Fourier transform of sinc function pdf

Sinc function fourier transform. Fourier transform of sinc(wt). Fourier transform sinc function example. Fourier series of sinc function.

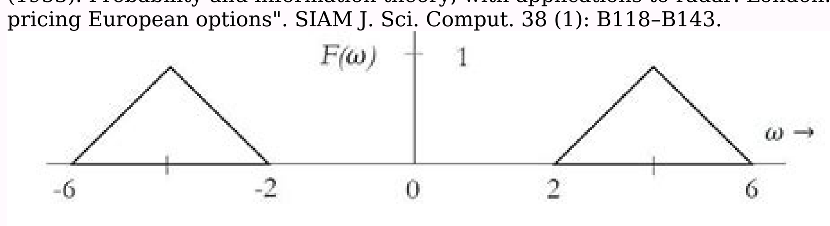
6,557 Let $f(x) = \text{sinc}(x)$.
We can rewrite $$ f(x) = \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x} = \frac{1}{2\pi}\frac{e^{i \pi x}-e^{-i \pi x}}{i x} = \frac{1}{2 \pi}\int \limits_{- \pi}^{\pi}e^{i \omega x} \,d \omega = \mathcal{F}^{-1}(1_{[-\pi, \pi]}).$$ Related videos on Youtube 06 : 11 Sinc Function 48 : 00 Lecture 26 | Fourier Transform (Rect & Sinc) | Signals & Systems 05 : 36 Lecture on Fourier Transform of Sinc Function 19 : 39 Lecture 23 | Fourier Transform of Rect & Sinc Function 08 : 46 Fourier Transform Duality Rect and Sinc Functions Iain Explains Signals, Systems, and Digital Comms 11 Comments Let us consider the sinc function: \begin{equation} {\rm{sinc}}(x)= \begin{cases} \frac{ \sin(\pi x)}{\pi x} \qquad &x ot= 0,\\ 1\qquad & x=0, \end{cases} \end{equation} What is the fourier transform, so-defined: $$\int_{-\pi}^{\pi} f(x) e^{-\imath k x}dx$$ of sinc function? I can't calculate this integral: $$\int_{-\pi}^{\pi} {\rm{sinc}}(x) e^{-\imath k x}dx$$ Any suggepstion please? Sine the function is not in L^1, you can only do it in the distributional sense. The answer, if I remember correctly, is $\chi_{[-1,1]}$, the characteristic function. Then $\mathcal{F} \left(\text{sinc}(x)\right)=\mathcal{F}\left(\mathcal{F}^{-1}(1_{[-\pi, \pi]})\right)=1_{[-\pi, \pi]}$?
Yes @Mark. That's it. Special mathematical function defined as sin(x)/x "Sinc" redirects here. For the designation used in the United Kingdom for areas of wildlife interest, see Site of Importance for Nature Conservation. For the signal processing filter based on this function, see Sinc filter. In mathematics, physics and engineering, the sinc function, denoted by sinc(x), has two forms, normalized and unnormalized.[1] SincPart of the normalized sinc (blue) and unnormalized sinc function (red) shown on the same scaleGeneral informationGeneral definition sinc x = { sin x x , x ≠ 0 1 , x = 0 {\displaystyle \operatorname {sinc} x={\begin{cases}{\dfrac {\sin x}{x}},&x\neq 0\\[2px]1,&x=0\end{cases}}} Motivation of inventionTelecommunicationDate of solution1952Fields of applicationSignal processing, spectroscopyDomain, Codomain and ImageDomain R {\displaystyle \mathbb {R} } Image [− 0.217234 … , 1] {\displaystyle \left[-0.217234\ldots ,1\right]} Basic featuresParityEvenSpecific valuesAt zero1Value at +∞0Value at −∞0Maxima1 at x = 0 {\displaystyle x=0} Minima − 0.21723 … {\displaystyle -0.21723\ldots } at x = ± 4.49341 … {\displaystyle x=\pm 4.49341\ldots } Specific featuresRoot π k , k ∈ Z ≠ 0 {\displaystyle \pi k,k\in \mathbb {Z} _{\neq 0}} Related functionsReciprocal { x csc x , x ≠ 0 1 , x = 0 {\displaystyle {\begin{cases}x\csc x,&x\neq 0\\1,&x=0\end{cases}}} Derivative sinc ′ x = { cos x − sinc x x , x ≠ 0 0 , x = 0 {\displaystyle \operatorname {sinc} 'x={\begin{cases}{\dfrac {\cos x-\operatorname {sinc} x}{x}},&x\neq 0\\0,&x=0\end{cases}}} Antiderivative ∫ sinc x d x = Si (x) + C {\displaystyle \int \operatorname {sinc} x\,dx=\operatorname {Si} (x)+C} Series definitionTaylor series sinc x = ∑ k = 0 ∞ (− 1) k x 2 k (2 k + 1) ! {\displaystyle \operatorname {sinc} x=\sum _{k=0}^{\infty }{\frac {\left(-1\right)^{k}{x}^{2k}}{\left(2k+1\right)!}}} The sinc function as audio, at 2000 Hz (±1.5 seconds around zero). In mathematics, the historical unnormalized sinc function is defined for x ≠ 0 by sinc x = sin x x . {\displaystyle \operatorname {sinc} x={\frac {\sin x}{x}}.} Alternatively, the unnormalized sinc function is often called the sampling function, indicated as Sa(x).