Sample Undergraduate Course Notes

Module: Geotechnical Engineering

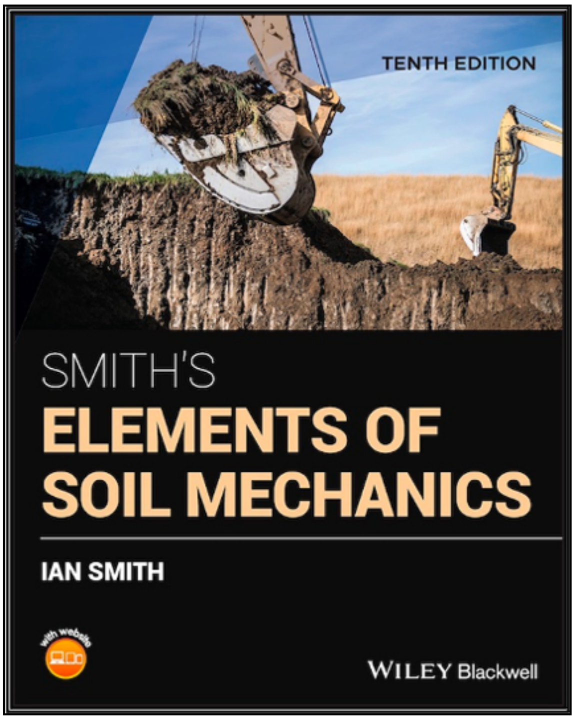
Prof Ian Smith

Session 2021/22

Blank sections in notes are completed in class.

These course notes have been written and compiled by Professor Ian Smith, specifically for the students of xxx university.

Additional support and reading should be gained from Smith's Elements of Soil Mechanics, 10th Edition.



Specifically, refer to the following chapters:

Chapter 2 Permeability and flow of water in soils

Chapter 3 Stresses in the ground

Chapter 4 Shear strength of soils

Chapter 6 Eurocode 7

Chapter 8 Lateral earth pressure

Chapter 9 Retaining structures

Visit the book's website for free downloads…

www.wiley.com/go/smith/soilmechanics10e

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Contents:

SECTION 1 – PERMEABILITY AND FLOW OF WATER

1.1 The hydrological cycle

Changes in in-situ soil water content occur as a result of variations in meteorological and hydrological conditions. As meteorological conditions change, the soil water regime is modified by the hydrological cycle. This results in the changes in soil moisture content, as well as fluctuations in the ground water flow and changes in the ground water level. The hydrological processes involved are shown in Figure 1.1.

