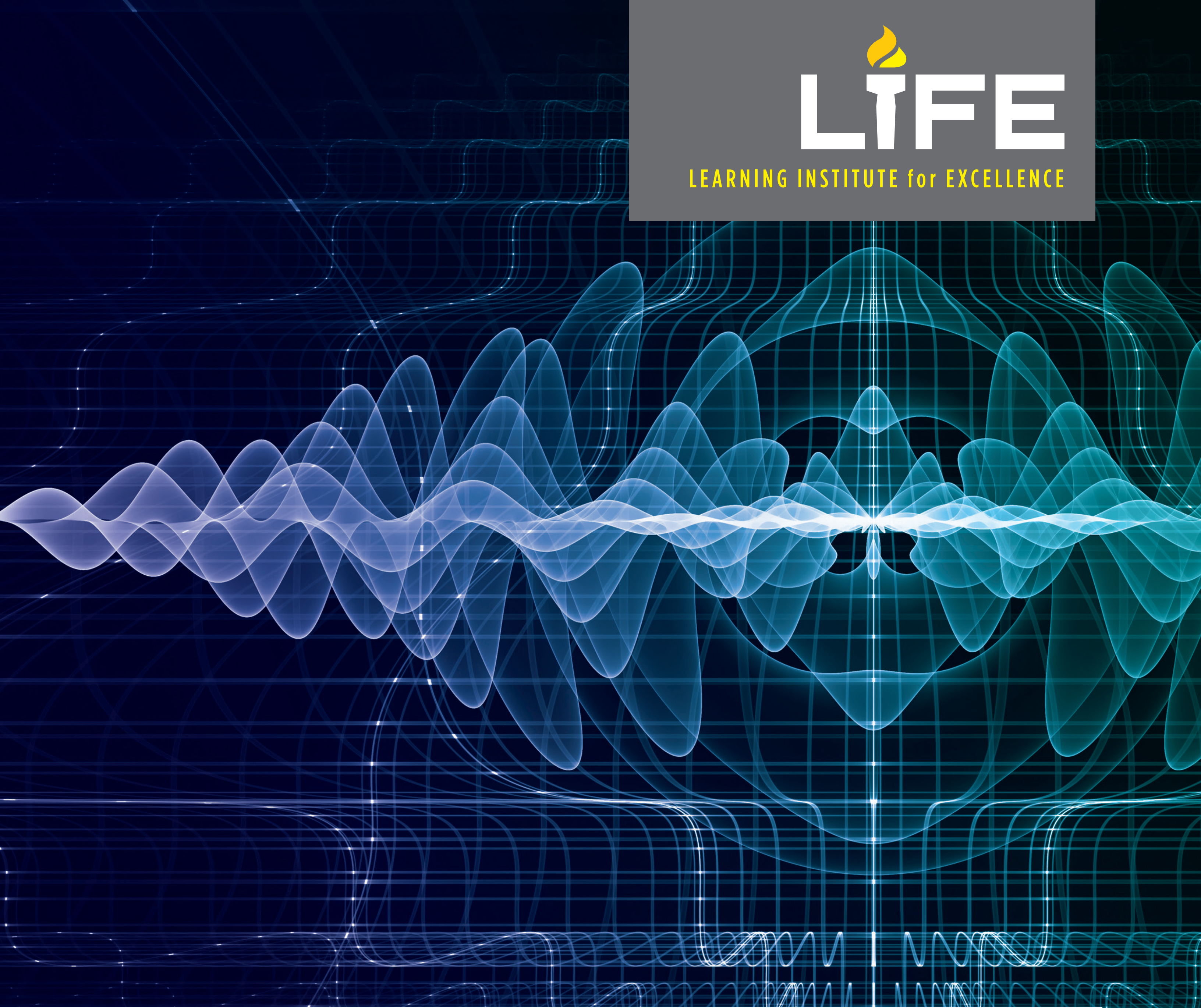


LIFE

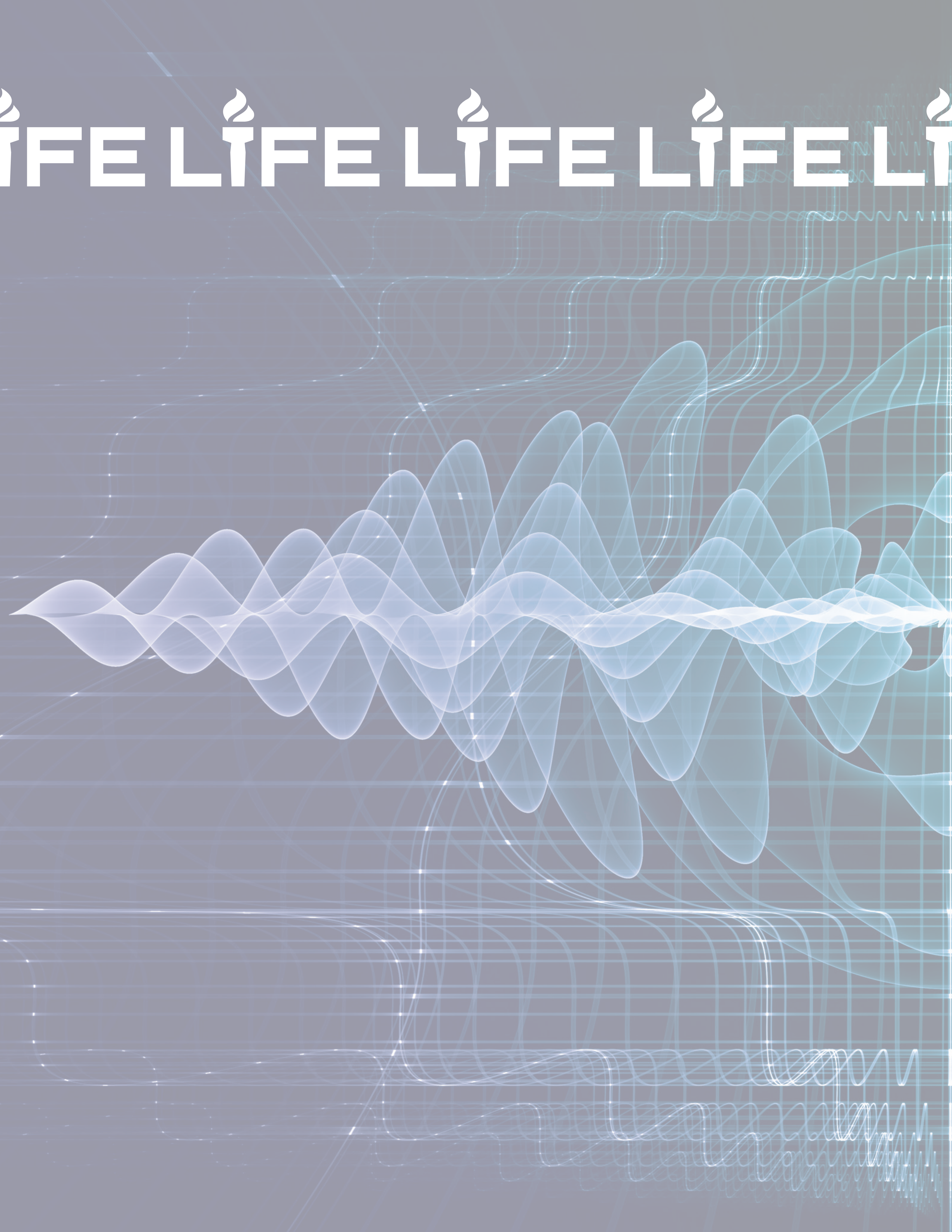
LEARNING INSTITUTE for EXCELLENCE

Tools for Mastering **Mathematics**

For Teachers, Students, Curriculum Writers, Administrators and Those Helping Students Excel in Mathematics and Beyond

Amos Tarfa & Nathan Jersett

IGNITE LIFELONG LEARNING


LIFE LIFE LIFE

Tools for Mastering Mathematics

Summary:

Using the tools and resources we have been given, to teach students to gain a love for learning that helps them to master the material we present to them so that they become lifelong learners and individuals who make an impact in an age where people constantly ask about the relevance of the material they are learning.

Article to Math Teachers, Students, Curriculum Writers, Administrators and anyone who is willing to help students excel in Mathematics and beyond

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CONNECTING WITH INDIVIDUALS WHO ENJOY DOING MATHEMATICS- By getting this material, you are now in touch with a group of individuals who enjoy doing Mathematics. We want you to send your questions and concerns and we will provide the extra resources you will need to excel in Mathematics and much more.

Contents

PART 1:

What is Mathematics?
How should students excel in it?

Table of contents

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Introduction

This is an article about learning and mastering principles in Mathematics. It also gives teachers some ideas that could be used to inspire students as they proceed in their education. One of the things our students need today is Math coaches, people who are willing and ready to inspire them to excel. For some students, math is not exciting. They sometimes think that they are incapable and that math is just too hard. In this article, I want to show how Math can be used as a tool to help students excel in academics and beyond. Math is a fascinating subject, just like many other subjects, however, many students <u>do not know</u> what math is or how they are supposed to excel in it so they conclude that there is nothing fascinating about Mathematics. Many people never get to see what Math really is and why they should appreciate it. People appreciate many other subjects but then they either despise or give up trying to master Mathematics. My goal is to show how a better understanding of Mathematics can help students appreciate it more. If they appreciate it, they are more likely to seek ways to master it. I agree people can master it without appreciating it but this does not always happen.

What is this thing called Mathematics?
Metaphors that could be used to describe Mathematics

Mathematics deals with patterns as well as many other things. Mathematics is a field of study that deals with concepts, numbers, variables, critical thinking, relationships and problem solving skills. I have decided to use some examples to describe what this thing called Mathematics is.

1. <u>Math is a sport or a game with rules:</u> It has rules, techniques, lessons and it challenges you to grow. The rules in basic Mathematics remain the same and are precise. Anytime a student gets a Math problem wrong, they should ask themselves "What Math rule did I break?" What principle did I misuse? Math is not a guessing game so one should pay attention to every rule that is discussed.

Students should remember to seek mastery in every concept discussed. Just like in any game, Mathematics studies patterns and we succeed when we understand the patterns.

2. <u>It is like riding a bicycle:</u> In 1999, I learned to ride a bicycle. It involved practice and hard work. I had to learn different things; how to turn, how to stop, how to start moving and so on. Math also involves learning principles; addition, subtraction, multiplication table. This is the first thing they have in common. The second thing is that if you stopped riding a bicycle for a period of time and then you started later, you will not have to start from the very beginning. This also happens in Mathematics (I will encourage students to keep practicing regularly). If for some reason students do not have contact with formal Mathematics lessons (we have to remember that we do Math everyday), once they have mastered a concept, it will probably be easier to relearn it when the time comes. Students can review the material and they will start remembering what they mastered in the past. Students need to remember to do corrections to the problems that they get wrong. Even if a student got a 98 percent on a test, the corrections will be helpful for them later on.

3. <u>Studying Math is like a building a house:</u> You need a solid foundation and then you need to master principles in every section. The foundation of a house is very important. Students should seek to master the concepts that you are being taught because they will need those concepts when they move to the next layer.

4. <u>It is a tool, which can be used to derive more tools:</u> You can think of the first set of topics in Mathematics as tools which will be used to derive more topics later on. Students should make sure their tools are always in good shape so they can create more tools when the need arises.

5. <u>It is an art:</u> You are allowed to be creative in Math as long as you obey the rules that govern the problems you are solving. Here is an example from Trigonometry. When you are asked to find the missing length in a 45-45 -90 triangle, you can use your knowledge of this

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special triangle to solve the problem or you can use the Pythagorean theorem. I have given you two options but you will notice that you can also use trigonometric ratios to find the missing side. We can use three different approaches to one problem. Whatever approach you use, seek to be consistent and make sure that you follow the rules.

If Math can be described with the terms I have listed above, I find it interesting that people can choose to dislike such a field of study. Mathematics is an exciting field and I hope more students can see what Mathematics is, so they appreciate it and learn more about the wonderful design in our universe. As they better understand Mathematics, they can excel in certain aspects of science as well.

Integrated Math Curriculum from Nigeria

I went to high school in Nigeria and we use a math curriculum which is similar to that which is used in many countries but not the one here in the United States. The curriculum followed an integrated approach and so there were no class titles for the Math classes students took. This was because there was some element of each section of Math each year (some Algebra, some Geometry, some lessons in Statistics). This was the basic structure, and it assumed that students in 10th grade understood all the concepts from 9th grade and below and students in 11th grade understood the concepts from 10th grade and below and students in 12th grade understood all the concepts from the previous grades. Every student took Math courses from 1st grade to 12th grade, however students had a choice to take Advanced Math (which was a separate class) in 10th, 11th and 12th grade. Duplication is mostly absent in the curriculum and students could always go back to their previous books if they had questions.

It is key to note that the integrated curriculum is a good way to learn Math independent of the countries that use it. It works not because of those who use it but because it helps to tie things together. The word integrated here is referring to the fact that we tie different strands of Math together every year.

Brief description of the sequence in America

In 2012, I decided to review and do an in-depth study of the math curriculum used in the United States. The students go through Math in a certain order. In elementary school, they go through a certain curriculum which puts students in certain classes based on their grade level and when they get to the beginning of middle school, they take Arithmetic and Pre-Algebra classes. Then they move to Algebra 1 around 8th grade in certain states like Minnesota. After Algebra 1, they take Geometry and then Algebra 2 (or the other way, Algebra 2 then Geometry). After Algebra 2 or Geometry, they take Pre-Calculus or Trigonometry and then Calculus 1, 2 and 3 (This is the general order describing the sequence of Math classes in the United States. Not everyone takes Calculus in high school).

Here is a summary of the order:

Elementary School Math → Pre-Algebra → Algebra 1 → Geometry → Algebra 2 → Pre-Calculus/Trigonometry → Calculus

In each of these classes you will find elements of Algebra and Geometry; however students might not know this. I have met students who think they will not use Algebra as much after Algebra 1 because Algebra 1 is an independent Math section. This could become a problem for students who think themes in Math are not connected. I have seen students who say they are in Algebra 1 and they do not need Pre-Algebra. The challenge is that some do not know how to find the Lowest Common Multiple which could be useful when solving rational expressions.

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What is unique about Mathematics? How will this program help students make a difference?

We need to seek opportunities that will help us show students the potential they have and why they need to develop that potential. There are many people waiting for our children to flourish. If they flourish, they can make an impact in the lives of those around them. Because Math is precise and straightforward, (theoretically speaking) we can help students learn the rules and then move on to discussions about how to make an impact. Math teaches us critical thinking skills. The rules of basic Math are the same everywhere, and we just need to teach the students what these rules are.

I believe that one day the difference between students in Math class will not be in their skills but in their speed. I believe we can equip students by giving them the tools and skills they need to excel and then we will notice that everyone can work when the time for work comes. Now some students struggle with certain skills, and yet we have them take a test even though we know that they have not digested the material. What if we gave students the opportunity to work with the tools available first to achieve mastery within a reasonable time frame? One of the tools might be working in groups after they have had the opportunity to review the material themselves. Giving the students the opportunity to work in groups will create an atmosphere that will allow the students who fully understand the concept to help teach those who need some more practice to master it. This will then allow them to be tested on speed not skill. I will present a list of other tools later in this document.