[2] In digital signal processing and information theory, the normalized sinc function is commonly defined for x ≠ 0 by sinc x = sin (π x) π x . {\displaystyle \operatorname {sinc} x={\frac {\sin(\pi x)}{\pi x}}.} In either case, the value at x = 0 is defined to be the limiting value sinc 0 := lim x → 0 sin (a x) a x = 1 {\displaystyle \operatorname {sinc} 0:=\lim _{x\to 0}{\frac {\sin(ax)}{ax}}=1} for all real a ≠ 0 (the limit can be proven using the squeeze theorem). The normalization causes the definite integral of the function over the real numbers to equal 1 (whereas the same integral of the unnormalized sinc function has a value of π). As a further useful property, the zeros of the normalized sinc function are the nonzero integer values of x. The normalized sinc function is the Fourier transform of the rectangular function with no scaling. It is used in the concept of reconstructing a continuous bandlimited signal from uniformly spaced samples of that signal. The only difference between the two definitions is in the scaling of the independent variable (the x axis) by a factor of π. In both cases, the value of the function at the removable singularity at zero is understood to be the limit value 1. The sinc function is then analytic everywhere and hence an entire function. The function has also been called the cardinal sine or sine cardinal function.[3][4] The term sinc /ˈsɪŋk/ was introduced by Philip M. Woodward in his 1952 article "Information theory and inverse probability in telecommunication", in which he said that the function "occurs so often in Fourier analysis and its applications that it does seem to merit some notation of its own",[5] and his 1953 book Probability and Information Theory, with Applications to Radar.[6][7] The function itself was first mathematically derived in this form by Lord Rayleigh in his expression (Rayleigh's Formula) for the zeroth-order spherical Bessel function of the first kind. Properties The local maxima and minima (small white dots) of the unnormalized, red sinc function correspond to its intersections with the blue cosine function. The zero crossings of the unnormalized sinc are at non-zero integer multiples of π, while zero crossings of the normalized sinc occur at non-zero integers. The local maxima and minima of the unnormalized sinc correspond to its intersections with the cosine function. That is, sin(ξ)/ξ = cos(ξ) for all points ξ where the derivative of sin(x)/x is zero and thus a local extremum is reached. This follows from the derivative of the sinc function: d d x sinc (x) = cos (x) − sinc (x) x . {\displaystyle {\frac {d}{dx}}\operatorname {sinc} (x)={\frac {\cos(x)-\operatorname {sinc} (x)}{x}}.} The first few terms of the infinite series for the x coordinate of the n-th extremum with positive x coordinate are x n = q − q − 1 − 2 3 q − 3 − 13 15 q − 5 − 146 105 q − 7 − ⋯ , {\displaystyle x_{n}=q-q^{-1}-{\frac {2}{3}}q^{-3}-{\frac {13}{15}}q^{-5}-{\frac {146}{105}}q^{-7}-\cdots ,} where q = (n + 1 2) π , {\displaystyle q=\left(n+{\frac {1}{2}}\right)\pi ,} and where odd n lead to a local minimum, and even n to a local maximum. Because of symmetry around the y axis, there exist extrema with x coordinates −xn. In addition, there is an absolute maximum at ξ0 = (0, 1). The normalized sinc function has a simple representation as the infinite product: sin (π x) π x = ∏ n = 1 ∞ (1 − x 2 n 2) {\displaystyle {\frac {\sin(\pi x)}{\pi x}}=\prod _{n=1}^{\infty }\left(1-{\frac {x^{2}}{n^{2}}}\right)} The cardinal sine function sinc(z) plotted in the complex plane from -2-2i to 2+2i and is related to the gamma function Γ(x) through Euler's reflection formula: sin (π x) π x = 1 Γ (1 + x) Γ (1 − x) .