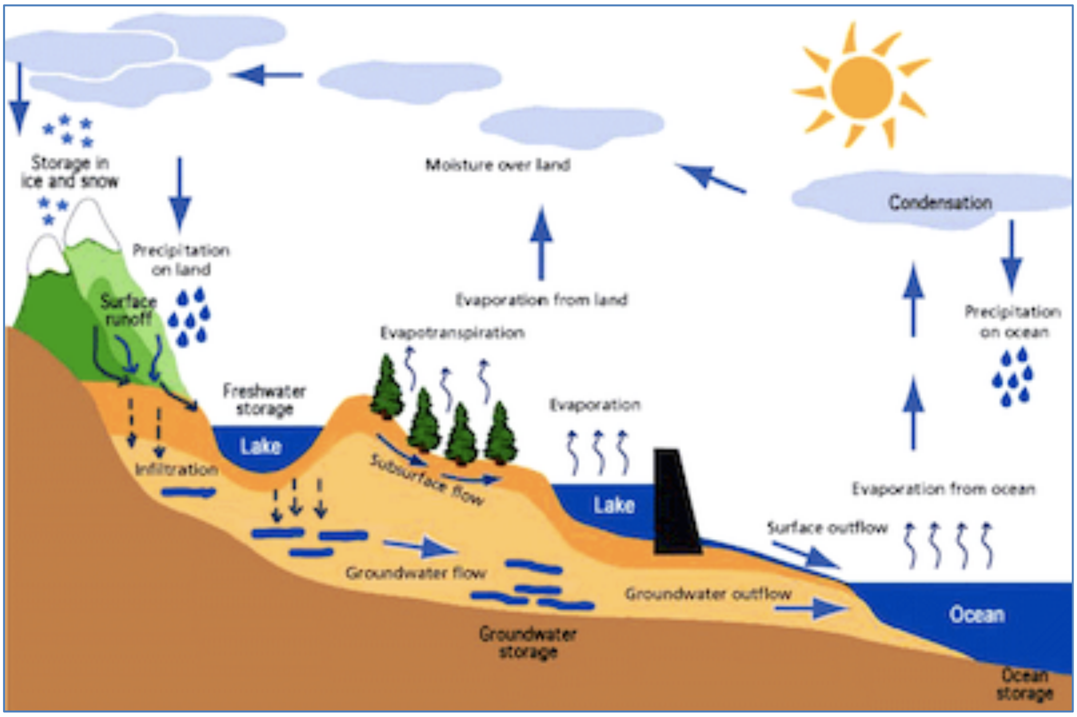


Figure 1.1. The hydrological cycle.

The cyclic order of events depicted by Figure 1.1 (i.e. the movement of water from the sea to the atmosphere and thereafter by precipitation to the land and back to the sea) is generally true. However, the hydrological cycle is not as simple as the figure suggests and there are several factors that confuse the rather simplistic concept shown in the figure:

- the cycle can 'short-circuit' at several stages. e.g. precipitation may fall directly onto the sea;
- the period of the cycle is variable;
- the intensity and frequency of the cycle is geographically and climatic dependent, and
- the various parts of the cycle can be quite complicated and variable (e.g. percolation may not always follow infiltration).

1.2 Subsurface water

This is the term used to define all water found beneath the Earth's surface. The main source of subsurface water is rainfall, which percolates downwards to fill up the voids. Subsurface water can be split into two distinct zones: *saturation zone and aeration zone.*

1.2.1 Saturation zone

This is the depth throughout which all the voids are filled with water under hydrostatic pressure. The upper level of this water is known as the water table, phreatic surface or groundwater level, and water within this zone is called phreatic water or groundwater.

The water table tends to follow in a more gentle manner than the topographical features of the surface above as shown in Figure 1.2. At groundwater level, the hydrostatic pressure is zero. The water table is not constant but rises and falls with variations of rainfall, atmospheric pressure, temperature, etc., whilst coastal regions are affected by tides. When the water table reaches the surface, springs, lakes, swamps, and similar features can be formed.

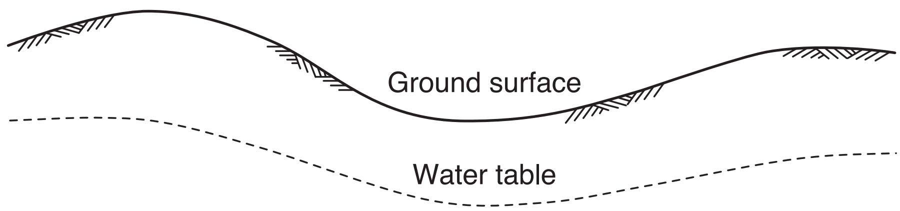


Figure. 1.2. Tendency of the water table to follow the earth's surface.

1.2.2 Aeration zone

This zone occurs between the water table and the surface, and can be split into three sections.

Capillary fringe

Owing to capillarity, water is drawn up above the water table into the voids of the soil. Water in this fringe can be regarded as being in a state of negative pressure, i.e. at pressure values below atmospheric. The minimum height of the fringe is governed by the maximum size of the voids within the soil. The maximum height of the fringe is governed by the minimum size of the voids. Between the minimum and maximum heights the soil is partially saturated.
Terzaghi and Peck (1948) give an approximate relationship between the maximum height and the grain size for a granular soil:

where C is a constant depending upon the shape of the grains and the surface impurities (varying from 10.0 to 50.0 mm^2) and D_{10} is the effective size expressed in millimetres.

Intermediate belt

As rainwater percolates downward to the water table, a certain amount is held in the soil by the action of surface tension, capillarity, adsorption and chemical action. The water retained in this manner is termed held water and is deep enough not to be affected by plants.

Soil belt

This zone is constantly affected by precipitation, evaporation and plant transpiration.