Why should we teach Mathematics differently?

Here are some ways we can help our students to master principles in Mathematics

1. <u>Reduce Duplication:</u> Students are sometimes required to learn a topic several times which reduces the time to introduce new concepts. We need to assign a series of questions to help the students understand the concept enough so that when they take the next class they don't need to relearn the concept from scratch. Students start learning about solids in middle school; however Geometry textbooks sometimes cover topics on volumes of cylinders. What if we focused on Volume and areas for a period of time with application lessons and gave the students the opportunity to learn the material thoroughly? Let us encourage them to learn the material well now. Is there a place for duplication, review and repetition in our math program? Yes, we should allow students to review concepts as much as they need the review without making the review our focus. This is where students have to own their learning and admit that they need to review certain concepts. Whenever we are teaching a lesson and we come across a topic that uses another topic within it, we need to take that opportunity to review. Some students may need more review. We should have a reference book students can always go back to. This will be similar to an English student having a dictionary. **We have put together booklets with examples and practice for specific concepts for students. These books will act as a resource for students in case they ever get stuck on a particular topic. They can take the book that addresses that topic to look at more examples for that topic. Any student at any school could use the books as a supplement to whatever textbook they may be using. Teachers could also use these books to give more examples if the students have questions. These booklets can also act as a textbook for students.**

2. <u>Lack of Mastery:</u> Many students move into the next class without mastering essential concepts. Some students do not like graphing; others do not like adding fractions. However, they are allowed to move on to the next class. What if we made students learn the rules for graphing and taught them how to master the skills that are essential? If a student learns to graph functions in Algebra 1, they will most likely do better in Geometry and Algebra 2 as opposed to if they do

not master graphing. I believe that when students master concepts, they will be more willing to apply themselves to the next set of concepts. I know that not everyone likes graphing even if they know how to graph, however it is very good that they know this important tool. We need to encourage students to master skills as they go on.

3. Misunderstanding: Many students do not understand what Math is. Many question if there are any applications. We need to start teaching applications of Math concepts. Also, we should include opportunities for critical thinking and reasoning, so students see what Math really is. We need to develop clear goals for the students so they can work toward those goals. Many students do not have expectations, and they leave a Math class without reaching mastery in all or some of the concepts necessary for the next class. We need to lay out all the goals for the math course a student takes so they are aware of the tools they will need for the next course.

Students think the concepts we teach only apply to certain numbers or terms. They do not know that the mechanism stands valid not matter what the contents of the problem are. For example, when we use the rules of exponents, it does not matter what the numbers or letters are. We can add exponents even when variables are involved. The rules work for numbers as well as letters.

CURRICULUM DESIGN AND DEVELOPMENT: Can we design a Math Curriculum that guarantees that if a student understands a particular level necessary for the next level, that they will understand the new level? It is key to note that understanding the lower level only shows that you are ready for the next level not that you will find it very easy. I think students will find the next level easier compared to if they did not master the previous level. Also can we show students how to use tools that they have gained to create what needs to be made using those tools? Can we encourage creativity in students so that they can think of several ways of solving problems and looking for the most efficient ways? These skills are very important and students can use them in whatever field they go into. Can we help students see the relevance in the material we present to them so they see why they should care?

TEACHERS: The Role of a Teacher is similar to that of a coach instead of a person who just gives information. Students have access to materials on the internet and all over their neighborhoods, what they need is direction and guidance as they investigate the materials they have. Teachers provide the guidance, support and encouragement necessary to help them flourish.

Let us remember that every student has talent and potential. Let us not conclude that a student is incapable, or that their past shows certain things that cannot be changed. Let us keep encouraging our students to get better.

Students- Fundamental principles that need to be taught in Mathematics and questions students should consider whenever they are given a problem

1. What have I been given?
2. Do I understand what the question is asking?
3. What am I solving for?
4. What do I know?
5. What tools do I use to solve this problem?
6. What mechanism is in question? It is not about the numbers or letters, there is a mechanism that can be used to solve the problem.
7. What should my answer look like?
8. What does my answer mean?

9. How can I check my answer?
10. Does the answer I found make sense?
11. Are there more efficient ways to solve this problem?

Our goal should be to get students to get an interest in learning for life. When they are interested in the material it changes everything.

Ways to get students interested in learning Mathematics

1. Teach the application of the concept.
2. Teach about the History of the people behind the concept. Why did they work so hard to show us these principles?
3. Projects – Hands-on
4. Documentaries and movies about Mathematics
5. Reading projects. Maybe read an articles on the beauty of Mathematics
6. Field Trips to places that use Mathematics
7. Provide opportunities for them to teach others what they learn
8. Ask students to set goals and then give rewards to the students who accomplish the goals they set.
9. Introduce them to the Philosophy of Mathematics and Logic
10. Create Challenging and doable situations so students are challenged in something they can do. Make sure you equip the students with all the tools they will need.
11. Show them that they can do it. Confidence is a concern for many students. They think they are incapable. They forget that there is nothing wrong in trying again.
12. Remove the language barrier in Mathematics. Mathematics involves reasoning and the answer to the problems they solve should make sense.

As you have noticed, I tried to write curriculum without duplication that also comes with tools students can use to review topics. We can review units occasionally to make sure students have not forgotten the concept, but when we review we will review all relevant concepts within that theme. Maybe we can go through the topics several times in one year by going back to earlier topics later in the year and reviewing them. When a student moves on to the next grade, they should not have to review the topic for as long as a student who is encountering the topic for the first time. Students should be constantly reminded of the interconnectivity of topics in Mathematics as they go through any grade level. My plan is to have students master each concept before they move on within a reasonable time frame. Sometimes topics are not as connected so you can come back to them if the student does not master the concept within that time frame. Schools can decide what this will look like. Students also get to learn about the Mathematicians behind the concepts they are learning. They also get to think through the concepts that we discuss. No matter how many math lessons the students learn every year, we need to integrate lessons in Math history and some time to learn the application of the material they are learning. If they are given time to digest the material they will start excelling in the next set of topics. I tried this with a few students over the summer and they all wanted to do more problems to master the concept. I also tried using this method with students during the school year and they also wanted more problems, because they had mastered concepts they previously thought were very difficult. As students start mastering the concepts in Mathematics, they will begin to see the connection between Math and other subjects. Math teachers can now become Math coaches as they inspire their students to face challenges, think, process, and excel. Math is a mental sport and students need to see that the more problems they do on a certain concept, the better they will become.

If we get a straight forward inventory for the students, they can start working through the topics as they master each concept. I made an inventory for students, which covers topics from 5th grade all the way to Pre-Calculus. I also have an inventory for elementary school.

STUDENTS: Why should you excel in Mathematics or why should you want to excel?

Are we ever going to use this? Are we ever going to use Math? When are we going to use this information?

I have been asked this question several times throughout my tutoring and teaching career. Before I answer this famous question, I will use an analogy with sports to clarify certain concepts. When are we ever going to use sports? When are we ever going to use basketball? When are we ever going to use volleyball, soccer, tennis? The answer is, we might never use the skills in a particular sport outside the time when we play that sport. So why do we play sports? We play sports to gain other things not necessarily the sport itself. The sport is a tool that gives us the opportunity to exercise and use what we have been blessed with. The reason we play sports is not because of the sport itself, we also play because of certain things that it gives us, things like exercise for the body, skills with working with a team and much more. This is the case with Mathematics. The comparison used to explain sports and Math is only used to show that there are certain things we do, not because we want to gain something from the thing we are doing but instead because of the other products of the activity which may or may not include the activity in question. When you play soccer, you are simply moving a soccer ball around. This activity alone is not the reason people play. We gain a lot from playing besides simply moving the soccer ball. We learn discipline in sports, organization, planning, critical thinking skills and we also learn these in Mathematics.

As I mentioned in the previous paragraph, there is much more we can learn from both sports and academics. Let me turn to Mathematics and state that we use Math every day in different activities. We might not solve rules of exponents in our heads but we do use basic math at least every day. Students do need to learn the concepts partly because of standardized tests, Chemistry concepts, Physics concepts and other Math related fields.

Math teaches us to reason in a systematic approach. We learn to put forth propositions and also to respond to questions people ask. Math gives us a tool to use for this. We learn how to lay our points in a systematic manner.

Math teaches us how to give reasons for the propositions we make. The reason why our propositions are valid is because of an established principle that has been stated earlier. We learn this in Mathematics and we can also see this in many areas of life.

We make arguments every day but how do we know that our arguments are valid? We need to be able to justify what we say not just by our words but also by our actions.

This analogy should help to explain my point (My point is that there are many things we do which have no benefit in and of themselves but they provide us with a way to learn tools we will need for life). In the movie, the Karate Kid, we see that the teacher makes the student pick up his coat many times. There was a reason for this but the student was not fully aware. Could it be possible that the systematic methods in Math are for a purpose you are not fully aware of yet?

I agree that there are certain topics that are difficult in Mathematics and many students do not like proofs but this does not mean that students should give up easily. Teachers need to do their best to find resources that will help the students understand the concept better. This might be a different textbook, a website, videos or investigations. Let us all do our best to learn even when we do not understand why we are learning what we are learning.

<u>Summary of some of the skills and lessons we can learn from Math</u>

1. Hard work as we give our best to better understand a concept.

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2. Discipline: Learning to set aside time to practice.
3. Diligence
4. Growth
5. Ability to think critically

STUDENTS: How to excel in Mathematics

1. Take good notes in class
2. Talk to other classmates who are paying attention to what is happening in the class.
3. Read the examples from the lesson before coming to class.
4. Do practice problems daily. If you ask someone for help, make sure you understand what they explain to you. Make sure you are able to explain it back to them. Practice is key in Mathematics and any other sport or art.
5. Talk to the teacher when you do not understand the topic. Sometimes listening to the explanation twice helps.
6. When taking tests, make sure you are aware of the time assigned. If you do not understand a particular problem, go through the remaining questions and come back to the ones that you do not understand.
7. Use the tools and resources available to you. Remember you can watch videos online and do practice problems. You can also print out worksheets to get extra practice
8. Teach others what you learn even if you have not mastered it.

Why do we do practice problems in Mathematics

1. To help us better understand the concept.
2. It teaches us how to work hard and how to be disciplined.

How do we know when we have practiced enough? When we master the concept. How do we know when we have mastered the concept? When we can talk about it with confidence to others and teach them thoroughly. There are other ways to know when you have mastered a concept but this is one way.

The Math Ambassador Program

This is a program that is designed to encourage students to teach their peers what they learn as they learn it. Learning is a lifelong endeavor. Math Ambassadors seek to teach their peers the relevance of Mathematics, the importance of tapping into their potential and the importance of making an impact for the right reason.

As students learn a specific topic, we want to encourage them to share their knowledge with others.

TEACHERS: If students master the material before the school is up, what do we do?

If students master the material before the school year is up, we can focus on career exploration and volunteer opportunities. Many students are seeking opportunities to explore careers and we can help them explore as we free up time as they master the material. We need to teach students the importance of service. This math program will allow for students to job shadow.

We can also give the students opportunities to read books. We can have them read inspirational books. The Math coach can make a list of books the students can pick from. As they master concepts, we can reward them with a book.

If a student needs more time over the summer to master a concept, then let us provide them with the opportunity to work on a few problems a day during the summer.

Conclusion

When should we start Mastering concepts in Mathematics?

Today is the day to start learning, growing and seeking ways to make an impact. Let us start encouraging students in ways that will help them achieve mastery.

I am excited to see what happens as we seek ways to help students succeed so they can encourage others to succeed as well. Thank you for reading through this article.

INSPIRATIONAL MATHEMATICS
Short Note on techniques and tools for teachers

May 2014

"What students need fundamentally in academics today is not information, they need inspiration and vision. They can get information from the library as well as Google. Once they are inspired and hardworking, the technology and coaching/teaching available will help them excel in whatever they choose to study. With the technology and tools available, teachers can help students master concepts in Mathematics like never before. We need to constantly seek ways to get them to care about excelling. Teachers can provide students with the tools they need to have a vision and the necessary inspiration"

-Amos Tarfa [Founder, Learning Institute For Excellence (LIFE), LLC]

What can we do to help our students enjoy doing mathematics as well as help them master the concepts they are expected to learn?

There are different types of students. We have students who have a good attitude towards learning mathematics and some who do not have a good attitude. It is important to note that a person's attitude towards something will affect how much they are willing to sacrifice in order to master principles within the thing in question. In this document, I am proposing that there are certain techniques that can help students appreciate mathematics, adjust their attitude and gain the skills they need to excel in it. It is important to note that not every student will change their attitude right away but these tools might help inspire more students to care about learning. What are these tools, techniques and recommendations?

Teachers need to seek ways to inspire their students. This could be done by telling them what it takes to excel and showing them that they can always improve as they keep working hard. This step is very important because if a student thinks that they are incapable then it will be difficult to help them appreciate and work hard in the subject/topic before them. How can we inspire students?

1. We can inspire students by telling them about things that happened in **math history.** We can help students by showing them how great mathematicians learned from other mathematicians. We can also use **real world applications** to help the students see why they should care. We can also inspire students by using **quotes** from people who worked hard.

2. Solid Logical and Consistent Content: When students know that there is hope for them, we want to make sure we provide them with solid content including examples and sufficient practice problems. Students need to be reminded that they can do more problems if they have not mastered the concept before them. The content needs to flow logically from something they have learned or will be learning. When a student does not know what prime numbers are, it will be difficult for them to prime factorize.

3. Study Skills and Organization: It is interesting to note that even if a student has solid content and a willingness to learn, they will be hindered if they do not have a structure in place. Students need to have a daily plan even when they do not have assignments. There is always room to learn more on a particular topic.

4. Assessment: When a student masters a concept, they should be able to excel when they are given a test. If the student does not do well with tests, we can examine this issue and see how to help them handle this. Students also need to know how to handle standardized tests so we need to create opportunities for them to take sample problems and work on them.

5. <u>Discussions, Projects and Presentations:</u> Students should be given opportunities to share what they are learning. Maybe they could do a presentation on a mathematician they like or on a math concept they find fascinating like the Fibonacci Sequence. We should provide students with the tools they need to teach one another. Eventually we might see them working in groups after school or during lunch.

6. <u>Summer:</u> When students see what it takes to succeed, they might be willing to spend 20 minutes a day during the summer reviewing concepts. This will help them start the new school year with some tools ready to excel again.

7. <u>Review Days:</u> Students can be given opportunities to review the concepts they have been learning. Maybe they can work in groups if the atmosphere is right.

For the teacher:

The key to excelling in inspiring students is finding the balance and knowing when to adjust each of the tools mentioned. When is it ok to spend more time on tests or when is it ok to spend time talking about study skills and good organizational skills?

Example: When you notice that your students are not doing their assigned work, it might be a good time to teach them how to schedule time and how to follow through on their assignments.

PART 2:

Tools for Mastering Mathematics
Mathematics Reference Book

Table of Contents

Place Value

What is the difference between 1,000,000 and 1? It is obvious that they look different, but those 0's mean a lot. In fact, the difference is understood by using place value. Each 0 and each 1 means something in the place that you find it. That is because each place the number is put is understood to have a value.