TABLE 3.1
Short Table of Fourier Transforms

	$g(t)$	$G(f)$	
	$e^{-at}u(t)$	$\frac{1}{a+j2\pi f}$	$a>0$
2	$e^{at}u(-t)$	$\frac{1}{a-j2\pi f}$	$a>0$
3	$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+(2\pi f)^2}$	
4	$te^{-at}u(t)$	$\frac{1}{(a+j2\pi f)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a+j2\pi f)^{n+1}}$	
6	$\delta(t)$		
7	1	$\delta(f)$	
8	$e^{j2\pi f_0 t}$	$\delta(f-f_0)$	
9	$\cos 2\pi f_0 t$	$0.5[\delta(f+f_0)+\delta(f-f_0)]$	
10	$\sin 2\pi f_0 t$	$j0.5[\delta(f+f_0)-\delta(f-f_0)]$	
11	$u(t)$	$\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$	
12	$\operatorname{sgn} t$	$\frac{2}{j2\pi f}$	
	$\cos 2\pi f_0 t\, u(t)$	$\frac{1}{4}[\delta(f-f_0)+\delta(f+f_0)]+\frac{j2\pi f}{(2\pi f_0)^2-(2\pi f)^2}$	
14	$\sin 2\pi f_0 t\, u(t)$	$\frac{1}{4j}[\delta(f-f_0)-\delta(f+f_0)]+\frac{2\pi f_0}{(2\pi f_0)^2-(2\pi f)^2}$	
15	$e^{-at}\sin 2\pi f_0 t\, u(t)$	$\frac{2\pi f_0}{(a+j2\pi f)^2+4\pi^2 f_0^2}$	
16	$e^{-at}\cos 2\pi f_0 t\, u(t)$	$\frac{a+j2\pi f}{(a+j2\pi f)^2+4\pi^2 f_0^2}$	
	$\Pi\left(\frac{t}{\tau}\right)$	$\tau\operatorname{sinc}(\pi f\tau)$	
18	$2B\operatorname{sinc}(2\pi Bt)$	$\Pi\left(\frac{f}{2B}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\pi f\tau}{2}\right)$	
20	$B\operatorname{sinc}^2(\pi Bt)$	$\Delta\left(\frac{f}{2B}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT$	$f_0\sum_{n=-\infty}^{\infty}\delta(f-nf_0)$	
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-2(\sigma\pi f)^2}$	

{\displaystyle {\frac {\sin(\pi x)}{\pi x}}={\frac {1}{\Gamma (1+x)\Gamma (1-x)}}.} Euler discovered[8] that sin (x) x = ∏ n = 1 ∞ cos (x 2 n) , {\displaystyle {\frac {\sin(x)}{x}}=\prod _{n=1}^{\infty }\cos \left({\frac {x}{2^{n}}}\right),} and because of the product-to-sum identity[9] Domain coloring plot of sinc z = sin z/z ∏ n = 1 k cos (x 2 n) = 1 2 k − 1 ∑ n = 1 2 k − 1 cos (n − 1 / 2 2 k − 1 x) , ∀ k ≥ 1 , {\displaystyle \prod _{n=1}^{k}\cos \left({\frac {x}{2^{n}}}\right)={\frac {1}{2^{k-1}}}\sum _{n=1}^{2^{k-1}}\cos \left({\frac {n-1/2}{2^{k-1}}}x\right),\quad \forall k\geq 1,} Euler's product can be recast as a sum sin (x) x = lim N → ∞ 1 N ∑ n = 1 N cos (n − 1 / 2 N x) . {\displaystyle {\frac {\sin(x)}{x}}=\lim _{N\to \infty }{\frac {1}{N}}\sum _{n=1}^{N}\cos \left({\frac {n-1/2}{N}}x\right).} The continuous Fourier transform of the normalized sinc (to ordinary frequency) is rect(f): ∫ − ∞ ∞ sinc (t) e − i 2 π f t d t = rect (f) , {\displaystyle \int _{-\infty }^{\infty }\operatorname {sinc} (t)\,e^{-i2\pi ft}\,dt=\operatorname {rect} (f),} where the rectangular function is 1 for argument between −1/2 and 1/2, and zero otherwise. This corresponds to the fact that the sinc filter is the ideal (brick-wall, meaning rectangular frequency response) low-pass filter. This Fourier integral, including the special case ∫ − ∞ ∞ sin (π x) π x d x = rect (0) = 1 {\displaystyle \int _{-\infty }^{\infty }{\frac {\sin(\pi x)}{\pi x}}\,dx=\operatorname {rect} (0)=1} is an improper integral (see Dirichlet integral) and not a convergent Lebesgue integral, as ∫ − ∞ ∞ | sin (π x) π x | d x = + ∞ . {\displaystyle \int _{-\infty }^{\infty }\left|{\frac {\sin(\pi x)}{\pi x}}\right|\,dx=+\infty .} The normalized sinc function has properties that make it ideal in relationship to interpolation of sampled bandlimited functions: It is an interpolating function, i.e., sinc(0) = 1, and sinc(k) = 0 for nonzero integer k. The functions xk(t) = sinc(t − k) (k integer) form an orthonormal basis for bandlimited functions in the function space L2(R), with highest angular frequency ωH = π (that is, highest cycle frequency fH = 1/2). Other properties of the two sinc functions include: The unnormalized sinc is the zeroth-order spherical Bessel function of the first kind, j0(x). The normalized sinc is j0(πx). where Si(x) is the sine integral, ∫ 0 x sin (θ) θ d θ = Si (x) . {\displaystyle \int _{0}^{x}{\frac {\sin(\theta)}{\theta }}\,d\theta =\operatorname {Si} (x).