The various zones are illustrated in Figure 1.3

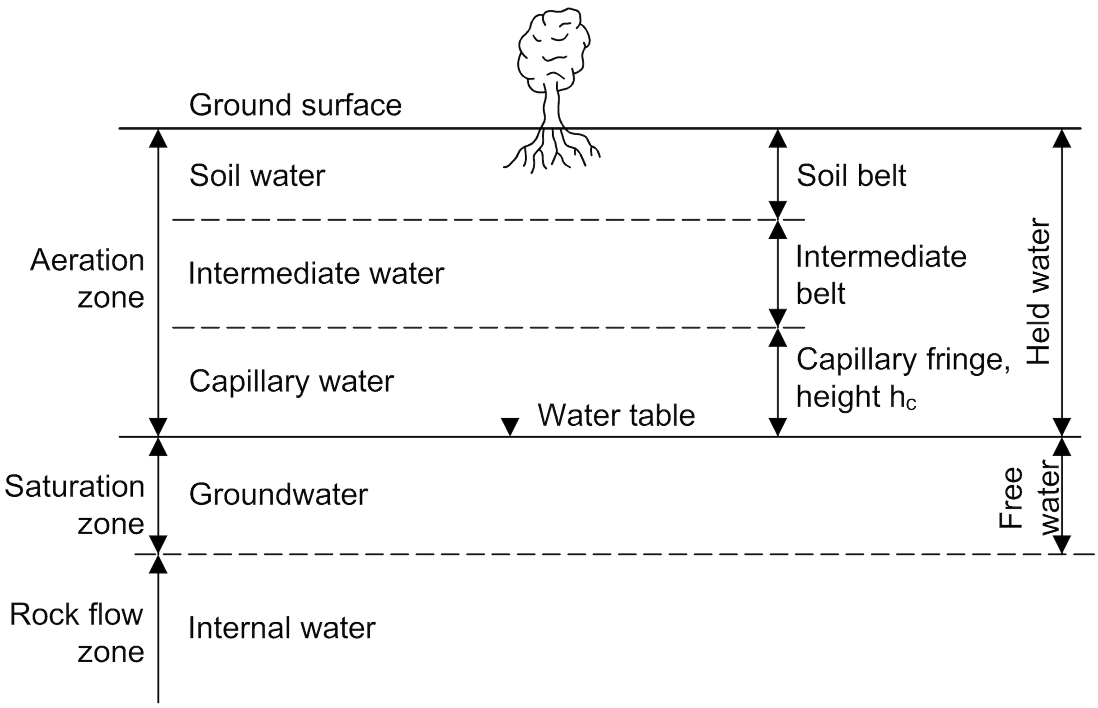


Figure. 1.3. Types of subsurface water.

1.3 Flow of water through soils

The voids of a soil are connected together and form continuous passageways for the movement of water brought about by rainfall infiltration and transpiration of plants etc.

When rainfall falls on the soil surface, some of the water infiltrates the surface and percolates downward through the soil. This downward flow results from a gravitational force acting on the water. During flow, some of the water is held in the voids in the *aeration zone* and the remainder reaches the groundwater table and the *saturation zone*. In the aeration zone, flow is said to be *unsaturated*. Below the water table, flow is said to be *saturated*.

1.3.1 Saturated flow

The water within the voids of a soil is under pressure. This water, known as pore water, may be static or flowing. Water in saturated soil will flow in response to variations in hydrostatic head within the soil mass. These variations may be natural or induced by excavation or construction.

1.3.2 Hydraulic or hydrostatic head

The head of water acting at a point in a submerged soil mass is known as the hydrostatic head and is expressed by Bernoulli's equation:

Hydrostatic head = Velocity head + Pressure head + Elevation head

In seepage problems atmospheric pressure is taken as zero and the velocity is so small that the velocity head becomes negligible; the hydrostatic head is therefore taken as:

Excess hydrostatic head

Water flows from points of high to points of low head. Hence flow will occur between two points if the hydrostatic head at one is less than the hydrostatic head at the other, and in flowing between the points the water experiences a head loss equal to the difference in head between them. This difference is known as the excess hydrostatic head.

1.3.3 Seepage velocity

The conduits of a soil are irregular and of small diameter – an average value of the diameter is $D_{10}/5$. Any flow quantities calculated by the theory of pipe flow must be in error and it is necessary to think in terms of an average velocity through a given area of soil rather than specific velocities through particular conduits.

If Q is the quantity of flow passing through an area A in time t, then the average velocity (v) is:

This average velocity is sometimes referred to as the seepage velocity.

1.4 Darcy's law of saturated flow

In 1856, Darcy showed experimentally that a fluid's velocity of flow through a porous medium was directly related to the hydraulic gradient causing the flow, i.e.

$$v \propto i$$

where I = hydraulic gradient (the head loss per unit length), or

$$v = ki$$

where k is a constant involving the properties of both the fluid and the porous material.