Since our number system is base 10, each place that is different from the exact center, the decimal place or the ones place, will be some power of 10. Furthermore, the factors of 10 that are in a number may be multiplied by one of the 10 digits our number system uses: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

Every number we write is actually a combination of digits multiplied by factors of 10. We can, therefore, break up the number back into its parts. To do this, you need to understand the different powers of ten that are within the place value. For now, lets look at a few powers of ten within a number.

What are the place values in the number 1,526,049?

Number	Place Value	Power of Ten
1	Million	1,000,000
5	Hundred Thousand	100,000
2	Ten Thousand	10,000
6	Thousand	1,000
0	Hundred	100
4	Ten	10
9	One	1

So, the number 1,526,049 is the same as $1 \cdot 1,000,000 + 5 \cdot 100,000 + 2 \cdot 10,000 + 6 \cdot 1,000 + 0 \cdot 100 + 4 \cdot 10 + 9 \cdot 1$.

The simple operations you do on numbers–addition, subtraction, multiplication, and division–all deal with changing the place values. When I say 15 + 26, I actually mean two 10s and six 1s in addition to one 10 and five 1s. How do we add it? Well, count how many ones you have now, but don't go above nine 1s (since this is the largest digit we have). When we have nine 1s and have an additional 1, then we move this whole group into the 10s place value, as $9 + 1 = 10$. So, in our example, we have nine 1s and two 1s, so we take the nine and one of the two and make a 10 out of it, leaving us with just one 1 left. Then, we count how many 10s there are. We have two 10s and added one 10, as well as getting an additional 10 from the 1s. This gives us four 10s, and a final count of four 10s with one 1. This is 41.

What does a multiplication problem like $3 \cdot 25$ mean? Well, it is saying, we have 3 different quantities of 25. Each quantity of 25 is two 10s and five 1s, so adding the 1s together gives a set of 10 and five 1s. Then adding the 10s together gives seven 10s. The grand total is 75.

Decimal Place Value

What are the place values of decimals? Let's look at 12,345.56789. We have already talked about the place values of 12,345, these follow the normal rules, but what about the .56789 portion?

Number	Place Value	Power of Ten
5	Tenths	0.1
6	Hundredths	0.01
7	Thousandths	0.001
8	Ten Thousandths	0.0001
9	Hundred Thousandths	0.00001

You can break up the decimal portion just as easily as the rest of the numbers, in the same way. More place value names and powers of ten are shown in the next section.

Powers of Ten

Powers of Ten are not only important in understanding the place value, but also for writing in scientific notation, using the Metric System, quickly reducing fractions, and more.

...	...	...
Billionth	10^{-9}	0.000000001
Hundred Millionth	10^{-8}	0.00000001
Ten Millionth	10^{-7}	0.0000001
Millionth	10^{-6}	0.000001
Hundred Thousandth	10^{-5}	0.00001
Ten Thousandth	10^{-4}	0.0001
Thousandth	10^{-3}	0.001
Hundredth	10^{-2}	0.01
Tenth	10^{-1}	0.1
One	10^{0}	1
Ten	10^{1}	10
Hundred	10^{2}	100
Thousand	10^{3}	1,000
Ten Thousand	10^{4}	10,000
Hundred Thousand	10^{5}	100,000
Million	10^{6}	1,000,000
Ten Million	10^{7}	10,000,000
Hundred Million	10^{8}	100,000,000
Billion	10^{9}	1,000,000,000
Trillion	10^{12}	1,000,000,000,000
Quadrillion	10^{15}	1,000,000,000,000,000
Quintillion	10^{18}	1,000,000,000,000,000,000
...	...	...
Googol	10^{100}	10,000,000,000,000,000,000,000,000,000,000,000, 000,000,000,000,000,000,000,000,000,000,000,000, 000,000,000,000,000,000,000,000,000,000,000
...	...	...
Googolplex	$10^{10^{100}}$	...
...	...	...

Rounding and Estimation

Rounding

We round to make numbers easier to use. The way to round is to find the place value you want to round, then look at the number to the right. If that number is 5 or higher add one to the number in the place value you want then change all of the numbers to the right of this place to 0. If the number is 4 or less, then just change all of the numbers to the right of the place to 0.

Example 1:
What do we get if we round 363,755 to the hundreds place? The hundreds place is the 7 in this number. Then, to round this number, we need to look at the number to the right of the 7: the number 5. Is this number 5 or higher? Yes, so we need to add one to the number 7 (giving 8) and change all of the numbers to the right to 0. Doing this gives 363,800.

Example 2:
What do we get if we round 4,567.823 to the tenths place? The tenths place is the 8 in this number. So, the number to the right of this place is a 2, which is 4 or less. This means that we do nothing but change all numbers to the right of this place to 0. This gives the number $4{,}567.800 = 4{,}567.8$. If there are zeros after the last nonzero digit, then it is acceptable to drop them, but only if they are to the right of the decimal. This is due to the fact that they will not change the place values of other digits.

Estimation

Sometimes we need to make numbers more manageable by estimating their value. This can be done by either knowing their exact value and rounding to a certain spot or picking what a number is by an educated guess. Estimation is used to quickly error check certain calculations, quickly do calculations by making the numbers easier to handle, and even just make numbers easier to remember. Think about it, which is easier to remember: 3 million or 3,124,645.897?

Estimation problems can be really easy to find, and as you handle more and more problems using estimation, you will be a better judge of what estimations are good guesses or bad guesses. Let's do a simple estimation problem, how many ice cream cones will you have eaten in your lifetime? To make it easier, let's assume you live until 100 years old. Even though you may have had more ice cream cones when you are younger, you probably eat less when you are older, and even though you probably eat a lot of cones during the summer, you will eat less when you are in winter. So, balancing it all to one amount per year, let's say you have 20 ice cream cones per year, every year of your life. Then, all you need to do is $100 \cdot 20$ which is 2000 ice cream cones. Of course, depending on the person and lifestyle, this can change, but it does a decent job giving a general number.

Addition Table

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

Table 1: Addition Table from 1 - 20

Multiplication Table

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400

Table 2: Multiplication Table from 1 - 20

Order of Operations and Number Chart

Order of Operations: PEMDAS

1st: (P) Parentheses
2nd: (E) Exponents
3rd: (MD) Multiplication and Division (left to right)
4th: (AS) Addition and Subtraction (left to right)

Whenever you need to solve an expression that has only numbers, you always follow this order.

Number Chart:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Properties of Real Numbers

Additive Identity:

$$5 + 0 = 5$$
$$a + 0 = a$$

Inverse Property of Addition:

$$2 + (-2) = 0$$
$$c + (-c) = 0$$

Multiplicative Identity:

$$9 \cdot 1 = 9$$
$$h \cdot 1 = h$$

Inverse Property of Multiplication:

$$4 \cdot \frac{1}{4} = 1$$
$$m \cdot \frac{1}{m} = 1$$

Zero Property of Multiplication:

$$38 \cdot 0 = 0$$
$$n \cdot 0 = 0$$

Commutative Property of Addition:

$$\begin{aligned} 5 + 3 &= 3 + 5 \quad \text{and both sides equal 8} \\ x + y &= y + x \end{aligned}$$

Commutative Property of Multiplication:

$$\begin{aligned} 3 \cdot 4 &= 4 \cdot 3 \quad \text{and both sides equal 12} \\ p \cdot r &= r \cdot p \end{aligned}$$

Associative Property of Addition:

$$\begin{aligned} (2 + 3) + 4 &= 2 + (3 + 4) \quad \text{becomes} \\ 5 + 4 &= 2 + 7 \quad \text{and both sides equal 9} \\ (a + b) + c &= a + (b + c) \end{aligned}$$

Associative Property of Multiplication:

$$\begin{aligned} (2 \cdot 3) \cdot 4 &= 2 \cdot (3 \cdot 4) \quad \text{becomes} \\ 6 \cdot 4 &= 2 \cdot 12 \quad \text{and both sides equal 24} \\ (a \cdot b) \cdot c &= a \cdot (b \cdot c) \end{aligned}$$

Distributive Property:

$$\begin{aligned} 2 \cdot (5 + 6) &= 2 \cdot 5 + 2 \cdot 6 \quad \text{becomes} \\ 2 \cdot 11 &= 10 + 12 \quad \text{and both sides equal 22} \\ a \cdot (b + c) &= a \cdot b + a \cdot c \end{aligned}$$

Integer Arithmetic

Integer Multiplication (remember, two numbers next to each other, in parentheses, are multiplying):

$$+a \cdot +b = +a \cdot b$$
$$+a \cdot -b = -a \cdot b$$
$$-a \cdot +b = -a \cdot b$$
$$-a \cdot -b = +a \cdot b$$

Examples:

$$2 \cdot 4 = 8$$
$$(-3)(5) = -15$$
$$-8 \cdot -2 = 16$$
$$7(-3) = -21$$

Integer Addition:
If both numbers are +, then just add them: 7 + 3 = 10
If the bigger number is + and the smaller number is -, then subtract like usual: 10 - 4 = 6
If the bigger number is - and the smaller number is +, then subtract the smaller from the bigger and give the answer a negative sign: -8 + 3 = ? Well, 8 - 3 = 5, then the answer is -5.
If both numbers are -, then add the numbers and give the answer a negative number: -4 - 6 = ? Well, 4 + 6 = 10, then the answer is -10.

Whenever two signs are right next to each other, follow the integer multiplication rules to see how the signs combine:

$$+(+ = ++ = +,$$
$$-(+ = -+ = -,$$
$$-(- = -- = +,$$
$$+(- = +- = -.$$

Examples:

$$-5 + 4 = 4 - 5 = -1$$
$$-6 + 8 = 8 - 6 = 2$$
$$12 - (-10) = 12 + 10 = 22$$
$$-5 - 1 = -6$$
$$2 + 8 = 10$$
$$-1 - (-6) = -1 + 6 = 5$$

Divisibility Rules

For number x to be divisible by number y, when you take $\frac{x}{y}$, you must get a whole number as a result, not a fraction or decimal.

2	Even numbers are divisible by 2. Even numbers are those that end in 0, 2, 4, 6, or 8.
3	Add the digits, if the addition is divisible by 3, then the original number is divisible by 3. 363? $3 + 6 + 3 = 12$, $12 \div 3 = 4$, Yes. 373? $3 + 7 + 3 = 13$, $13 \div 3 = 4.333\ldots$, No.
4	Look at the number in the last two digits spot, if this number is divisible by 4, then the original number is divisible by 4. 1,524? $24 \div 4 = 6$, Yes. 863? $63 \div 4 = 15.75$, No.
5	If the number ends with 0 or 5, then it is divisible by 5.
6	If the number is divisible by 2 and 3, then it is divisible by 6.
7	Double the last digit and subtract it from the number that remains, if the subtraction is 0 or divisible by 7 then the original number is divisible by 7. 98? $8 \cdot 2 = 16$, $9 - 16 = -7$, $-7 \div 7 = -1$, Yes. 164? $4 \cdot 2 = 8$, $16 - 8 = 8$, $8 \div 7 = 1.142\ldots$, No.
8	If the number in the last three digits spot is divisible by 8, then the original number is divisible by 8. 51,200? $200 \div 8 = 25$, Yes. 81,106? $106 \div 8 = 13.25$, No.
9	Add the digits, if the addition is divisible by 9, then the original number is divisible by 9. 864? $8 + 6 + 4 = 18$, $18 \div 9 = 2$, Yes. 1,502? $1 + 5 + 0 + 2 = 8$, $8 \div 9 = 0.888\ldots$, No.
10	If the number ends with 0, then it is divisible by 10.
11	If you sum up every second digit then subtract the rest of the digits added up, and if the answer is 0 or divisible by 11, then the original number is divisible by 11. 1,232? $2 + 2 - (1 + 3) = 0$, Yes. 653? $5 - (6 + 3) = -4$, $-4 \div 11 = 0.808\ldots$, No.
12	If the number is divisible by 3 and 4, then it is divisible by 12.

Prime Number Properties

Prime numbers are numbers that are only divisible by themselves and 1. The first prime number is 2, and we only consider numbers greater than 2 to be candidates to be prime numbers. To see if a number is prime, you need to see if this number is divisible by all of the previous prime numbers. This is an easy process for small, one to two digit numbers, but can be a hassle for larger numbers.

All whole numbers (excluding 0 and 1) can be factorized into prime numbers. In fact, every number has a unique factorization, meaning when you multiply different combinations of prime numbers together, you will get different numbers every single time.

Prime numbers example, between 2 and 100:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Since all even numbers are divisible by 2, then the only even number that can be prime is 2. The number 9 is not prime because $9 \div 3 = 3$. The number 15 is not prime because $15 \div 3 = 5$. The number 21 is not prime because $21 \div 3 = 7$. This process can be repeated for every number not shown above.

Prime Factorization: To prime factorize a number, keep dividing by the smallest divisible prime number possible, until the division equal 1. Then, the prime factorization of the original number is all of those divisible prime numbers multiplied (don't actually do the multiplication, just state them all with a multiplication sign between each number).

Example: What is the prime factorization of 180?

The number 180 is even, so the smallest prime number that can go into it is 2. So, $180 \div 2 = 90$. This answer is also even, $90 \div 2 = 45$. The number 45 is not divisible by 2, but can be divided by 3 (the next smallest prime number), so $45 \div 3 = 15$. This is also divisible by 3, so $15 \div 3 = 5$. Finally, $5 \div 5 = 1$. Since this answer is 1, we have finished. So, taking all of the primes we divided by, the prime factorization of 180 is:

$$180 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5.$$

This can be easily checked by multiplying the prime factors and seeing that the answer is 180.

Basics of Set Theory

In order to talk about Classification of Real Numbers, we need to have an understanding of the notation used. But, before we can talk about the notation, we need to understand what a set is.

A set is a collection of things, a group. A set can be a group of numbers, of items, of people, of anything really. For Mathematics, it is most common for us to use numbers though, so that will be the main focus. You can name the sets, and whenever you mention the letter, you mean to say all of the items in the set. It is important to note that sets do not have repetition inside. If there is an item in the set, then there cannot be another of the exact same item in that same set. You have to have distinguishable items. There are a few ways of describing a set:

1st way:

$$\text{List: } S = \{3, 4, 10, 18, 29, 8, 23\} \quad \text{or} \quad H = \{3, 6, 9, 12, 15, 18, ...\}$$

The list method actually itemizes what is in the set. Each item (separated by a comma) is called an element of the set. The list method will either state all of the elements, or it will show a pattern so that the reader can find the rest of the elements, as seen in set H as it contains all multiples of 3.