} λ sinc(λx) (not normalized) is one of two linearly independent solutions to the linear ordinary differential equation x d 2 y d x 2 + 2 d y d x + λ 2 x y = 0. {\displaystyle x{\frac {d^{2}y}{dx^{2}}}+2{\frac {dy}{dx}}+\lambda ^{2}xy=0.} The other is cos(λx)/x, which is not bounded at x = 0, unlike its sinc function counterpart. Using normalized sinc, ∫ − ∞ ∞ sin 2 (θ) θ 2 d θ = π ⇒ ∫ − ∞ ∞ sinc 2 (x) d x = 1 , {\displaystyle \int _{-\infty }^{\infty }{\frac {\sin ^{2}(\theta)}{\theta ^{2}}}\,d\theta =\pi \quad \Rightarrow \quad \int _{-\infty }^{\infty }\operatorname {sinc} ^{2}(x)\,dx=1,} ∫ − ∞ ∞ sin (θ) θ d θ = ∫ − ∞ ∞ (sin (θ) θ) 2 d θ = π . {\displaystyle \int _{-\infty }^{\infty }{\frac {\sin(\theta)}{\theta }}\,d\theta =\int _{-\infty }^{\infty }\left({\frac {\sin(\theta)}{\theta }}\right)^{2}\,d\theta =\pi .} ∫ − ∞ ∞ sin 3 (θ) θ 3 d θ = 3 π 4 . {\displaystyle \int _{-\infty }^{\infty }{\frac {\sin ^{3}(\theta)}{\theta ^{3}}}\,d\theta ={\frac {3\pi }{4}}.} ∫ − ∞ ∞ sin 4 (θ) θ 4 d θ = 2 π 3 . {\displaystyle \int _{-\infty }^{\infty }{\frac {\sin ^{4}(\theta)}{\theta ^{4}}}\,d\theta ={\frac {2\pi }{3}}.} The following improper integral involves the (not normalized) sinc function: ∫ 0 ∞ d x x n + 1 = 1 + 2 ∑ k = 1 ∞ (− 1) k + 1 (k n) 2 − 1 = 1 sinc (π n) . {\displaystyle \int _{0}^{\infty }{\frac {dx}{x^{n}+1}}=1+2\sum _{k=1}^{\infty }{\frac {(-1)^{k+1}}{(kn)^{2}-1}}={\frac {1}{\operatorname {sinc} ({\frac {\pi }{n}})}}.} Relationship to the Dirac delta distribution The normalized sinc function can be used as a nascent delta function, meaning that the following weak limit holds: lim a → 0 sin (π x a) π x = lim a → 0 1 a sinc (x a) = δ (x) . {\displaystyle \lim _{a\to 0}{\frac {\sin \left({\frac {\pi x}{a}}\right)}{\pi x}}=\lim _{a\to 0}{\frac {1}{a}}\operatorname {sinc} \left({\frac {x}{a}}\right)=\delta (x).} This is not an ordinary limit, since the left side does not converge. Rather, it means that lim a → 0 ∫ − ∞ ∞ 1 a sinc (x a) φ (x) d x = φ (0) {\displaystyle \lim _{a\to 0}\int _{-\infty }^{\infty }{\frac {1}{a}}\operatorname {sinc} \left({\frac {x}{a}}\right)\varphi (x)\,dx=\varphi (0)} for every Schwartz function, as can be seen from the Fourier inversion theorem. In the above expression, as a → 0, the number of oscillations per unit length of the sinc function approaches infinity. Nevertheless, the expression always oscillates inside an envelope of ±1/πx, regardless of the value of a. This complicates the informal picture of δ(x) as being zero for all x except at the point x = 0, and illustrates the problem of thinking of the delta function as a function rather than as a distribution. A similar situation is found in the Gibbs phenomenon. Summation All sums in this section refer to the unnormalized sinc function. The sum of sinc(n) over integer n from 1 to ∞ equals π − 1/2: ∑ n = 1 ∞ sinc (n) = sinc (1) + sinc (2) + sinc (3) + sinc (4) + ⋯ = π − 1 2 . {\displaystyle \sum _{n=1}^{\infty }\operatorname {sinc} (n)=\operatorname {sinc} (1)+\operatorname {sinc} (2)+\operatorname {sinc} (3)+\operatorname {sinc} (4)+\cdots ={\frac {\pi -1}{2}}.} The sum of the squares also equals π − 1/2:[10][11] ∑ n = 1 ∞ sinc 2 (n) = sinc 2 (1) + sinc 2 (2) + sinc 2 (3) + sinc 2 (4) + ⋯ = π − 1 2 . {\displaystyle \sum _{n=1}^{\infty }\operatorname {sinc} ^{2}(n)=\operatorname {sinc} ^{2}(1)+\operatorname {sinc} ^{2}(2)+\operatorname {sinc} ^{2}(3)+\operatorname {sinc} ^{2}(4)+\cdots ={\frac {\pi -1}{2}}.