1.5 Coefficient of permeability, k

In soils we are generally concerned with water flow: the constant k is determined from tests in which the permeant is water and k is known as the coefficient of permeability.

Provided that the hydraulic gradient is less than 1.0, as is the case in most seepage problems, the flow of water through a soil is linear and Darcy's law applies, i.e.

$$V = ki$$

or

$$Q = Atki$$

or

q = Aki (where q = quantity of unit flow $\left(= \frac{Q}{t}\right)$

From this latter expression a definition of k is apparent: the coefficient of permeability is the rate of flow of water per unit area of soil when under a unit hydraulic gradient.

1.6 Determination of permeability in the laboratory

1.6.1 The constant head permeameter

The test is described in BS 1377: Part 5 and the apparatus is shown in Figure 1.4. Water flows through the soil under a head which is kept constant by means of the overflow arrangement. The head loss, *h*, between two points along the length of the sample, distance *l* apart, is measured by means of a manometer.

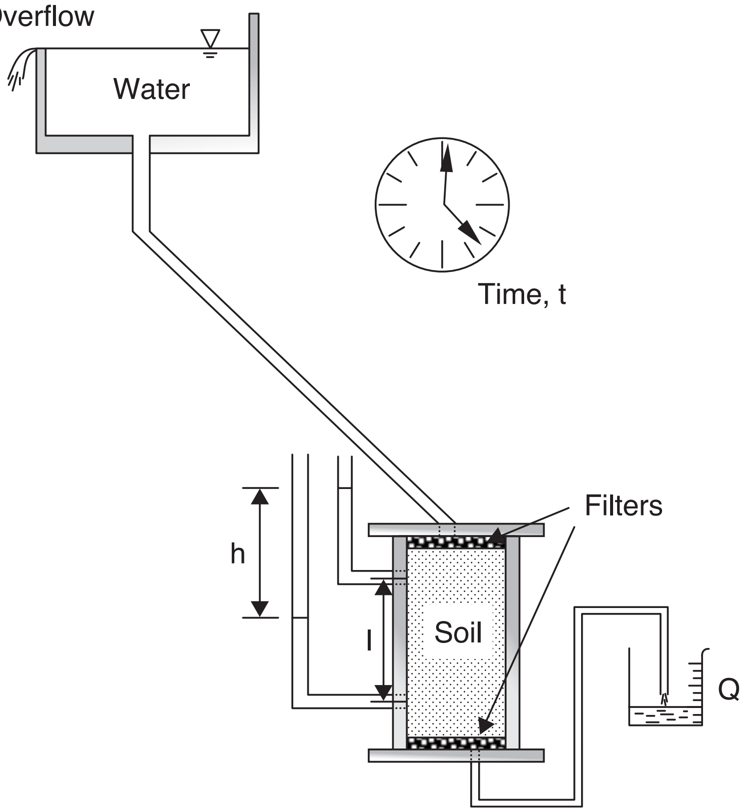


Figure 1.4. The constant head permeameter.

From Darcy's law: q = Aki

The unit quantity of flow, $q = \frac{Q}{t}$

The hydraulic gradient, $i = \frac{h}{l}$

and A = area of sample

Hence k can be found from the expression

$$k = \frac{q}{Ai} \qquad \text{or} \qquad k = \frac{Ql}{tAh}$$

A series of readings can be obtained from each test and an average value of k determined. The test is suitable for gravels and sands and could be used for many fill materials.

Example 1.1: Constant head permeameter

In a constant head permeameter test the following results were obtained:

Duration of test = 4.0 min
Quantity of water collected = 300 ml
Head difference in manometer = 50 mm
Distance between manometer tappings = 100 mm
Diameter of test sample = 100 mm

Determine the coefficient of permeability in m/s.

Solution:

1.6.2 The falling head permeameter

A sketch of the falling head permeameter is shown Figure 1.5. In this test, which is suitable for silts and some clays, the flow of water through the sample is measured at the inlet. The height, h_1, in the stand-pipe is measured and the valve is then opened as a stop clock is started. After a measured time, t, the height to which the water level has fallen, h_2, is determined.