2nd way:

$$\text{Set Builder: } G = \{3 \cdot x + 4 \mid x \in S\}$$

The set builder method describes a rule for the elements, and the rule has a condition on it. The rule is found before the vertical bar, while the condition is after. A lot of the time, you will see the symbol $\in$. This stands for "is an element of". So, in our set G, the variable x is an element of S, or the set from the first example (the set $\{3, 4, 10, 18, 29, 8, 23\}$). This mean you use the rule for each of the elements in S to find all of the elements of G. Thus:

$$G = \{3 \cdot (3) + 4,\ \ 3 \cdot (4) + 4,\ \ 3 \cdot (10) + 4,\ \ 3 \cdot (18) + 4,\ \ 3 \cdot (29) + 4,\ \ 3 \cdot (8) + 4,\ \ 3 \cdot (23) + 4\}$$
$$G = \{13, 16, 34, 58, 91, 28, 73\}$$

3rd way:
The last way to describe a set is just to describe the set using words. For instance, set J is the set of all numbers ending with 1. Another example is set M is the set of all pens of the world. As you see, both are just described in words, but the set J can also be described as a list and as a set builder. Set M can also be described using a list, but it would be a lot harder. When making sets of anything besides numbers, you will always use the list method or the descriptive method.

Set Arithmetic
There are some operations that we can do with sets. The most basic operations is the union and the intersection. These are done between two sets, just like addition and subtraction are done between two numbers. The result of the union and the intersection is a single, new set, just like the result of addition and subtraction is a single, new number.

Union
The union, or $\cup$, is an operation that is like addition of sets. When you take a union of two sets, you are taking all of the elements in the first and taking all of the elements in the second, and putting them into a new set. Remember, you do not repeat elements inside a set, as seen below in the last example.

Examples:

$$\{2,3,5\} \cup \{1,6,7\} = \{1,2,3,5,6,7\}$$
$$\{1,3,5,7,9,11,...\} \cup \{0,2,4,6,8,10,...\} = \{0,1,2,3,4,5,6,7,8,9,10,11,...\}$$
$$\{15,30,45,60\} \cup \{10,20,30,40,50,60\} = \{10,15,20,30,40,45,50,60\}$$

Intersection
The intersection, or $\cap$, is an operation that is like subtraction of sets. When you take an intersection of two sets, you are taking only the elements that are in both sets, and putting them into a new set. As always, do not repeat elements.
Examples:

$$\{1,3,5,7,9\} \cap \{2,3,4,5,6,7,8\} = \{3,5,7\}$$
$$\{...,-5,-3,-1,1,3,5,...\} \cap \{...,-15,-10,-5,0,5,10,15,...\} = \{...,-25,-15,-5,5,15,25,...\}$$
$$\{-8,-4,2,5,19,22\} \cap \{-3,-2,-1,0,1,2,3\} = \{2\}$$

Empty Set
In mathematics, 0 is a key number for many operations. It gives us a mathematical sense of nothing. This is also true for set theory, except the concept of 0 is called the empty set ($\varnothing$). The empty set is a set with no elements in it:

$$\text{Empty Set: } \varnothing = \{\}$$

Make sure you remember that the empty set is not $\{\varnothing\}$. This is a completely different thing, and is specifically used in certain situations. The empty set, $\varnothing$, is the set containing nothing, while $\{\varnothing\}$ is the set containing the set that contains nothing. Very different.

Examples of the empty set in action:

$$\{3,4,5,6\} \cup \varnothing = \{3,4,5,6\}$$
$$\{2,3,4\} \cap \{9,10,11\} = \varnothing$$

Classification of Real Numbers

There are many different ways to group members within the whole Real Numbers system and each grouping is called a subset. Some of the most important subsets are the ones we use to classify the Real Numbers. Each has a special symbol for a quick and understandable reference as these subsets are commonly used. There are 5 main subsets.

Natural (Counting) Numbers: $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$

Whole Numbers: $\mathbb{W} = \{0, 1, 2, 3, 4, 5, \ldots\}$

Integers: $\mathbb{Z} = \{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$

Rational Numbers: $\mathbb{Q} = \{\frac{a}{b} \mid \text{a,b} \in \mathbb{Z}\}$
These numbers can always be written as a decimal that terminates or keeps repeating the same pattern.

Irrational Numbers: $\mathbb{I} = \{\text{x} \mid \text{x is an infinite decimal that does not continuously repeat}\}$
These numbers can not be written as a fraction like the Rational Numbers can.

With these set being defined, the Real Numbers can be defined as:
Real Numbers: $\mathbb{R} = \mathbb{Q} \cup \mathbb{I}$
The Real Numbers are all the numbers in the Rational and Irrational sets.

Parity of Numbers
The numbers within the Integer subset can be classified further into having one of two parities, either even or odd. Even parity numbers are numbers that, when divided by 2, result in an integer. Odd parity numbers are the rest of the integers. Example:

$$10 \div 2 = 5, \qquad 10 \text{ is even}$$
$$9 \div 2 = 4.5, \qquad 9 \text{ is odd}$$

Even numbers: $\{..., -6, -4, -2, 0, 2, 4, 6, ...\} = \{2 \cdot x \mid x \in \mathbb{Z}\}$
Odd numbers: $\{..., -5, -3, -1, 1, 3, 5, ...\} = \{2 \cdot x + 1 \mid x \in \mathbb{Z}\}$

Least Common Multiple and Greatest Common Factor

Least Common Multiple (LCM)
The multiples of a number is that number multiplied by the Natural Numbers, or 1, 2, 3, 4, 5, . . . So, for example, the multiples of 3 and 6 are:

$$3 \cdot 1, \;\; 3 \cdot 2, \;\; 3 \cdot 3, \;\; 3 \cdot 4, \;\; 3 \cdot 5, ... = 3, 6, 9, 12, 15, ...$$
$$6 \cdot 1, \;\; 6 \cdot 2, \;\; 6 \cdot 3, \;\; 6 \cdot 4, \;\; 6 \cdot 5, ... = 6, 12, 18, 24, 30, ...$$

Now, the least common multiple (LCM) is a number that is the smallest multiple in common of two or more numbers. In the case of 3 and 6 above, 6 is the LCM. You can easily find the LCM by listing out a fair number of multiples of each number, then beginning from the left side, and, starting with the smallest multiple, check if that particular multiple is in all of the lists. If the first multiple is not common in each list, then look at the next multiple. Keep doing this until you find the first multiple that is common in each list. This is the least common multiple.

Greatest Common Factor (GCF)
The factors of a number is all of the Whole Numbers that, when multiplied by another Whole Number, give the original number back. Normally, you would not be looking for factors of negative numbers (as you can just take the negative out as a factor of -1), and you can't find factors of numbers besides Integers. For instance, the factors of 56 are: 1, 2, 4, 7, 8, 14, 28, and 56. These can be checked pretty easily, as:

$$1 \cdot 56 = 56$$
$$2 \cdot 28 = 56$$
$$4 \cdot 14 = 56$$
$$7 \cdot 8 = 56$$

Factors will always be Whole Numbers, and you will always multiply a factor by another factor to get the original number, so that means you will always have an even number of factors (unless looking at 1 or a square–these result in an odd number of factors where the center factor is the square root).

So, the GCF is a number that is the biggest factor in common of two or more numbers. The first step to finding the GCF is to find all of the factors of each of the numbers, then begin at the right (instead of the left in LCM) and start with the largest number. Check each list to see if it is a common factor, if not, then move on to the next highest factor in all of the lists. If there is no number greater than 1 that is common in each of the lists, then 1 is the GCF.
Example:

$$56 : 1, 2, 4, 7, 8, 14, 28, 56$$
$$80 : 1, 2, 4, 5, 8, 10, 16, 20, 40, 80$$
$$8 \text{ is the GCF}$$

Fraction Arithmetic

Simplifying (Reducing) Fractions:
1st way:
Prime factorize numerator and denominator. For each one to one occurrence of a number in the top and bottom, those two numbers cancel. Meaning, they can be divided out and you would still have an equivalent fraction.
Example 1:

$$\frac{100}{120} = \frac{2\cdot 2\cdot 5\cdot 5}{2\cdot 2\cdot 2\cdot 3\cdot 5} = \frac{2}{2}\cdot\frac{2}{2}\cdot\frac{5}{5}\cdot\frac{5}{2\cdot 3} = 1\cdot 1\cdot 1\cdot\frac{5}{6} = \frac{5}{6}.$$

As you could see, after dividing out all common prime factors you must multiply the remaining numbers in the numerator and the denominator. So, the fraction $\frac{5}{6}$ is $\frac{100}{120}$ most simplified form.
Example 2:

$$\frac{300}{30} = \frac{2\cdot 2\cdot 3\cdot 5\cdot 5}{2\cdot 3\cdot 5} = \frac{2}{2}\cdot\frac{3}{3}\cdot\frac{5}{5}\cdot\frac{2\cdot 5}{1} = 1\cdot 1\cdot 1\cdot\frac{10}{1} = 10$$

2nd way:
Find a common factor in the top and bottom, one that you can see easily. Then, divide both the numerator and the denominator by this common factor. Keep doing this until there is no common factor between the top and the bottom.
Example 1:

$$\frac{100}{120} = \frac{100\div 10}{120\div 10} = \frac{10}{12} = \frac{10\div 2}{12\div 2} = \frac{5}{6}$$

Since both the numerator and denominator numbers had a zero on the end, there must be a factor of 10 in each, dividing this out resulting in a fraction with a factor of 2 in the top and bottom. Dividing the 2 out then gives us a fraction with no common factor in the top and bottom, so this must be the most simplified form of the fraction.
Example 2:

$$\frac{300}{30} = \frac{300\div 30}{30\div 30} = \frac{10}{1} = 10$$

Conclusion:
Between the two methods, the first is the more rigorous way. It will always give you the most simplified fraction at the end. The only issue is that you must prime factorize the numbers, which can be a problem when working with large numbers. The 2nd method works well for large numbers, but you have to keep an eye out for any common factors. If there is one common factor, no matter how small, then it is not in simplest form.

What seems to work the best is to do a little of both. For large numbers or very easy to see factors (such as a 0 on the end of both numbers) find the common factor and divide it out. If you get stuck with that, then prime factorize the numbers and divide out common primes.

Multiplying Fractions:
To multiply fractions, all you need to do is multiply straight across the numerators and denominators, and then reduce the resulting fraction:

$$\frac{5}{6}\cdot\frac{4}{3} = \frac{5\cdot 4}{6\cdot 3} = \frac{20}{18} = \frac{20\div 2}{18\div 2} = \frac{10}{9}$$

$$\frac{2}{8}\cdot\frac{8}{4} = \frac{2\cdot 8}{8\cdot 4} = \frac{16}{32} = \frac{16\div 16}{32\div 16} = \frac{1}{2}$$

It can be useful to simplify the fractions before doing the multiplication. To do this, you can first simplify each fraction individually, but you can actually simplify them before fully multiplying the numerator and denominator. Look for a common factor in the numerator of the first fraction and the denominator of the second fraction, then divide it out of them. Keep doing this until there are no more common factors. Then, do the same with the denominator of the first fraction and the numerator of the second.

$$\frac{5}{6} \cdot \frac{4}{3} = \frac{5}{6 \div 2} \cdot \frac{4 \div 2}{3} = \frac{5}{3} \cdot \frac{2}{3} = \frac{5 \cdot 2}{3 \cdot 3} = \frac{10}{9}$$

$$\frac{2}{8} \cdot \frac{8}{4} = \frac{2 \div 2}{8} \cdot \frac{8}{4 \div 2} = \frac{1}{8} \cdot \frac{8}{2} = \frac{1}{8 \div 8} \cdot \frac{8 \div 8}{2} = \frac{1}{1} \cdot \frac{1}{2} = \frac{1}{2}$$

To do this the easiest way, try simplifying the individual fractions first, then simplify before multiplying, and then finally multiply it out. There should not be any more simplification needed after all of that.

Dividing Fractions:
To divide fractions, you need to take the first fraction and multiply it by the reciprocal (or flipped) of the second fraction.

$$\frac{4}{11} \div \frac{5}{11} = \frac{4}{11} \cdot \frac{11}{5} = \frac{4}{11 \div 11} \cdot \frac{11 \div 11}{5} = \frac{4}{1} \cdot \frac{1}{5} = \frac{4}{5}$$

Compound fractions (below) are common in higher mathematics, but you treat them the same as the problem just shown, multiply by the reciprocal of the bottom fraction.

$$\frac{\frac{9}{10}}{\frac{3}{16}} = \frac{9}{10} \cdot \frac{16}{3} = \frac{9 \div 3}{10} \cdot \frac{16}{3 \div 3} = \frac{3}{10 \div 2} \cdot \frac{16 \div 2}{1} = \frac{3}{5} \cdot \frac{8}{1} = \frac{24}{5}$$

Adding Fractions:
<u>Like Denominator:</u> You must have the same denominators in both fractions to add them; there is no other way to do it (unless you convert to decimals first). If you do in fact have like denominators, then all you need to do is add the tops and keep the denominator on bottom:

$$\frac{5}{3} + \frac{4}{3} = \frac{5+4}{3} = \frac{9}{3} = 3$$

Always make sure to simplify the resulting answer.

<u>Unlike Denominator:</u> If you want to add two fractions with different denominators, then you must make it the same denominator. The easiest ways to go about this are either multiplying by the other denominator or multiplying to make the Least Common Multiple (LCM).

Multiplying by other denominator:

$$\frac{7}{6} + \frac{1}{4} = \frac{4}{4} \cdot \frac{7}{6} + \frac{1}{4} \cdot \frac{6}{6} = \frac{28}{24} + \frac{6}{24} = \frac{34}{24} = \frac{17}{12}$$

Multiply by LCM: Find LCM of both of the denominators, then multiply by a factor that makes each denominator that number:

$$\frac{7}{6} + \frac{1}{4} = \frac{2}{2} \cdot \frac{7}{6} + \frac{1}{4} \cdot \frac{3}{3} = \frac{14}{12} + \frac{3}{12} = \frac{17}{12}$$

Doing the LCM method gets to the most simplified form the fastest, but requires more work up front. Multiplying by the other denominator is the simplest to see, but can require a lot more work later.

Subtracting Fractions:
Subtracting fractions is the exact same process as adding fractions, except you must subtract the numerators instead of adding them.

Converting between Fractions, Decimals, and Percents

Converting from Fractions to Decimals
To convert from fractions to decimals, use long division to divide the numerator by the denominator.

Converting from Decimals to Fractions
To convert from decimals to fractions, you must notice a couple things first. If the decimal never repeats a pattern infinitely, then you cannot convert the decimal to a fraction. If, at any point, the decimal starts to repeat a pattern infinitely or comes to a stop (or terminates), then you can change the decimal into a fraction. Follow the instructions for each case of decimal:

Terminating Decimals
To change a terminating decimal into a fraction, move the decimal all the way to the very right end of the number, counting as you go along. Then, you must divide the new number by 10 to the power of the number of digits you passed. Finally, simplify the resulting fraction.
Example:

$$\begin{aligned} &84.6785 \rightarrow 846785. \quad \text{The decimal passed 4 digits, so,} \\ &84.6785 = \frac{846785}{10^4} = \frac{846785}{10000} = \frac{169537}{2000}. \end{aligned}$$

Infinitely Repeating Decimals
To change an infinitely repeating decimal into a fraction, you need to do a little trick to it. First, make x equal to your decimal. Then, find out for how many digits the pattern repeats. Create a new equation by multiplying the x by 10 to the power of the number of digits repeated and making it equal to the decimal multiplied by the same power of 10. If you subtract this new equation by the equation x equal to the decimal, the repeating pattern should cancel out, and you can solve easily for x. Finally, if there still is a terminating decimal in the numerator, multiply and divide by the same factor of 10 such that the decimal cancels, and then simplify.
Example 1:

$$\begin{aligned} x &= 0.33333333... = 0.\overline{3} \quad \text{only 1 digit is repeated, so,} \\ 10^1 \cdot x &= 10^1 \cdot 0.\overline{3} \quad \text{which is,} \\ 10x &= 3.\overline{3}. \end{aligned}$$

Doing the subtraction gives:

$$\begin{array}{rl} 10x & = 3.\overline{3} \\ - \quad x & = 0.\overline{3} \\ \hline 9x & = 3 \end{array}$$

Solving for x gives $x = \frac{3}{9} = \frac{1}{3}$.