} When the signs of the addends alternate and begin with +, the sum equals 1/2: ∑ n = 1 ∞ (− 1) n + 1 sinc (n) = sinc (1) − sinc (2) + sinc (3) − sinc (4) + ⋯ = 1 2 . {\displaystyle \sum _{n=1}^{\infty }(-1)^{n+1}\,\operatorname {sinc} (n)=\operatorname {sinc} (1)-\operatorname {sinc} (2)+\operatorname {sinc} (3)-\operatorname {sinc} (4)+\cdots ={\frac {1}{2}}.} The alternating sums of the squares and cubes also equal 1/2:[12] ∑ n = 1 ∞ (− 1) n + 1 sinc 2 (n) = sinc 2 (1) − sinc 2 (2) + sinc 2 (3) − sinc 2 (4) + ⋯ = 1 2 , {\displaystyle \sum _{n=1}^{\infty }(-1)^{n+1}\,\operatorname {sinc} ^{2}(n)=\operatorname {sinc} ^{2}(1)-\operatorname {sinc} ^{2}(2)+\operatorname {sinc} ^{2}(3)-\operatorname {sinc} ^{2}(4)+\cdots ={\frac {1}{2}},} ∑ n = 1 ∞ (− 1) n + 1 sinc 3 (n) = sinc 3 (1) − sinc 3 (2) + sinc 3 (3) − sinc 3 (4) + ⋯ = 1 2 . {\displaystyle \sum _{n=1}^{\infty }(-1)^{n+1}\,\operatorname {sinc} ^{3}(n)=\operatorname {sinc} ^{3}(1)-\operatorname {sinc} ^{3}(2)+\operatorname {sinc} ^{3}(3)-\operatorname {sinc} ^{3}(4)+\cdots ={\frac {1}{2}}.} Series expansion The Taylor series of the unnormalized sinc function can be obtained from that of the sine (which also yields its value of 1 at x = 0): sin x x = ∑ n = 0 ∞ (− 1) n x 2 n (2 n + 1) ! = 1 − x 2 3 ! + x 4 5 ! − x 6 7 ! + ⋯ {\displaystyle {\frac {\sin x}{x}}=\sum _{n=0}^{\infty }{\frac {(-1)^{n}x^{2n}}{(2n+1)!}}=1-{\frac {x^{2}}{3!}}+{\frac {x^{4}}{5!}}-{\frac {x^{6}}{7!}}+\cdots } The series converges for all x. The normalized version follows easily: sin π x π x = 1 − π 2 x 2 3 ! + π 4 x 4 5 ! − π 6 x 6 7 ! + ⋯ {\displaystyle {\frac {\sin \pi x}{\pi x}}=1-{\frac {\pi ^{2}x^{2}}{3!}}+{\frac {\pi ^{4}x^{4}}{5!}}-{\frac {\pi ^{6}x^{6}}{7!}}+\cdots } Euler famously compared this series to the expansion of the infinite product form to solve the Basel problem.
Higher dimensions The product of 1-D sinc functions readily provides a multivariate sinc function for the square Cartesian grid (lattice): sincC(x, y) = sinc(x) sinc(y), whose Fourier transform is the indicator function of a square in the frequency space (i.e., the brick wall defined in 2-D space). The sinc function for a non-Cartesian lattice (e.g., hexagonal lattice) is a function whose Fourier transform is the indicator function of the Brillouin zone of that lattice. For example, the sinc function for the hexagonal lattice is a function whose Fourier transform is the indicator function of the unit hexagon in the frequency space.
For a non-Cartesian lattice this function can not be obtained by a simple tensor product. However, the explicit formula for the sinc function for the hexagonal, body-centered cubic, face-centered cubic and other higher-dimensional lattices can be explicitly derived[13] using the geometric properties of Brillouin zones and their connection to zonotopes. For example, a hexagonal lattice can be generated by the (integer) linear span of the vectors u 1 = [1 2 3 2] and u 2 = [1 2 − 3 2] . {\displaystyle \mathbf {u} _{1}={\begin{bmatrix}{\frac {1}{2}}\\{\frac {\sqrt {3}}{2}}\end{bmatrix}}\quad {\text{and}}\quad \mathbf {u} _{2}={\begin{bmatrix}{\frac {1}{2}}\\-{\frac {\sqrt {3}}{2}}\end{bmatrix}}.