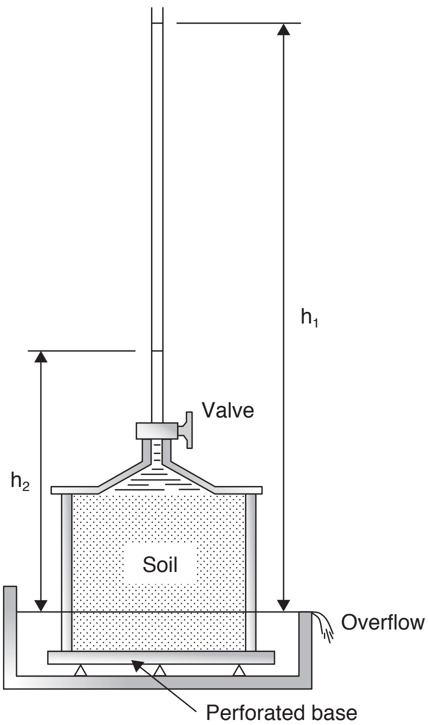


Figure 1.5. The falling head permeameter.

k is given by the formula:

$$k = \frac{al}{At} ln \frac{h_1}{h_2}$$

where

A = cross-sectional area of sample
a = cross-sectional area of standpipe
l = length of sample.

During the test, the water in the standpipe falls from a height h_1 to a final height h_2.

Let h be the height at some time, t.

Consider a small time interval, *dt*, and let the change in the level of h during this time be -*dh* (negative as it is a drop in elevation).

The quantity of flow through the sample in time *dt* = -*adh* and is given the symbol dQ. Now

dQ = Aki dt

$$= Ak\frac{h}{l} dt = -adh$$

or

$$dt = -\frac{al}{Ak}\frac{dh}{h}$$

Integrating between the test limits:

$$\int_0^t dt = -\frac{al}{Ak} \int_{h_1}^{h_2} \frac{1}{h}\, dh$$

i.e.

$$t = -\frac{al}{Ak} ln \frac{h_2}{h_1} = \frac{al}{Ak} ln \frac{h_1}{h_2}$$

or

$$k = \frac{al}{At} ln \frac{h_1}{h_2}$$

Example 1.2: Falling head permeameter
An undisturbed soil sample was tested in a falling head permeameter. The results were:

Initial head of water in standpipe = 1500 mm
Final head of water in standpipe = 283 mm
Duration of test = 20 minutes
Sample length = 150 mm
Sample diameter = 100 mm
Stand-pipe diameter = 5 mm

Determine the permeability of the soil in m/s.

Solution:

1.7 Graphical solution to flow problems: Flow nets

As we saw in Section 1.4, the flow of water in soils is governed by Darcy's Law, which states simply that the velocity of flow is proportional to the hydraulic gradient, *i* causing the flow:

$v \propto i$

therefore… v = ki

where k = the coefficient of permeability of the soil.

The hydraulic gradient, *i* is the head loss per unit length:

$$i = \frac{h}{l}$$

Solutions for two-dimensional flow problems can be established using flow nets. A flow net is formed from two orthogonal sets of curves:

- *equipotentials* connecting points of equal total head *h*
- *flow lines* indicating the direction of seepage down a hydraulic gradient

A classic flow net is shown in Figure 1.6.