Example 2:

$$\begin{aligned} x &= 27.12758585858... = 27.127\overline{58} \quad \text{2 digits are repeated, so,} \\ 10^2 \cdot x &= 10^2 \cdot 27.127\overline{58} \quad \text{which is,} \\ 100x &= 2712.7\overline{58}. \end{aligned}$$

Doing the subtraction gives:

$$
\begin{array}{rl}
100x & = 2712.758\overline{58} \\
- \quad x & = \quad 27.127\overline{58} \\
\hline
99x & = 2685.631
\end{array}
$$

Solving for x gives $x = \frac{2685.631}{99}$. However, I haven't yet converted the number. I need to remove the decimal out of the numerator to fully convert the decimal. The last step is to multiply top and bottom (to keep the numbers equal) by 10 to the power of the number of digits to the right of the decimal. In this case, I need to multiply by $10^3 = 1000$. Doing so,

$$\frac{2685.631}{99} \cdot \frac{1000}{1000} = \frac{2685631}{99000}. \quad \text{This is the simplest form.}$$

Converting from Decimals to Percents
To convert from decimals to percents, multiply the decimal by 100, and add a % on the end.

Converting from Percents to Decimals
To convert from percents to decimals, just divide by 100 and take off the %.

Converting from Percents to Fractions
To convert from percents to fractions, convert the percent to a decimal, then convert the decimal to a fraction.

Converting from Fractions to Percents
To convert from fractions to percents, first convert the fraction to a decimal, then convert the decimal to a percent.

Basic Units of Measure

We measure things to describe the natural world. So, what are things we need to know? It can be said that there are only 7 things that are fundamental in order to describe all the different parts of nature, and those seven are length (or distance), mass, time, temperature, electric current, amount of light, and an amount of a substance. There is a problem though, the United States utilizes a different system of units than most of the rest of the world. The main system that is used is the International System of Units (SI for short). Therefore, we need to know how to convert between the different systems. But first, here is the 7 fundamental measurements and their units:

	American	SI
Length	Feet (ft)	Meters (m)
Mass	Pounds (lbs)	Kilograms (kg)
Time	Seconds (s)	same
Temperature	Fahrenheit (°F)	Kelvin (K)
Electric Current	Ampere (A)	same
Amount of Light	Candela (cd)	same
Amount of a Substance	Mole (mol)	same

There is some caution that must be noted with this list:

- Most of the time, other countries use Celsius (°C) as their unit of measure for temperature, but Kelvin is the unit that scientists use.
- Electric current is measured in amperes (amps for short), but an Amp is based on the amount of charge (an electron charge, e) in an area over time.
- The SI unit of mass is kilogram, which is actually a measure of amount of matter, while the pound is the measure of Earth's gravity affecting an amount of matter, or weight. What this means is that your weight will change from planet to planet, since each has a different force of gravity, but your mass will not change.

Even though there are only 7 fundamental units, there are many, many things in the natural world that need to be described and measured. We measure these things by multiplying and dividing different combinations of the fundamental units. The other measurements we will cover here is area, volume, speed, as well as more units of length and mass.

Length:
There are different units of measure for length because there are different distances ranging from smaller than microscopic to larger than astronomical. Here are the most commonly used units of the American and SI system:

American	SI
12 inches (in) = 1 ft	1000 millimeters (mm) = 1 m
1 yard (yd) = 3 ft	100 centimeters (cm) = 1 m
1 mile (mi) = 5280 ft	1 kilometer (km) = 1000 m

Mass:
The most common mass measurements are lbs for the American and kg for the SI, but there are two smaller units that can be easier to use sometimes, and that is the ounce (oz) and gram (g).

$$16 \text{ oz} = 1 \text{ lb} \qquad 1000 \text{ g} = 1 \text{ kg}$$

Area:

While length is the measure of a one dimensional space, area is the measure of a two dimensional space. A basic example of area is the area of a rectangle, which is equal to the measure of its one dimensional length multiplied by the measure of its one dimensional width. Not only do the measurements get multiplied, but the units get multiplied as well. It is important to use the same unit of measure when doing math. So, if our rectangle was 6 ft long and 5 ft wide, then its area is 30 ft^2 read as 30 feet squared.

There is generally no change in unit (besides the square on the unit) for area, but it is important to know that area unit conversions result in bigger changes than regular length measures, which you will see below.

American	SI
144 in^2 = 1 ft^2	1,000,000 mm^2 = 1 m^2
1 yd^2 = 9 ft^2	10,000 cm^2 = 1 m^2
1 acre = 43,560 ft^2	1 km^2 = 1,000,000 m^2

The unit mi^2 is not used very much in the American units, but the acre is used frequently for plots of land. But, for your information, 1 mi^2 = 27,878,400 ft^2.

Volume

Volume is an interesting measurement, as there are a lot of different units. The reason for this is that the physical world we live in is three dimensional, so that is the most frequent type of object we use. For instance, in cooking there are about 7 commonly used units! But, we will not be covering all of this. We are only considering the most commonly used volume measurements in math and science.

American	SI
1728 in^3 = 1 ft^3	1,000,000,000 mm^3 = 1 m^3
1 yd^3 = 27 ft^3	1,000,000 cm^3 = 1 m^3
1 acre foot = 43,560 ft^3	1 km^3 = 1,000,000,000 m^3
1 (liquid) gal = 231 in^3	1 cm^3 = 1 mL
1 (dry) gal = 268.8 in^3	1 liter (L) = 1000 mL

Though the acre foot is not used frequently by the general public, it would be the biggest commonly used measure for the American system. A mile cubed is just too big of a number. Also, the SI system uses liters and milliliters the most, but knowing the meter conversions will help when trying to find how much liquid a solid shape can hold.

Speed:

Speed is the measurement of a distance changing over time. Speed tells us how fast we were moving by dividing the distance traveled by the time traveled. Time is measured in both seconds (s) and hours (hr).

$$1\ \frac{\text{mi}}{\text{hr}} = 1.467\ \frac{\text{ft}}{\text{s}} \qquad 3.6\ \frac{\text{km}}{\text{hr}} = 1\ \frac{\text{m}}{\text{s}}$$

Conversions:

Now that you have been introduced to the most common units used in the American and SI systems, we need to show you how to convert between them. This is done with a conversion ratio, which there is generally one ratio for each type of measurement (length, area, volume, etc.). Here are the ratios:

Type of Unit	Conversion Ratio
Length	3.281 ft = 1 m
Mass	2.205 lbs = 1 kg
Area	use length measurement
Volume	use length measurement
Speed	use length measurement and 3600 s = 1 hr

A simple example of a unit conversion is to change inches to meters. The conversion we have above is for feet to meters, so the only way we can use this ratio is if we first change the inches to feet (ratio given above). So, what we do is place every ratio we need as fractions that are being multiplied. We need to place every ratio in a way that cancels the units we do not want, the way this happens depends on the original measurement. So, let's convert 50 inches to meters:

$$50 \text{ in} = \frac{50 \text{ in}}{1} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ m}}{3.281 \text{ ft}} = \frac{50 \cdot 1 \cdot 1 \text{ in} \cdot \text{ft} \cdot \text{m}}{1 \cdot 12 \cdot 3.281 \text{ in} \cdot \text{ft}} = \frac{50 \cancel{\text{in}} \cdot \cancel{\text{ft}} \cdot \text{m}}{39.372 \cancel{\text{in}} \cdot \cancel{\text{ft}}} = \frac{50}{39.372} \text{ m} = 1.27 \text{ m}$$

As you can see, when you multiplied the fractions together, you get to cancel common units on the top and bottom, and in the end, you should always have the unit you desire, in the case above, meters. Now, we will go over another example, this time from one speed to another: from 10 meters per second (meters divided by seconds) to feet per hour.

$$\begin{aligned} 10 \frac{\text{m}}{\text{s}} &= \frac{10 \text{ m}}{1 \text{ s}} \cdot \frac{3.281 \text{ ft}}{1 \text{ m}} \cdot \frac{60 \text{ s}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \\ &= \frac{10 \cdot 3.281 \cdot 60 \cdot 60 \text{ m} \cdot \text{ft} \cdot \text{s} \cdot \text{min}}{1 \cdot 1 \cdot 1 \cdot 1 \text{ s} \cdot \text{m} \cdot \text{min} \cdot \text{hr}} = \frac{118116 \cancel{\text{m}} \cdot \text{ft} \cdot \cancel{\text{s}} \cdot \cancel{\text{min}}}{1 \cancel{\text{s}} \cdot \cancel{\text{m}} \cdot \cancel{\text{min}} \cdot \text{hr}} = 118116 \frac{\text{ft}}{\text{hr}} \end{aligned}$$

What is important is that you always make sure you do whatever multiplication you need to get the desired unit, as shown above we needed to convert to minutes and then to hours. Still, we always need to multiply the ratios in a way that will cancel the units, one is above the fraction bar and the same is below.

Properties of Exponents

Exponents:
Exponents have a base and a power. If you see this: a^b, then a is the base and b is the power. The expression a^b means take the number a and multiply it by itself b times. Example:

$$2^3 = 2 \cdot 2 \cdot 2 = 8$$
$$4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1024$$

Properties of Exponents:

$$a^m \cdot a^p = a^{m+p}$$
$$(a^n)^o = a^{n \cdot o}$$
$$\frac{b^q}{b^r} = b^{q-r}$$
$$c^{-d} = \frac{1}{c^d}$$
$$x^0 = 1$$
$$y^1 = y$$
$$(i \cdot j)^k = i^k \cdot j^k$$
$$(\frac{i}{j})^k = \frac{i^k}{j^k}$$
$$m^{\frac{n}{o}} = \sqrt[o]{m^n} = (\sqrt[o]{m})^n$$

Squared and Cubed Tables

Number	Squared	Cubed
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	1331
12	144	1728
13	169	2197
14	196	2744
15	225	3375
16	256	4096
17	289	4913
18	324	5832
19	361	6859
20	400	8000

Properties of Logarithms

Logarithms:
Logarithms have a base and an argument. If you see this: $\log_a(b)$, then a is the base of the logarithm and b is the argument. The expression $\log_a(b)$ is asking what exponential power, for example c, can you put on a such that $a^c = b$. Logarithms are the inverse functions of exponential functions with the variable in the power (unlike radicals where the variable is in the base). Examples:

$$\log_2(8) = 3$$
$$\log_4(1024) = 5$$

Properties of Logarithms:
First and foremost, you cannot take the logarithm of a negative number, no matter the base.

$$log_a(x \cdot y) = log_a(x) + log_a(y)$$
$$log_b(\frac{f}{g}) = log_b(f) - log_b(g)$$
$$log_c(z^w) = w \cdot log_c(z)$$
$$log_p(o) = \frac{log_m(o)}{log_m(p)}$$
$$log_n(n) = 1$$
$$log_n(1) = 0$$

Properties of Radicals

Radicals:
Radicals are mathematical statements that involve roots. The most common root used is called the square root, and it is written short hand as $\sqrt{a}$, but the long version is written $\sqrt[2]{a}$. For the square root, 2 is the degree. There are many types of roots than just the square root, and they are either written as $\sqrt[x]{y}$ or $y^{\frac{1}{x}}$, and it is said that the x is the degree of the root. What radicals mean is if $\sqrt[a]{b} = c$, then $c^a = b$. Radicals are the inverse functions of exponential functions where the variable is in the base (unlike logarithms where the variable is in the power).
Examples:

$$\sqrt{4} = \sqrt[2]{4} = \pm 2$$
$$\sqrt[3]{64} = 4$$

Properties of Radicals:
-If you take an even degree root of a positive number, you will get a Real Number.
-If you take an even degree root of a negative number, you will get a Complex Number, not a Real Number.
-If you take an odd degree root, no matter the sign on the number, you will get a Real Number.

$$(\sqrt[c]{b})^a = \sqrt[c]{b^a} = b^{\frac{a}{c}}$$
$$\sqrt[a]{b} \cdot \sqrt[a]{c} = \sqrt[a]{b \cdot c}$$
$$d\sqrt[a]{b} \cdot e\sqrt[a]{c} = d \cdot e \cdot \sqrt[a]{b \cdot c}$$
$$g\sqrt[x]{y} + h\sqrt[x]{y} = (g + h) \cdot \sqrt[x]{y}$$
$$\sqrt[m]{\sqrt[n]{z}} = \sqrt[(m \cdot n)]{z}$$
$$\sqrt[a]{1} = 1$$
$$\sqrt[b]{0} = 0$$

It is important to note that there are some differences between the answer to even and odd square roots. The first way to see this is to take the square and cube of both 2 and -2:

Square:
$$2^2 = 2 \cdot 2 = 4$$
$$(-2)^2 = -2 \cdot -2 = 4$$
Cube:
$$2^3 = 2 \cdot 2 \cdot 2 = 8$$
$$(-2)^3 = -2 \cdot -2 \cdot -2 = -8$$

As you can see, when you square (or take any even power for that matter) a number and the negative of that number, you get the exact same answer. However, when you cube (or take any odd power) a number and the negative of that number, you get two distinct answers. So, when taking a square root (or even root), you are asking, "What number must I square to get this number?" Well, there are two numbers that can do the job: the number and its negative. We write this with a $\pm$ in front of the number, take the positive number as an answer and take the negative number as an answer. If you take an odd root, you only get one number as a result, and it will always have the same sign as the number that you are rooting.

Basics of Algebra

The first thing we need to define in order to understand Algebra is a variable. A variable is generally used to denote an unknown value, and you normally use a letter to describe that unknown. Variables can actually be one of two things, a constant or a changing value.

Constant vs. Changing Value:

Numbers can be called constants, or in other words they do not change their value. The number 3 has the value 3. It can never change its value, and thus is constantly worth 3 units. Now, let's say we had a variable x. What is its value? We don't know the exact value, but if someone tells us that x is a constant, then we know that it can only have one value. However, if someone told us that x is not a constant variable, then x can take on many, many different values, and we don't know what it is at present. It is important, though, that with any variable you can treat it as a number, meaning you can add variables, subtract variables, multiply and divide variables, but you just cannot find the actual answer to that operation.