} Denoting ξ 1 = 2 3 u 1 , ξ 2 = 2 3 u 2 , ξ 3 = − 2 3 (u 1 + u 2) , x = [x y] , {\displaystyle {\boldsymbol {\xi }}_{1}={\tfrac {2}{3}}\mathbf {u} _{1},\quad {\boldsymbol {\xi }}_{2}={\tfrac {2}{3}}\mathbf {u} _{2},\quad {\boldsymbol {\xi }}_{3}=-{\tfrac {2}{3}}(\mathbf {u} _{1}+\mathbf {u} _{2}),\quad \mathbf {x} ={\begin{bmatrix}x\\y\end{bmatrix}},} one can derive[13] the sinc function for this hexagonal lattice as sinc H (x) = 1 3 (cos (π ξ 1 ⋅ x) sinc (ξ 2 ⋅ x) sinc (ξ 3 ⋅ x) + cos (π ξ 2 ⋅ x) sinc (ξ 3 ⋅ x) sinc (ξ 1 ⋅ x) + cos (π ξ 3 ⋅ x) sinc (ξ 1 ⋅ x) sinc (ξ 2 ⋅ x)) . {\displaystyle {\begin{aligned}\operatorname {sinc} _{\text{H}}(\mathbf {x})={\tfrac {1}{3}}{\big (}&\cos \left(\pi {\boldsymbol {\xi }}_{1}\cdot \mathbf {x} \right)\operatorname {sinc} \left({\boldsymbol {\xi }}_{2}\cdot \mathbf {x} \right)\operatorname {sinc} \left({\boldsymbol {\xi }}_{3}\cdot \mathbf {x} \right)\\&{}+\cos \left(\pi {\boldsymbol {\xi }}_{2}\cdot \mathbf {x} \right)\operatorname {sinc} \left({\boldsymbol {\xi }}_{3}\cdot \mathbf {x} \right)\operatorname {sinc} \left({\boldsymbol {\xi }}_{1}\cdot \mathbf {x} \right)\\&{}+\cos \left(\pi {\boldsymbol {\xi }}_{3}\cdot \mathbf {x} \right)\operatorname {sinc} \left({\boldsymbol {\xi }}_{1}\cdot \mathbf {x} \right)\operatorname {sinc} \left({\boldsymbol {\xi }}_{2}\cdot \mathbf {x} \right){\big)}.\end{aligned}}} This construction can be used to design Lanczos window for general multidimensional lattices.[13] See also Anti-aliasing filter – Mathematical transformation reducing the damage caused by aliasing Borwein integral – Type of mathematical integrals Dirichlet integral – Integral of sin(x)/x from 0 to infinity. Lanczos resampling – Application of a mathematical formula List of mathematical functions Shannon wavelet Sinc filter – Ideal low-pass filter Trigonometric functions of matrices – Important functions in solving differential equations Trigonometric integral – Special function defined by an integral Whittaker–Shannon interpolation formula – Signal (re-)construction algorithm Winkel tripel projection – Pseudoazimuthal compromise map projection (cartography) Sinhc function References

^ Olver, Frank W. J.; Lozier, Daniel M.; Boisvert, Ronald F.; Clark, Charles W., eds. (2010), "Numerical methods", NIST Handbook of Mathematical Functions, Cambridge University Press, ISBN 978-0-521-19225-5, MR 2723248.
^ Singh, R. P.; Sapre, S. D. (2008). Communication Systems, 2E (illustrated ed.). Tata McGraw-Hill Education. p. 15. ISBN 978-0-07-063454-1. Extract of page 15 ^ Weisstein, Eric W.
"Sinc Function". mathworld.wolfram.com. Retrieved 2023-06-07. ^ Merca, Mircea (2016-03-01). "The cardinal sine function and the Chebyshev–Stirling numbers". Journal of Number Theory. 160: 19–31. doi:10.1016/j.jnt.2015.08.018. ISSN 0022-314X. ^ Woodward, P. M.; Davies, I. L. (March 1952). "Information theory and inverse probability in telecommunication" (PDF). Proceedings of the IEE - Part III: Radio and Communication Engineering. 99 (58): 37–44. doi:10.1049/pi-3.1952.0011. ^ Poynton, Charles A. (2003). Digital video and HDTV. Morgan Kaufmann Publishers. p. 147. ISBN 978-1-55860-792-7. ^ Woodward, Phillip M.
(1953). Probability and information theory, with applications to radar. London: Pergamon Press. p. 29. ISBN 978-0-89006-103-9. OCLC 488749777. ^ Euler, Leonhard (1735). "On the sums of series of reciprocals". arXiv:math/0506415. ^ Luis Ortiz-Gracia; Cornelis W. Oosterlee (2016). "A highly efficient Shannon wavelet inverse Fourier technique for pricing European options". SIAM J. Sci. Comput. 38 (1): B118–B143.

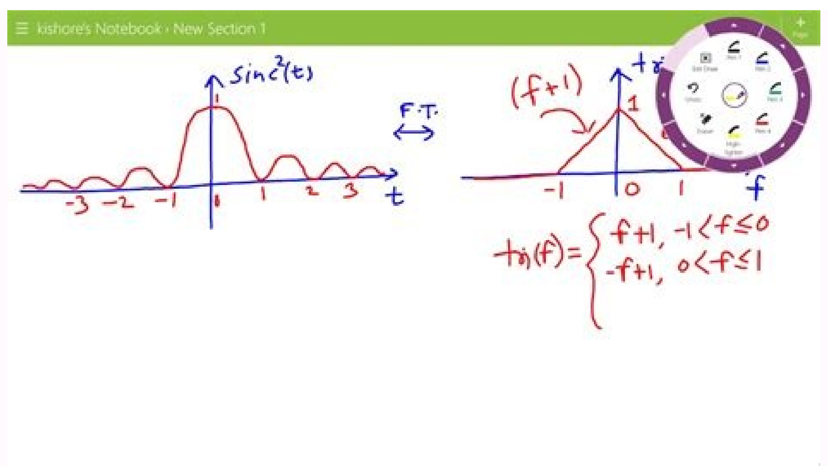


where: $\Delta(t)$ is the Unit Triangle Function $\Delta\left(\frac{t}{\tau}\right) \Leftrightarrow \frac{\tau}{2} sinc^2\left(\frac{\omega\tau}{4}\right)$

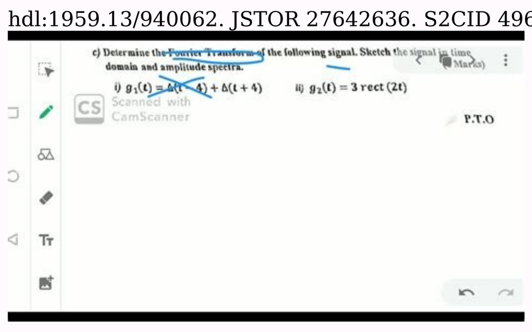
doi:10.1137/15M1014164. ^ "Advanced Problem 6241". American Mathematical Monthly. Washington, DC: Mathematical Association of America.