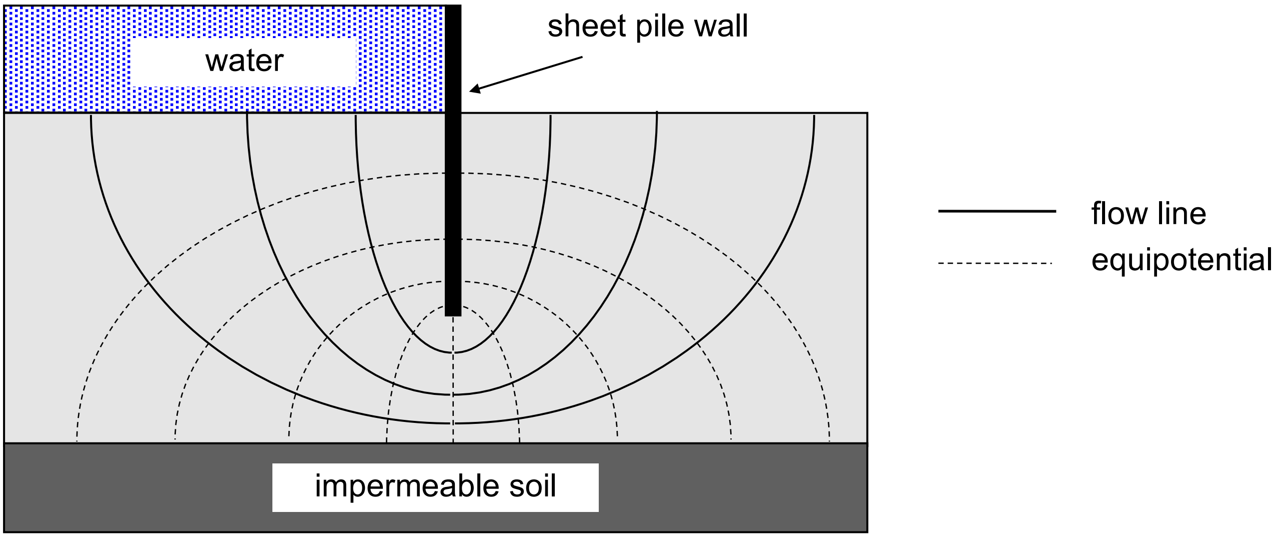


Figure 1.6. Flow under a sheet pile wall.

If standpipe piezometers were inserted into the ground with their tips on a single equipotential then the water would rise to the same level in each standpipe. (The pore pressures would be different because of their different elevations.)

There can be no flow along an equipotential, because there is no hydraulic gradient. The flow lines define channels along which the volume flow rate is constant.

1.7.1 Boundary conditions

A surface on which the total head is fixed (for example, from the level of a river, pool, reservoir) is an equipotential. A surface across which there is no flow (for example, an impermeable soil layer or an impermeable wall) is a flow line.

For the situation shown in Figure 1.5, with flow occurring under a sheet pile wall, the axis of symmetry must also be an equipotential.

1.7.2 Calculation of flow

Consider an element from a flow channel of length l between equipotentials which indicate a fall in total head Δh and between flow lines b apart (Figure 1.7).

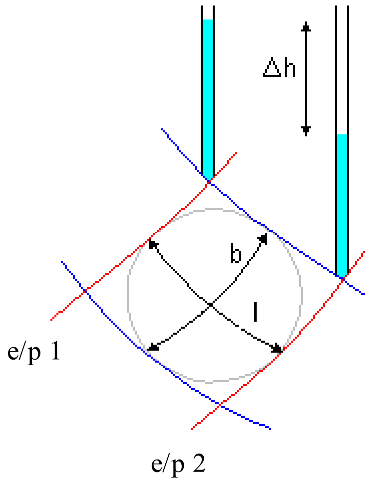


Figure 1.7. Flow net element.

The average hydraulic gradient is

and for unit width of flow net the volume flow rate is

$$q = kb\frac{\Delta h}{l}$$

There is an advantage in displaying or sketching flownets in the form of curvilinear 'squares' so that an imaginary circle can be inscribed within each four-sided figure bounded by two equipotentials and two flow lines. Then $b = l$ and $q = k\Delta h$ so the flow rate through the flow channel is the permeability multiplied by the uniform interval Δh between equipotentials.

1.7.3 Calculation of total flow

For a complete problem, the flownet is drawn with the overall head drop, *h* divided into N_d equal intervals with N_f flow channels:

i.e. $\Delta h = h / N_d$

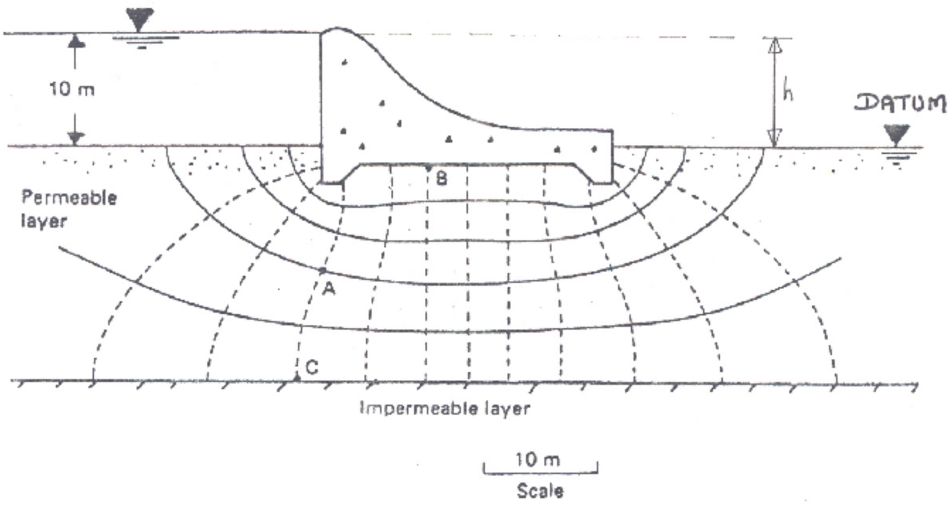
Then the total flow rate per unit width is:

The number of potential drops (N_d) is simply equal to the number of equipotentials minus one. The number of flow channels (N_f) is easily counted from the flow net. In the example shown in Figure 1.6:

1.7.4 Anisotropic soil

The flow net of Figure 1.6 is typical for an isotropic, homogenous soil. Isotropic soils have same value of permeability in all directions. Usually however, soils are not isotropic, but are anisotropic in nature, meaning that the permeability in the vertical direction is different from the permeability in the horizontal direction.

Example 1.3: Seepage beneath concrete dam



For the flow net shown:

(a) If k = 0.01 mm/sec, determine the seepage loss of the dam in m^3/day.

(b) What is the total head at (i) A, (ii) B, and (iii) C ?

Solution:

1.8 Quick condition and piping

If the flow is upward then the water pressure tends to lift the soil element. If the upward water pressure is high enough the effective stresses in the soil disappear, no frictional strength can be mobilised and the soil behaves as a fluid. This is the *quick condition* and is associated with piping instabilities around excavations and with liquefaction events in or following earthquakes. This phenomenon was witnessed in the tragedy in Palu, Indonesia in the September 2018 earthquake.

1.8.1 Critical hydraulic gradient

The quick condition occurs at a critical upward hydraulic gradient i_c, when the seepage force just balances the buoyant weight of an element of soil. This can be explained with reference to Figure 1.8.

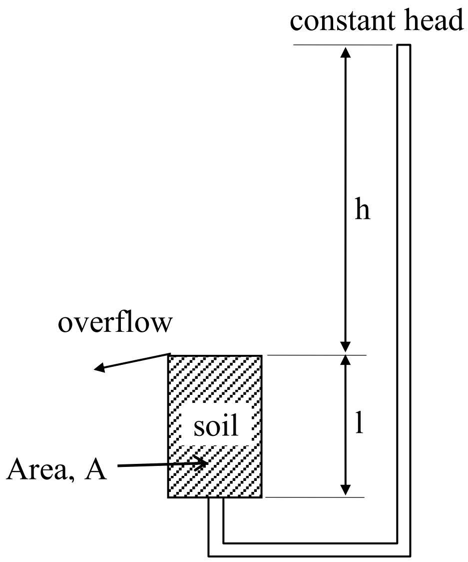


Figure 1.8. Constant head of flow.

The excess head, *h* leads to a corresponding excess hydrostatic pressure acting on the base of the sample equal to $\gamma_w h$. The soil is on the point of being washed out when the upward forces equal the downward forces.

downward forces = effective unit weight x volume

$= \gamma' \times Al$

$= \gamma_w \frac{G_s - 1}{1+e} Al$

upward forces $= h\gamma_w A$

i.e. $h\gamma_w A = \gamma_w \frac{G_s - 1}{1+e} Al$

Therefore, $\frac{h}{l} = \frac{G_s - 1}{1+e}$ .... this is the critical hydraulic gradient, i_c.

The critical hydraulic gradient is typically around 1.0 for many soils.

SECTION 2 – TOTAL AND EFFECTIVE STRESS

2.1 Total stress

The total vertical stress acting at a point below ground surface is due to the weight of ***everything*** lying above, e.g. soil, water and surface loading.

Consider the total stress at a point in the following situations:

(a) in a homogeneous soil mass
(b) in a multi-layered soil mass
(c) in a soil mass below the water table

2.1.1 Total stress in a homogeneous soil mass

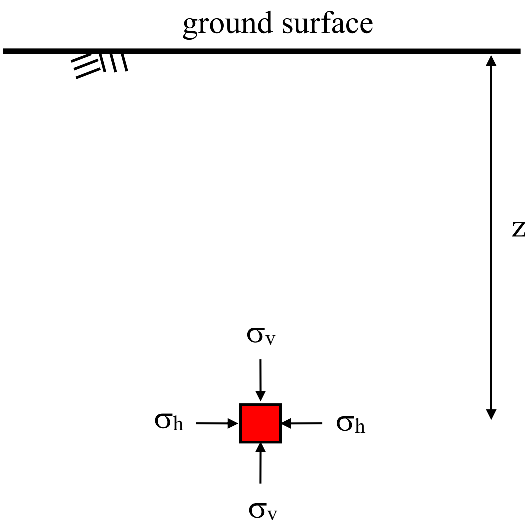


Figure 2.1. Total stress in a homogeneous soil mass.