Variables in Expressions and Equations:

When a variable is sitting right next to another variable or number, it means that the variable is being multiplied with the variable or number ($10 \cdot z = 10z, xy = x \cdot y$). The number 10 is called the coefficient of the variable, and every variable has a coefficient. If there is no number next to a variable, then it is understood that its coefficient is 1 ($m = 1 \cdot m$). Here are some examples of this fact and variables in action:

$$3x, \quad 4xy, \quad a^2 + 6b, \quad \frac{56x + 17y}{x^8 - z}, \quad a^m = b^n + c^{o-p}, \quad \text{etc.}$$

Combining Like Terms:

In these examples we cannot reduce the expressions and equations into any simpler statements, but there are times when you can. Variables can be added and subtracted when they are "like terms". This means that every term, or group of numbers and variables that are being multiplied and divided, that have the **exact** same variables can have their coefficients added or subtracted. This process is called combining like terms.

$$\begin{array}{rl}
3x - 7x + 2y & \text{Combine like terms.} \\
\underline{3x} + \underline{-7x} + 2y & \text{Here are two.} \\
3 + -7 = -4 & \text{Adding coefficients gives -4, producing,} \\
-4x + 2y & \text{No other like terms.}
\end{array}$$

Note that we changed the $3x - 7x$ into $3x + -7x$. This is very common practice, and it keeps the negative with the coefficient even if you move the number around by the commutative property.

$$\begin{array}{rl}
6a - 6b + 12ab - 14b + a & \text{Combine like terms.} \\
\underline{6a} - 6b + 12ab - 14b + \underline{a} & \text{Here are two.} \\
6 + 1 = 7 & \text{Adding the coefficients gives 7, producing,} \\
7a - 6b + 12ab - 14b & \text{There are still more like terms.} \\
7a + \underline{-6b} + 12ab + \underline{-14b} & \text{Here they are.} \\
-6 + -14 = -20 & \text{Adding the coefficients gives -20, producing,} \\
7a - 20b + 12ab & \text{No other like terms.}
\end{array}$$

As you can see, you need to combine all like terms possible to find the simplest form of the expression or equation. Plus, combining all terms that you can will often make the expression or equation much more organized and easier to handle. It is just good practice to always combine terms as much as you can.

Substitution of Value:

If you were given this expression $3x + \frac{15}{x}$, could you find the exact value of it? No. As stated above, you do not know the value of the variable x, thus you cannot do the operations. But, what if you were then given that $x = 3$? Now, we know what the value of x is, and since x is the only variable in the expression, that means we can solve it! How do we do this? For every x you see in the expression, substitute a 3 there instead. Commonly this is said as "plugging in" a 3 into the expression.

$$
\begin{array}{rl}
3x + \dfrac{15}{x} & \text{We know that } x = 3\text{, so we plug it in.} \\
3(3) + \dfrac{15}{(3)} & \text{We put the 3 in parentheses to show we plugged it in.} \\
3 \cdot 3 + \dfrac{15}{3} & \text{Now we just solve the expression.} \\
9 + 5 & \text{Adding gives our answer of 14.}
\end{array}
$$

By plugging in all variables, it turns the variable expression into one of only numbers, which can be simplified. You use the exact same process when confronted with even more complex problems.

Quick tip, if you have an expression or equation that has like terms, it is often easier to combine like terms first and then substitute the numbers in rather than doing the substitution first.

Basic Probability and Statistics

Probability is the study of randomness. Statistics is the science of gathering data about large groups of things and displaying results, generally the data is some kind of number. These two things are very closely tied since randomness is used in the studying of large groups.

The most basic thing you would learn in any Probability and Statistics course is how do you find the probability of something simple, like getting a heads on a coin flip or rolling a 4 on a six-sided die. The process to find the probability of getting a heads or 4 is the same for both since we are only rolling it once and we assume that nothing is affecting either situations. Take the number of ways you can flip or roll the answer and divide it by the total number of possible answers.

Probabilities will always (unless an error was made) be between 0 and 1. To get a 0 as a probability means there is no possible way that situation could ever happen. To get a 1 as a probability means it will always happen. Every possibility in between shows the likelihood of that situation happening.

Heads on a Coin:

On a coin there are two sides, heads and tails, therefore when you flip a coin you only get one of two things, either heads or tails. This means there is a total of two possible outcomes for any coin toss. Furthermore, there is only one way to get a heads on a coin: flipping it and it lands on heads. So, what is the probability of getting a heads on a coin flip?

$$\frac{\text{Possible ways of getting heads}}{\text{Total number of possible flips}} = \frac{1}{2}$$

This means that the probability of getting a heads on a coin flip is one half.

4 on a Die:

A die has 6 sides, numbered 1 through 6. Only one of those sides has a 4, so there is only one way of rolling a 4: landing with that side up. When you roll a die, there are six total possible rolls. So, what is the probability of getting a 4 on a die roll?

$$\frac{\text{Possible ways of getting a 4}}{\text{Total number of possible rolls}} = \frac{1}{6}$$

So, you will get a 4 on a die roll one sixth of the time.

Complete meaning of Probability:

With the examples above, what exactly does that mean if we have a probability of one half or one sixth? Does it mean that if we flipped a coin two times we would get a heads one time and a tails as another? Does it mean that if we roll a die six times, we will get a 4 only once? Not necessarily, no. Those instances might happen, but more likely they will not. However, what would happen if we flipped a coin and rolled a die 6,000 times? As long as each flip and roll is independent of each other, meaning each flip and roll has no effect on the next flip and roll, then all we need to do is multiply the total number of times we do the action by the probability that we found in the first place.

$$\text{Heads on a Coin: } 6,000 \cdot \frac{1}{2} = 3,000 \text{ heads}$$
$$\text{4 on a Die: } 6,000 \cdot \frac{1}{6} = 1,000 \text{ 4s}$$

If you actually flip a coin 6,000 times, you will find that you get very close to 3,000 heads as an outcome, same with 1,000 4s on a die. This is why probability is used for statistics, as a normal thing to study with statistics is people, which there are around 8 billion of them.

PART 3:

Inventory with a list of topics and themes in Mathematics

These topics come from the following branches of Mathematics: Arithmetic and Pre-Algebra, Algebra 1 and 2, Geometry and Trigonometry, and Pre-Calculus.

Table of Contents

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Fundamentals of Arithmetic and Number Theory

Counting, Calendars and Clocks
Numbers and Words—Writing words to Numbers and Numbers to words
Place Value
Estimation and Estimating Answers
Expanded Notation
Rounding Numbers
Addition of Whole Numbers
Subtraction of Whole Numbers
The Multiplication Table (1 to 15)
Multiplication of Whole Numbers
Division of Whole Numbers
Key Terms: Factors, Multiples, Divisor, Dividend, Quotient, Sum, Difference, Minuend, Addend, Product
Multiplying by Multiples and powers of 10
Dividing by multiples and powers of 10
Divisibility Rules (dividing by 2, 3,4,5,6,7,8,10,11,12) and Multiplication Patterns
Long Division and Division with remainders
Introduction to the Number Line
Comparison of Whole Numbers on the Number line
Classification of Real Numbers (Natural, Whole, Integers, Rational and Irrational)
Addition of Integers
Subtraction of Integers
Multiplication of Integers
Division of Integers
Comparison of Integers on the Number line
Ordering real numbers on a Number line (Whole Numbers and Integers)
Order of Operations (PEMDAS or BODMAS)
Using Significant Figures in Science and Mathematics
Introduction to Prime Numbers
Finding Factors of a number
Prime Factorization: Factor Trees and Factor Ladders
Lowest Common Multiple
Greatest Common Factor
Introduction to Sets and Set Notation
Venn Diagrams with 2 Circles
Venn Diagrams with 3 Circles
Introduction to word problems
Roman Numerals
Optional: Greek Alphabets and what they are used for in Mathematics and Science
Properties of Real Numbers

Fractions, Complex Fractions, Percent, Decimals

Identifying the Numerator and the Denominator, Proper fractions, Improper fractions, Mixed Numbers
Converting from Improper fractions to Mixed Numbers
Converting Mixed Fractions to Improper Fractions
Using Pie Charts
Basics of Fractions: Halves, Thirds, Fourths, Fifths… Tenths

Reducing Fractions and leaving your answer in the simplest form
Fraction Multiplication
Fraction Division
Introduction to Complex Fractions- Fraction Division II
Fraction Addition: Like Denominators
Fraction Subtraction: Like Denominators
Fraction Addition: Unlike Denominators
Fraction Subtraction: Unlike Denominators
Arithmetic Operations with several fractions
Converting from Fractions to Decimals
Converting from Decimals to Fractions
Converting from Fractions to Percents
Converting from Percents to Fractions
Converting from Percents to Decimals
Converting from Decimals to Percents
Addition of Mixed Fractions
Subtraction of Mixed Fractions
Multiplication of Mixed Fractions
Division of Mixed Fractions
The Number line and Fractions
Place Value in Decimals
Addition of decimals
Subtraction of decimals
Multiplication of decimals
Division of decimals
Comparing Decimals on a number line
Arithmetic Operations involving Decimals, Integers and Fractions (Integrated)
Working with fractions, decimals and number lines- Comparing decimals and fractions
Rounding decimal numbers
Working with Repeating Decimals
Introduction to Ratios and Proportions
Working with ratios and Punnett Squares in Biology
Category 1: Finding the Percent of a Number- 10% of 50=____
Category 2: 10% of ___ is 12
Category 3: ___% of 15 is 9
Solving equations involving fractions
Solving equations involving decimals
Applications of fractions, decimals and percents in business and daily life
Applications: Commission, Mark ups and Discounts
Percent Increase and Percent Decrease
Profit and loss
Calculating Percent error
Units of Measure and Conversions
Finding the unit price
Converting from one unit to another unit, Example- dollars to pounds
Using a chart to find equivalence, Example: 1 mile=5280 feet
Metric Measurement facts and Using Multiple Conversion Factors
Converting Rates, Converting from m/s to Km/s
Converting from m/s to Km/h

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Temperature Conversion

Rules of Exponents, Radicals, Roots

Base, Powers and Exponents
Rules of Exponents
Solving equations involving powers
Rational Exponents
Properties of Radicals
Properties of nth roots: Product property and Quotient Property of radicals
Deriving rules of exponents from other rules
Introduction to Scientific Notation
Working with large numbers: millions, billions, trillions (Distance of the earth to the sun)
Multiplication involving Scientific Notation
Division involving Scientific Notation
Key terms: Radicands and Roots
Perfect squares, Square roots, Cube roots, and Higher order roots
Approximating square roots
Simplifying radicals using prime factorization
Addition and subtraction of radicals (same radicand and index), Similar to Like Terms
Multiplication of radicals
Division involving radicals (roots have to be greater than 0 if they are in the denominator)
Using distribution with radicals
Rationalizing the Denominator- Case 1- working with just the radical
Rationalizing the Denominator-Case 2, when the radical is multiplying a constant or variable that is not 1
Rationalizing the Denominator- Case 3, Using Radical Conjugates to Rationalize the Denominator
Rationalizing a Variable Denominator
Factoring out common radicals
Solving Radical Equations
Finding the Domain and Range with Radical Functions
Determining Extraneous Solutions
Graphing Radical functions
Graphing Square root functions
Graphing Cubic root functions
Imaginary Numbers
Arithmetic Operations involving imaginary numbers

Expressions, Equations, Variables, Constants, Like Terms, Word Problems, Solving Linear Equations and Solving Systems of Equations

Introduction to Variables, constants, like terms, and coefficients
Why do we use variables? Simplifying expressions involving Like terms
Evaluating Expressions: Substituting numbers in for variables
Chemistry Application of evaluating expressions: Finding the Molar Mass
Solving 1-step Equations
2-step Equations
Multi-step equations
Reflexive Property a=a Symmetric Property a=b and b=a
Transitive Property If a=b and b=c then a=c

Substitution Property If a=b, then a may be replaced by b
Properties of Equality (Addition, Subtraction, Multiplication and Division)
Square root property of Equality
Zero Product Property of Equality
Cross Multiplication with equations, Cross Products (0 cannot occur in the denominator)
Solving Abstract equations, literal equations, working with Variables on both sides
Solving systems of equations (2) using Substitution
Solving systems of equations (2) using Elimination
Solving systems of equations (2) using Graphing
Solving Systems of equations: 3 equations, 3 Variables
Solving nonlinear systems of equations with 2 variables
Linear Programming
Solving multistep word Problems

Inequalities and Introduction to Absolute Value

Introduction to inequalities
Inequalities to Number lines /Graphing on a number line
Translating between words and inequalities
Properties of Inequalities (Addition and Subtraction)
Multiplication Property of Inequality for $c > 0$
Multiplication Property of Inequality for $c < 0$
Division Property of Inequality for $c > 0$
Division Property of Inequality for $c < 0$
Comparison Property of Inequality
Solving inequality word problems
Solving inequalities - 1 step
Solving inequalities - 2 steps
Compound Inequalities- inequalities combined with the word "AND" or "OR"
Solving and Graphing Compound Multistep Inequalities
Writing and graphing conjunctions- Solving conjunctions
Writing and graphing disjunctions- Solving disjunctions
Writing a Compound Inequality from a Graph
Solving Inequalities that are Always True or Always False
Graphing Linear Inequalities in 2 variables
Solving linear systems of inequalities- Solution of a system of linear inequalities
Graphing Systems of Linear Inequalities
Writing a Linear Inequality Given the Graph- dashed lines or solid lines
Solving Systems of inequalities with parallel Boundary Lines
Application: Travel and luggage
Introduction to Absolute Value and Number Line Connection
Solving Absolute Value Equations
Solving Equations and Inequalities with Absolute Value
Conjunctions and Disjunctions involving Absolute Value Functions
Extraneous Solutions
Absolute value inequalities
Solving Multi-Step Absolute-Value Equations
Solving Multi-Step Absolute-Value Inequalities
Writing Absolute Value as a Compound Inequality
Graphing Absolute-Value functions

Transforming and Translating the Absolute value function

<u>Graphing, Transformations, Functions and Variations</u>

Applications: Maps, Grids, Aviation, Architecture, Business, Economics
History: Rene Descartes
Introduction to the Coordinate Plane, Plotting points, x and y coordinates
The Slope formula
Example relating slope to Distance, Speed and Time
Determining Slope from a table
Determining slope from a graph
The Slope of a horizontal line and The Slope of a vertical Line
Standard form of a Linear equation, $Ax + By = C$, Slope-intercept form, Point-slope formula
Writing the equation of a line starting with a point and the slope
Graphing Linear Functions- Method 1: Using Table of Values
Graphing Linear Functions- Method 2: Sketching a graph using intercepts and slope
Graphing Linear Functions given the Standard form
Graphing a linear function given point-slope form
Writing the equation of a line from a graph
Interpreting a graph
Introduction to Functions and Relations, Examples of functions and relations in everyday life
Domain, Range, Independent Variable and Dependent Variable, input and output
Using Function Notation to evaluate functions- substituting values for x
The Horizontal Line Test and The Vertical Line Test
Finding the sum and difference of Functions
Multiplying and Dividing Functions Numerically and Algebraically
Composition of a Function
Inverse of a Function
Parent Functions: Cubic functions, Continuous functions, Discontinuous functions, and discrete functions
Analyzing Discrete and Continuous Functions
Even and Odd functions
Solving systems of equations by graphing
Application: Graphs in Nursing and Medicine
Introduction to Variation
Identifying, Writing, and Graphing Direct Variation
Identifying, Writing and Graphing Inverse Variation
Changing a Variation to an Equation- Constant of Variation
Identifying direct variation from ordered pairs or table of Values
Writing and solving a Direct Variation Equation
Graphing direct variation
Identifying inverse variation from ordered pairs of table of values
Writing and solving an Inverse Variation Equation
Graphing Inverse variation
Joint Variation
The ideal Gas law
General Gas laws
Experiments describing variations- Charles' law and Boyle's law
Comparing Direct and Inverse Variation
Introduction to Limits

