																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
18	247.5	18	247.5																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
19	261.25	19	261.25																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
20	275	20	275																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
21	288.75	21	288.75																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
22	302.5	22	302.5																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
23	316.25	23	316.25																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
24	330	24	330																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
25	343.75	25	343.75																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
26	357.5	26	357.5																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
27	371.25	27	371.25																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
28	385	28	385																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
29	398.75	29	398.75																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
30	412.5	30	412.5																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
31	426.25	31	426.25																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
32	440	32	440																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
33	453.75	33	453.75																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
34	467.5	34	467.5																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
35	481.25	35	481.25																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
36	495	36	495																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
37	508.75	37	508.75																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
38	522.5	38	522.5																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
39	536.25	39	536.25																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
40	550	40																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																	

Harmony of the spheres



Universal musical Key Solfege

Fig.8



X = Diatonic (dièse/bémol(#/b))

X = Chromatic (augmenter/diminuer(+/-))

For more details see the section R/D

Select and print at legal format the Fig.8b (image) then cut the two rulers along the lines shown and also cut the eight (8) windows that correspond to the intervals of music.

Then superimpose the part of the center on the top of the rule that suits you and position the 1st degree (Tonic) on the C, you will obtain the sequence

“ C-D-E-F-G-A-B-C ”

You can repeat the exercise in fig.3
or simply move the ruler by tone or semitone.

What is important here is that no matter the tone (tonal height) in which you play, the intervals will always be the same :

Major (which constitute the key frame of our tonal system) or minor.



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Cut

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To fold

C

↓

C

B

B^b

A[#]

A

A^b

G[#]

G

G^b

F[#]

F

E

E^b

D[#]

D

D^b

C[#]

C

C

↑

Use a plastic ruler as a support print the sheet in legal size

Cut

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Couper

Octave

Couper

Couper

Septième

Couper

Sixième

Quinte

Couper

Couper

Quarte

Tierce

Couper

Seconde

Couper

Tonique

Couper

Solfège

Couper

Octave

Cut

Cut

Seventh

Cut

Sixth

Fifth

Cut

Cut

Fourth

Third

Cut

Second

Cut

Tonic

Cut

Solfège

Above the ruler

Couper

Dessus de la règle

Fig.8b

Couper

Clé universelle musicale

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Plier

Do

↓

Do

Si

Si^b

La[#]

La

La^b

Sol[#]

Sol

Sol^b

Fa[#]

Fa

Mi

Mi^b

Ré[#]

Ré

Ré^b

Do[#]

Do

Do

↑

Utiliser une règle en plastique comme support imprimer la feuille en format légal

Plier

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Harmonie des sphères inc.

Mission

Promote the universal musical key throughout the world,
research and develop simple and accessible applications
to all, in order to facilitate the understanding of musical notions.

« Music it's of space-time »

Sylvain Lalonde founding president : Harmonie des sphères inc.

To contact us : sylvain.lalonde@me.com

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In memory of an exceptional mind

« Life is not a mistery but a science undisclosed »

« The Genesis of reality, Bernard de Montréal »

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