87 (6): 496–498. June–July 1980. doi:10.1080/00029890.1980.11995075. ^ Robert Baillie; David Borwein; Jonathan M. Borwein (December 2008).

	$f(x)$	$\hat{f}_s(w) = \mathcal{F}_s(f)$
1	$\begin{cases} 1 & \text{if } 0<x<a \\ 0 & \text{otherwise} \end{cases}$	$\sqrt{\frac{2}{\pi}}\left[\frac{1-\cos aw}{w}\right]$
2	$1/\sqrt{x}$	$1/\sqrt{w}$
3	$1/x^{3/2}$	$2\sqrt{w}$
4	x^{a-1} $(0<a<1)$	$\sqrt{\frac{2}{\pi}}\frac{\Gamma(a)}{w^a}\sin\frac{a\pi}{2}$
5	e^{-ax} $(a>0)$	$\sqrt{\frac{2}{\pi}}\left(\frac{w}{a^2+w^2}\right)$
6	$\frac{e^{-ax}}{x}$ $(a>0)$	$\sqrt{\frac{2}{\pi}}\arctan\frac{w}{a}$
7	$x^n e^{-ax}$ $(a>0)$	$\sqrt{\frac{2}{\pi}}\frac{n!}{(a^2+w^2)^{n+1}}\operatorname{Im}(a+iw)^{n+1}$ Im = Imaginary part
8	$xe^{-x^2/2}$	$we^{-w^2/2}$
9	xe^{-ax^2} $(a>0)$	$\frac{w}{(2a)^{3/2}}e^{-w^2/4a}$
10	$\begin{cases} \sin x & \text{if } 0<x<a \\ 0 & \text{otherwise} \end{cases}$	$\frac{1}{\sqrt{2\pi}}\left[\frac{\sin a(1-w)}{1-w}-\frac{\sin a(1+w)}{1+w}\right]$
11	$\frac{\cos ax}{x}$ $(a>0)$	$\sqrt{\frac{\pi}{2}}u(w-a)$
12	$\arctan\frac{2a}{x}$ $(a>0)$	$\sqrt{2\pi}\frac{\sinh aw}{w}e^{-aw}$

"Surprising Sinc Sums and Integrals". American Mathematical Monthly. 115 (10): 888–901. doi:10.1080/00029890.2008.11920606.



hdl:1959.13/940062. JSTOR 27642636. S2CID 496934. ^ Baillie, Robert (2008). "Fun with Fourier series".



arXiv:0806.0150v2 [math.CA]. ^ a b c Ye, W.; Entezari, A. (June 2012). "A Geometric Construction of Multivariate Sinc Functions". IEEE Transactions on Image Processing. 21 (6): 2969–2979. Bibcode:2012ITIP...21.2969Y.
doi:10.1109/TIP.2011.2162421. PMID 21775264.
S2CID 15313688. External links Weisstein, Eric W.
"Sinc Function". MathWorld. Retrieved from " More...Less... The sinc function , also called the "sampling function," is a function that arises frequently in signal processing and the theory of Fourier transforms. The full name of the function is "sine cardinal," but it is commonly referred to by its abbreviation, "sinc." There are two definitions in common use. The one adopted in this work defines where is the sine function, plotted above. This has the normalization This function is implemented in the Wolfram Language as Sinc[x]. When extended into the complex plane, is illustrated above.
An interesting property of is that the set of local extrema of corresponds to its intersections with the cosine function , as illustrated above. The derivative is given by and the indefinite integral by where is the sine integral. Woodward (1953), McNamee et al. (1971), and Bracewell (1999, p. 62) adopt the alternative definition The latter definition is sometimes more convenient as a result of its simple normalization, That variant also satisfies the sum In addition, the binomial coefficient satisfies which is essentially a restatement of the reflection relation of the gamma function (M. Somos, pers. comm., Oct. 26, 2006.) The sinc function is closely related to the spherical Bessel function of the first kind and, in particular, and is given in terms of the Meijer G-function as Let be the rectangle function, then the Fourier transform of is the sinc function The sinc function therefore frequently arises in physical applications such as Fourier transform spectroscopy as the so-called instrument function, which gives the instrumental response to a delta function input. Removing the instrument functions from the final spectrum requires use of some sort of deconvolution algorithm.