Applications: Wages, Stocks, Electric bill
Classification of graphs- Dependent and Independent
Introduction to Coordinate Geometry
Coordinate planes and Space
Coordinate Geometry: Distance Formula
Coordinate Geometry: Midpoint Formula
Coordinate Geometry: Verifying shapes (parallelograms, triangles) using coordinate Geometry
Coordinate Geometry: Finding Perimeters
Coordinate Geometry- Finding Areas
Parallel and Perpendicular Lines in Coordinate Geometry
Congruent segments
Perpendicular line through a point on the line
Introduction to transformations in functions and relations
Translations, Rotations, Reflections, and Dilations in a Coordinate Plane
Composite Transformations
Transformation Matrices
Understanding Piecewise functions
Graphing a Piecewise function
Step functions
Introduction to Analytic Geometry
Identifying conic sections and their applications
Equation of a circle, writing the Equation of a circle
Graphs of circles centered at the origin
Graphs of circles not centered at the origin
Making graphs and using equations of ellipses
Graphs of ellipses centered at the origin
Graphs of ellipses not centered at the origin
Writing an Ellipse Equation in standard form
Graphing hyperbolas
Graphs of Hyperbolas centered at the origin
Graphs of Hyperbolas, not centered at the origin
Deriving the equation of a hyperbola in Standard form
Parabolas
Recognize conics by their equations
Complete the square to graph conic sections
Hyperbolic Geometry
Polar Equations of Conic sections
Rotation of Axes
Introduction to Graphing with x, y and z coordinates

Polynomials

Understanding Polynomial functions, and the Standard form
Classification- monomials, binomial, trinomial, Degree of a monomial, leading coefficient
Adding and subtracting polynomials
(When subtracting polynomials, remember your parenthesis and sign change)
Multiplying polynomials, The FOIL method or the Box method
Monomials and the Distributive property
Factoring Polynomials
Factoring trinomials (when a is 1)

Factoring trinomials by using the G.C.F
Factoring trinomials (a is not 1)
Factoring Polynomials by grouping and Rearranging before grouping
Factoring with opposites
Factoring Polynomials of this type: $y^4 + y^2 + 1$-substituting a different variable
Factoring with two variables- $x^2 + 5xy + 6y^2$
Difference of 2 squares
Difference of 2 cubes
Addition of 2 cubes
Choosing a Factoring Method: Choosing the most efficient technique
Making graphs of a polynomial function
Determining End behavior of Polynomial Functions
Using Graphs to analyze Polynomial Functions
Local Maxima and minima and Transforming polynomial functions
Finding Polynomial roots
Finding higher-order polynomials
Solving Polynomial equations
Dividing a Polynomial by a Binomial
Dividing Polynomials- Long Division
Using synthetic division
Dividing a Polynomial with a Zero Coefficient- Placing a zero next to the relevant variables
Using the Remainder and Factor Theorems
Rational Root theorem
Multiplicity of roots
Descartes Rule of signs
Using the fundamental theorem of Algebra
Finding irrational Roots

Rational Equations, Expressions, Inequalities

Review of Rational Numbers
Excluded Values, remember you cannot have 0 in the denominator
Extraneous solutions
Simplifying Rational Expressions
Addition- like denominators
Addition- unlike denominators
Subtraction of rational expressions- like denominators
Subtraction of rational expressions- unlike denominators
Multiplication of rational expressions
Division of rational expressions
Solving Rational Equations
Rational inequalities
Using the Distributive property with rational expressions
Graphing Rational Functions and Slant Asymptotes
Finding the equation of a slant asymptote and Horizontal Asymptotes
Slant Asymptote with Quadratic and cubic terms
Identifying all Asymptotes and holes in a Rational Function
Partial Fractions
Solving a rational Proportion

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Quadratic functions, equations and inequalities

Quadratic functions
Quadratic term, linear team and constant term
Standard form of a Quadratic Equation, Vertex form of a Quadratic equation
Converting from the standard form to the vertex form
Identifying Characteristics of Quadratic functions
Characteristics of Parabolas
The Vertex of a Parabola
Solving Quadratic equations and the Zero Product Property
Converse of the Zero-Product Property
Graphing Quadratic Functions using a table of values
Graphing and Sketching Quadratic Functions using intercepts and a calculator
Determining the Maximum and Minimum of the quadratic function
Using the Quadratic Formula
Deriving the Quadratic Formula
Solving Quadratic Equations by Completing the Square
Writing Quadratic Equations from Roots
Finding the equation of a parabola
Zero of a quadratic function, x-intercepts, y-intercepts
Axis of symmetry, Axis of symmetry formula
Transformations on Quadratic Functions
Review: Solving Quadratic Equations by Factoring
Finding the Roots and Factoring Out the GCF
Double roots in Quadratics
Finding the discriminant
Interpreting and Analyzing the discriminant (Positive, negative and zero)
Solving a Cubic equation
Solving Quadratic Equations by Graphing
Solving Quadratic Equations by using Square Roots
Graphing and Solving Systems of Linear and Quadratic Equations
Solving quadratic inequalities
Extraneous solutions
Applications: Avalanches, path of a baseball, free fall in Physics
Deriving Quadratic Equations with Complex roots
Bonus: Solving a Quartic equation

Logarithms and the Exponential Function

Introduction and notation
Standard forms and Converting from Exponential to Logarithmic
Using the Property of Logarithms
Using Natural Log
Properties of Natural Log
Transcendental Functions
Logarithmic Functions
Exponential Functions
Identifying and Graphing Exponential functions, Table of values
Graphing Logarithmic functions
Reflections and Transformations of Exponential and Log graphs
Solving for the unknown using logarithms and Natural Log

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Evaluating Logarithmic Expressions
Finding Growth and Decay
Half Life
Solving exponential Equations and Inequalities
Solving Logarithmic Equations
Solving Logarithmic Inequalities
Applications in Compound Interest
Leonhard Euler and his role with "*e*"

<u>Matrices and Linear Algebra</u>

Application: Tables and calculators
Matrix Addition
Matrix Subtraction
Scalar Multiplication, Distributive Property
Commutative Property: It holds true in Matrix Addition but not in Matrix Multiplication
Matrix Multiplication- 2 x 2 and 2 x 2
Matrix Multiplication- 3 x 3 and 3 x 3
Matrix Multiplication - 3 x 2 and 2 x 3
Matrix Multiplication – 2 x 3 and 3 x 2
Matrix Multiplication – 1 x 3 and 3 x 1
Matrix Multiplication – 3 x 1 and 1 x 3
Finding the Determinant of a 2 x 2 Matrix
Finding the Determinant of a 3 x 3 Matrix
Use expansion by cofactors
Finding the area of triangles
Inverse of a 2 x 2 Matrix
Inverse of a 3 x 3 Matrix
Using the inverse to solve for variables
Creating Matrices from data given
Who was Cramer? Cramer's Rule: Using matrices to solve systems of equations
Matrix Algebra
Using the Determinant to find the area of a Triangle
Application: Converting data into matrix format
Matrix Transformation
Reduced Row Echelon Form
Gaussian Elimination
Using Calculators with matrices

<u>Sequences, Series and Integrated Themes and Topics</u>

Pascal's Triangle and Binomial Theorem, Finding the xth term of a Binomial
Binomial Probability, with and without a calculator
Golden Ratio
Cryptography
Complex Numbers and Simplifying complex expressions
De Moivre's theorem
Finding Arithmetic Sequences
Finding Arithmetic Series
Finding Geometric Sequences
Finding Geometric Series

Partial Sums
Infinite Series
Modular Arithmetic
Binary Numbers: Conversion of bases
Longitude and Latitude Calculations
Riemannian Geometry or Spherical Geometry
Absolute or Neutral Geometry
Topology
Project: Derive a Formula
Project: Choose a shape and explore possible theorems

<u>Statistics, Probability and Finding Measures of Central Tendency</u>

Organizing and Classifying Data, Representing data with a table, Tally marks
Representing data with a graph, plotting graphs
Histograms
Stem and leaf plots
Displaying Data in Stem-and-Leaf Plots and Histograms
Bar Charts
Relative frequency
Pie Charts
Line Graphs
Double line graphs
Using graphs to predict events
Applications of Statistics and Probability in Insurance and Medicine
Measure of Statistical dispersion
Measures of central tendency
Mean, Median, Mode, Range
Outliers, Quartiles, Box-and-whisker plot
Standard Deviation and Variance
Mean Deviation
Permutation and Combination
Using Sampling
Applying Counting principles and using a Tree Diagram
Probability, Definition of key terms- Experiment
Dependent Events and Independent Events
Outcomes and the Sample space
Mutually exclusive events and mutually inclusive events
Using the Fundamental Counting Principle
Conditional Probability
Experimental probability
Finding the line of best fit
Finding best fit models
Recognizing misleading data
Identifying misleading graphs- Look at the scale and see where it starts, check scales
Comparing data
Normal Distribution
Confidence Intervals
z-scores and t-tables
Application: Business, Planning, Keeping track

Population growth or decline
Analyzing the effects of bias in Sampling and Surveys
Scatter plot and Trend lines
Correlation and Regression
Positive correlation and Negative correlation
Calculating frequency distributions
Discrete events
Random Number Generator

<u>Right Triangles and Trigonometry</u>

Structure and parts of a Right Triangle
Pythagoras theorem
Converse of the Pythagoras theorem
Pythagoras triples
Applications of Trigonometry: Pilots, Control tower, Aviation, Ships
Understanding the unit circle and radian measures
Conversion: Angles and Radians
Coterminal Angles, Quadrantal Angles, Reference Angles
Angles of Rotation
Introduction to circular functions
Introduction to Trigonometric Functions
Formula for Sine, Cosine, Tangent, Cosecant, Secant and Cotangent
Solving for Missing Sides in a triangle
Solving for Missing Angles in a triangle (Sin^{-1}, Cos^{-1}, tan^{-1})
Special Triangles: 30^{O} -60^{O} -90^{O}
Special Triangles: 45^{O} -45^{O} -90^{O}
Angles of Elevation and Depression
Law of Sines
Law of Cosines
Graphing the Sine Function
Graphing the Cosine Function
Graphing the Tangent function
Graphing the Cosecant Function
Graphing the Secant Function
Graphing the Tangent Function
Transformations and translations of Trigonometric Functions
Combining Sine and Cosine on graphs
Finding inverse trigonometric functions
Graphing the Arcsine function
Graphing the Arccosine function
Graphing the Arctangent function
Graphing the Cosecant function
Graphing the Secant function
Graphing the Cotangent function
Translating Sine and Cosine Functions, Sinusoid Functions
Trigonometric Functions of any Real Number
Trigonometric Functions of any angle
Bearings and Distances
Introduction to Analytic Trigonometry

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Introduction to Trigonometric Identities
Using the Sum and Difference Identities
Pythagorean Trigonometric Identities
Finding double-angle
Half-angle identities
Deriving New Trigonometric Identities
Fundamental Trigonometric Identities
Cofunction Identities
Product-to-Sum Identities
Sum-to-Product Identities
Area of a Triangle using Sine
Heron's Formula
Verification of Trigonometric Identities
Solving Trigonometric Equations
Geometric Mean
Using polar Coordinates
Graphs of Polar Equations
Using Vectors, Vector Addition, Vector Decomposition
Head-to-tail Method
Right Triangle Congruence Theorems
Parametric equations

<u>Fundamentals of Geometry, Using Formulas</u>

Geometry History: Euclid, Euclidean vs. Non-Euclidean Geometry- Read Euclid's Book and Euler's Book
10 postulates from Euclid
Geometry Fundamentals: Undefined terms, Points, Lines, Planes
Intersecting lines, Intersecting Planes, Parallel planes, Perpendicular planes
Collinear Points, Non-collinear Points, Line Segments, Endpoints
Ruler Postulate, One-to-One Correspondence, Segment Addition Postulate
Common Shapes in Mathematics
What is a Formula? Basic Formulas in Geometry and Mathematics
Introduction to Angles
Using Tools in Geometry: Rulers, Protractors
Angle Addition Postulate
Right Angle, Acute angle, Obtuse angle
Reference Angles, 180 degrees, 360 degrees
Angle on a Straight line
Connection and Integration: Relate Analog Clocks to Angles
Parallel Lines and Transversals, Identifying pairs of angles
Complimentary and Supplementary Angles
Skew Lines and Applications
Using formulas in Geometry: Distance Formula
Using formulas in Geometry: Midpoint Formula

<u>Construction and Calculators</u>

Constructions in Geometry
Congruent triangles
Congruent angles

Confirming Pythagoras theorem with coordinate geometry
Construction: Drawing Perpendicular Lines using a Compass
Perpendicular bisectors and angle bisectors
Construction: Segments and Angles
Construction: Bisecting a straight line
Construction of loci
Geoboards
Scale factor and Scale drawings, Scale factor in Surface Area and Volume
Changing dimensions of shapes
Fractals, Koch Snowflake
Isometric Projection
Iteration
Explore Architecture and Estate Design

Shapes and Geometry: Quadrilaterals, Solids and Polygons

Introduction to Polygons
Application to Building and Design
Regular Polygons, 3 sides, 4 sides, 5, 6, 7, 8, 9, 10, 11, 12 sides
Concave and Convex polygons
Measure of Angles in a polygon
Exterior angle of a polygon
Finding the Perimeter of Regular Polygons
Finding the Area of Regular Polygons
Classification of quadrilaterals
Properties of quadrilaterals, Finding areas and perimeters of quadrilaterals
Properties of Parallelograms, Rectangles, rhombuses and Squares
Distinguishing types of Parallelograms
Determining if a Quadrilateral is a Parallelogram
Percent change of dimensions
Properties of Kites, Rhombus, Trapezoids and Trapeziums
Quadrilaterals on the Coordinate Plane
Using Geometry Software to work with shapes, solids and planes
Finding Areas and Perimeters of Composite Figures
Applying Similarity and Proportionality theorems
Indirect Measurements, Who was Thales of Miletus?
Area Ratios of Similar Figures and Congruent polygons
Areas of Combined Polygons
Effects of Changing Dimensions on Perimeter and Area
Maximizing Area and Volume
Finding Areas of Polygons Using Matrices
Golden Rectangle
Semi regular tessellations
Geometric Probability
Introduction to solids, parts of solids, representing solids, Orthographic Views
One point perspective Drawing
Finding surface areas and volumes of a prisms, Different types of prisms
Finding surface areas and volumes of cylinders
Finding surface areas and volumes of pyramids
Finding surface areas and volumes of cones and spheres

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Great Circles and Hemispheres
Frustums of Cones and Pyramids
Cross sections of solids
Volume ratios of Similar Solids
Symmetry of Solids and Polyhedra
Nets with respect to shapes
Tessellations, Regular Tessellations
Finding Surface Areas and Volumes of Composite solids
Platonic Solids, Regular Polyhedron
Practical problems, Physics and solids-finding the time it takes to fill up a cylinder
Project: Plan and build an estate for people to live in (Use all Geometric topics relevant)

<u>Circles</u>

Applications and History, Structure and parts of a circle, 360 degrees-Sumerians and Babylonians
Circumference of a circle
Area of a circle
Segment of a circle, Length of arc
Area of the sector
Inscribed Angles, Central angles and arc measure
Chords, tangents & secants
Circles and Inscribed Angles
Angles interior to Circles
Angles exterior to Circles
Writing the equation of a Circle
Determining Chord Length
Relating Arc Lengths and Chords, Intercepted arc
Circumscribed and Inscribed Figures
Finding the Areas of Circle Segments
Concentric Circles
Tangent Circles
Determining the Lengths of Segments Intersecting Circles
Finding Angle Measures in Inscribed Triangles
Finding Measures of Arcs and Inscribed Angles
Using Geometry Software with Circles

<u>Triangles: Introduction, Classification, Congruence and Similarity</u>

Applications in building bridges
Triangle Rigidity
Perimeter of a Triangle
Area of a Triangle
Classification of triangles by angle size
Classification of triangles by side lengths
Triangle theorems- Triangle Angle Sum Theorem
How do we know that the Triangle Angle Sum Theorem is true?
External Angle Theorem
Properties of Isosceles triangles, Properties of Equilateral triangles
Triangle Similarity, Similar triangles and proportions
Triangle congruence: SSS, SAS, ASA, AAS

Triangle similarity: AA, SSS, SAS
Using Proofs to show congruence
Altitudes, Orthocenters, medians, angle bisectors and perpendicular bisectors of triangles
Midsegment of a Triangle
Triangle inequalities
Fractals and Sierpinski Triangle
Euler Line

<u>Logic and Reasoning, Theorems, Proofs, Conjectures, Postulates, Properties and Corollaries concerning shapes, lines and angles</u>

Lessons from Logic and Reasoning: What is the point?
What is a Theorem?
What are Proofs? Starting with what you are given and moving towards the goal
Paragraph proofs
Coordinate proofs
2-column proofs
Flowchart proofs
Algebraic Proofs
What are Conjectures?
What is a Postulate?
What is a Corollary?
Postulates and theorems about points
Postulates and theorems about lines
Definition of parallel lines
Proving lines parallel, Theorems associated with Parallel lines
Postulates and theorems about planes, Coplanar and Non-Coplanar lines
Definitions: Congruence and Similarity, Properties of Congruence and Similarity
Reflexive Property of Congruence
Symmetric Property of Congruence
Transitive Property of Congruence
Transitive Property of Similarity
Symmetric Property of Similarity
Reflexive Property of Similarity
Rays and Angles, Angle Postulates
Protractor Postulate
Interior and Exterior Angles
Angle of Rotation
Relate Circle Graphs to Angles
Vertical Angles, Corresponding angles, Alternate Interior Angles, Alternate Exterior Angles
Same-side interior Angles (Consecutive Interior Angles)
Converse of Corresponding Angles Postulate
Converse of the Alternate Interior Angle Theorem
Converse of the Alternate Exterior Angle Theorem
Converse of the Same-side Interior Angle Theorem
Lessons in Symbolic Logic: What is the point?
Reasoning, Truth tables and Venn diagrams in Logic
Introduction to Truth Tables
Interpreting truth tables
Logical Implication

Making truth tables with different combinations
Bi-conditional statement
Logically equivalent Statements
Contrapositive, Converse, Conjunction, Contradiction, Negation, Inverse, Tautology
Law of Non contradiction
Law of the excluded middle
Using conditional statements
Compound Statements
Mathematical Induction
Using Inductive reasoning
Using Deductive reasoning
Laws of detachment & syllogism, The Antecedent and the Consequent
Truth Value, Truth and Validity
Modus tollens
Modus ponens
Logical Fallacies- Denying the antecedent
Affirming the consequent
Fallacies around us, Gently analyzing Arguments
Disproving conjectures with counterexamples
Direct Reasoning
Indirect Proofs and Proof by contradiction- infinite number of prime Numbers- Terrence Tao
Proof of Pythagoras theorem
Triangle Proofs
Circle Proofs

History of Math, People in Mathematics, Science and Philosophy and Study Skills

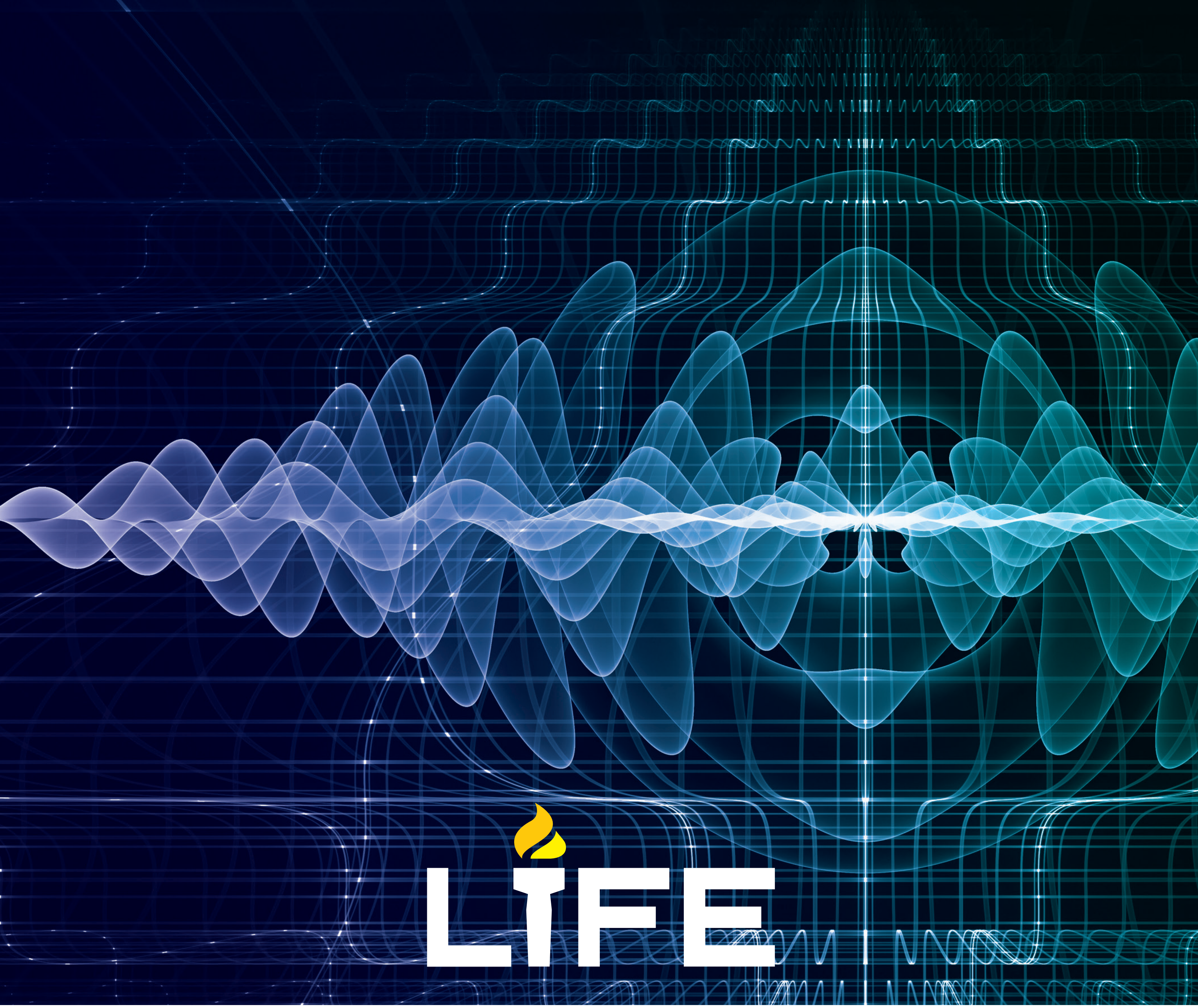
People in Philosophy, Science and Mathematics
Stories of Inspiration and Encouragement
History of Algebra (2000BC -)
History of Geometry (2000BC -)
History of Calculus (2000BC -)
Isaac Newton (1642-1727)
Blaise Pascal (1623-1662)
Gottfried Wilhelm von Leibniz (1646-1716)
Bertrand Russell (1872-1970)
Pythagoras of Samos (c. 570BC-c. 495BC)
Thales of Miletus (c. 624BC-c. 546BC)
Zeno of Elea (c. 490BC-c. 430BC)
Archimedes of Syracuse (c. 287BC-c. 212BC)
Chang Tshang (c. 200BC-c. 142BC)
Diophantus of Alexandria (Uncertain, probably, between AD 200 and 214 to 284 or 298)
Pappus of Alexandria (c. 340)
Euclid of Megara and Alexandria (mid 4th century BC- mid 3rd Century BC)
Socrates (470/469BC- 399BC)
Plato (Around 428/427BC or 424/434BC – 348/347 BC)
Aristotle of Stagira (384BC-322BC)
Leonardo Pisano (Fibonacci) - (c. 1170/1175AD- 1240AD)
Nicolaus Copernicus (1472-1543)
Girolamo Cardano (1501-1576)

John Napier (1550-1617)
Galileo Galilei (1564-1642)
Johannes Kepler (1571-1630)
Gerard Desargues (1591-1661)
Rene Descartes (1596-1650)
Pierre de Fermat (1601-1665)
John Brehaut Wallis (1616-1703)
Christiaan Hugens (1629-1695)
Jacob Bernoulli (1654-1705)
Johann Bernoulli (1667-1748)
Daniel Bernoulli (1700-1782)
Leonhard Euler (1707-1783)
Brook Taylor (1685-1731)
Alexis Claude Clairaut (1713-1765)
Johann Carl Friedrich Gauss (1777-1855)
James Clerk Maxwell (1831-1879)
Georg Cantor (1845-1918)
Oliver Heaviside (1850-1925)
Jules Henri Poincare (1854-1912)
Godfrey Harold Hardy (1877-1947)
Albert Einstein (1879-1955)
Kurt Gödel (1906-1978)
Alan Mathison Turing (1912-1954)
John Horton Conway (1937-)
Math History across the world- Africa, South America, Australia
Mathematics: the Middle East
Mathematics: India
Mathematics: China
Mathematics: Japan
Mathematics: France
Mathematics: Switzerland
Mathematics: England
Mathematics: Russia
Mathematics: Germany
Mathematics: Norway
Mathematics: Ireland
Mathematics: Scotland
Mathematics: USA
Mathematics: Holland
Mathematics: Hungary
Mathematics: Poland
Mathematics: England
Mathematics all over the world
A study on the History of Mathematics
Non-Euclidean Geometry, The work of Beltrami in 1868 in showing why Non- Euclidean Geometry is as logical as Euclidean Geometry

About the Authors

Amos Tarfa taught Science, History, Bible, Geometry, and Algebra 2 at Lakeview Christian Academy for 2 years. He graduated from the University of Wisconsin Superior with a degree in Chemistry and a minor in Philosophy in 2010. Amos has been a diligent Math student since elementary school. He took over 9 Math classes in high school and several in college. He worked at the Math lab at the University of Wisconsin Superior working with students in different math courses. In 2012, he started writing Math resources for students. He loves showing people why Mathematics is relevant. He is also interested in working with other teachers in finding efficient ways of teaching Mathematics. He plans to get a masters and a PhD in Math and Math Education.

Nathan Jersett is a graduate of the University of Wisconsin Superior. He has a degree in Mathematics with a minor in Physics.



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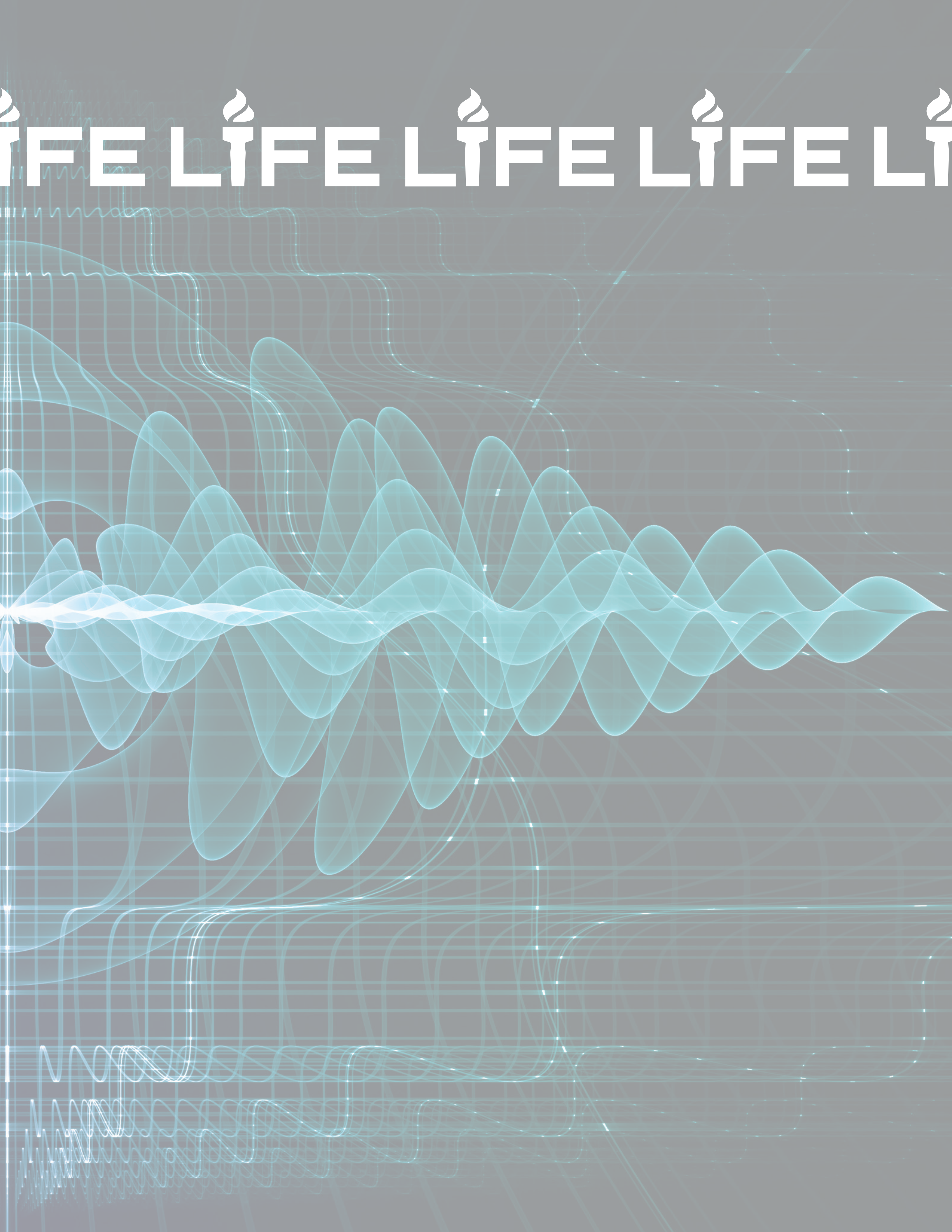
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