The sinc function can be written as a complex integral by noting that, for , and that and the integral both equal 1 for . The sinc function can also be written as the infinite product a result discovered in 1593 by Francois Viète (Kac 1959, Morrison 1995) and sometimes known as Euler's formula (Prudnikov et al. 1986, p. 757; Gearhart and Shulz 1990). It is also given by (Gearhart and Shulz 1990) and (Prudnikov et al. 1986, p. 757). Another product is given by (OEIS A118253; Prudnikov et al. 1986, p. 757), where is the constant from polygon circumscribing. Sums of powers of over the positive integers include The remarkable fact that the sums of and are equal appears to have first been published in Baillie (1978). Amazingly, the pattern of these sums being equal to plus a rational multiple of breaks down for the power , where the sum equals where The sinc function satisfies the identity Definite integrals involving the sinc function include After dividing out the constant factor of , the values are again 1/2, 1/2, 3/8, 1/3, 115/384, 11/40, 5887/23040, 151/630, 259723/1146880, ... (OEIS A049330 and A049331; Grimsey 1945, Medhurst and Roberts 1965). These are all special cases of the amazing general result where and are positive integers such that , , is the floor function, and is taken to be equal to 1 (Kogan; cf. Espinosa and Moll 2000). This spectacular formula simplifies in the special case when is a positive even integer to where is an Eulerian number (Kogan; cf. Espinosa and Moll 2000). The solution of the integral can also be written in terms of the recurrence relation for the coefficients The half-infinite integral of can be derived using contour integration. In the above figure, consider the path . Now write . On an arc, and on the x-axis, . Write where denotes the imaginary part. Now define where the second and fourth terms use the identities and . Simplifying, where the third term vanishes by Jordan's lemma. Performing the integration of the first term and combining the others yield Rearranging gives so The same result is arrived at using the method of complex residues by noting so Since the integrand is symmetric, we therefore have giving the sine integral evaluated at 0 as Borwein Integrals, Fourier Transform, Fourier Transform--Rectangle Function, Instrument Function, Jinc Function, Kilroy Curve, Sine, Sine Integral, Sinhc Function, Tanc Function Baillie, R.
"Advanced Problem 6241." Amer. Math. Monthly 85, 828, 1978.Baillie, R.; Henrici, P.; and Johnsonbaugh, R.
"Solution to Advanced Problem 6241." Amer. Math. Monthly 87, 496-498, 1980.Bracewell, R. "The Filtering or Interpolating Function, ." In The Fourier Transform and Its Applications, 3rd ed.
New York: McGraw-Hill, pp. 62-64, 1999.Brown, J. W. and Churchill, R. V. Fourier Series and Boundary Value Problems, 5th ed. New York: McGraw-Hill, 1993.Espinosa, O. and Moll, V. H.
"On Some Definite Integrals Involving the Hurwitz Zeta Function." . 11 Dec 2000.Finch, S. R.
Mathematical Constants. Cambridge, England: Cambridge University Press, 2003.Gearhart, W. B. and Schulz, H. S. "The Function ." College Math. J. 21, 90-99, 1990. A. H. R. "On the Accumulation of Chance Effects and the Gaussian Frequency Distribution." Phil. Mag. 36, 294-295, 1945.Higgins, J. R. "Five Short Stories About the Cardinal Series." Bull. Amer. Math. Soc. 12, 45-89, 1985.Kac, M. Statistical Independence in Probability, Analysis and Number Theory. Washington, DC: Math. Assoc. Amer., 1959.Kogan, S. "A Note on Definite Integrals Involving Trigonometric Functions." Unpublished manuscript, n.d.McNamee, J.; Stenger, F.; and Whitney, E. L. "Whittaker's Cardinal Function in Retrospect." Math. Comput. 25, 141-154, 1971.Medhurst, R. G. and Roberts, J. H. "Evaluation of the Integral ." Math. Comput.
19, 113-117, 1965.Morrison, K. E. "Cosine Products, Fourier Transforms, and Random Sums." Amer. Math. Monthly 102, 716-724, 1995.Prudnikov, A. P.; Brychkov, Yu. A.; and Marichev, O. I. Integrals and Series, Vol. 1: Elementary Functions. New York: Gordon & Breach, 1986.Sloane, N. J. A. Sequences A049330, A049331, and A118253 in "The On-Line Encyclopedia of Integer Sequences."Stenger, F. Numerical Methods Based on Sinc and Analytic Functions. New York: Springer-Verlag, 1993.Woodward, P. M. Probability and Information Theory with Applications to Radar. New York: McGraw-Hill, 1953.Zimmermann, P. "Int(sin(x)^k/x^k,x=0..infinity)." math-fun@cs.arizona.edu posting, 4 Mar 1997.Sinc Function Weisstein, Eric W.
"Sinc Function." From MathWorld--A Wolfram Web Resource. Subject classificationsMore...Less...