Total stress increases with depth and with unit weight.

Vertical total stress at depth *z,*

where γ = unit weight of the soil

The symbol for total stress may also be written σ_z, i.e. related to depth *z*. The unit weight will vary with the water content of the soil.

2.1.2 Total stress in a multi-layered soil mass

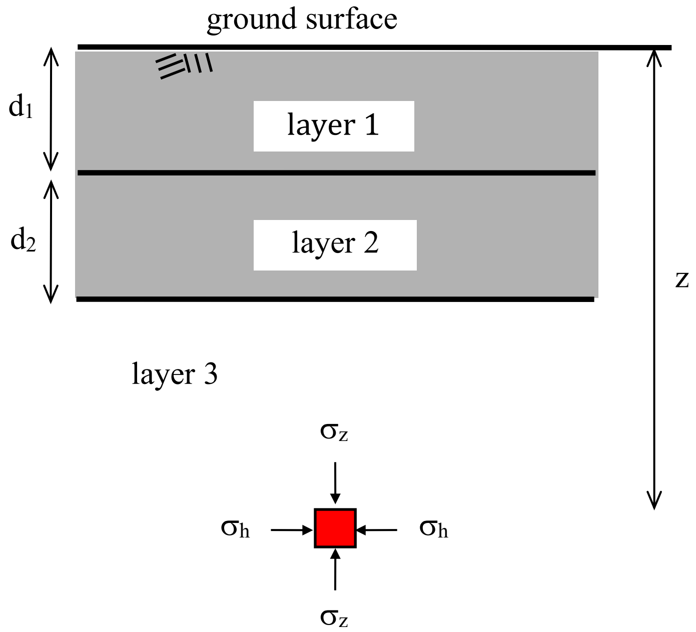


Figure 2.2. Total stress in a multi-layered soil mass.

The total stress at depth z is the sum of the weights of soil in each layer above.

Vertical total stress at depth z,

where γ_1, γ_2, γ_3, etc. = unit weights of soil layers 1, 2, 3, etc. respectively.

If a new layer is placed on the surface the total stresses at all points below will increase.

2.1.3 Total stress in a soil mass below the water table

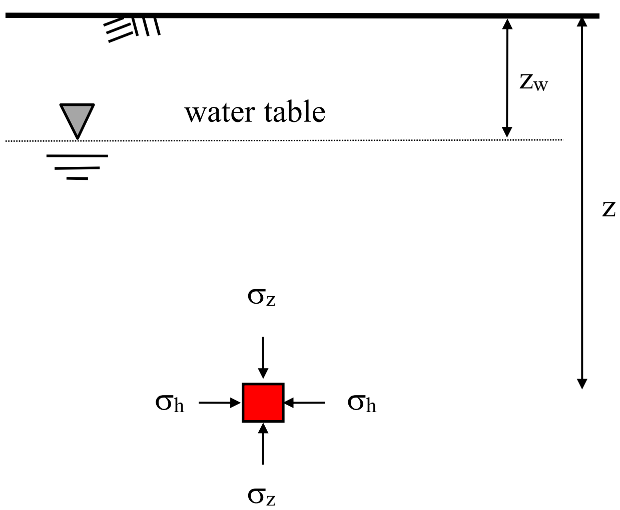


Figure 2.3. Total stress in a soil mass below the water table.

The soils above and below the water table are treated separately:

Below the GWT:

the soil is saturated

$\gamma = \gamma_{sat}$

Above the GWT:

The soil can vary from dry to saturated:

- sands and gravels soils may become dry
- clays may remain saturated with capillary water

Vertical total stress at depth z,

$$\sigma_z = \gamma z_w + \gamma_{sat}(z - z_w)$$

where

γ = unit weight of the soil lying above the water table.

If the water table level changes and the soil above it is not saturated, the total stresses below it will also change.

2.2 Pore water pressure

The water in the pores of a soil is called pore water. The pressure within this pore water is called pore water pressure (*u*). The magnitude of the pore water pressure depends on:

* the depth below the water table;
* the conditions of seepage flow.

Groundwater and hydrostatic pressure

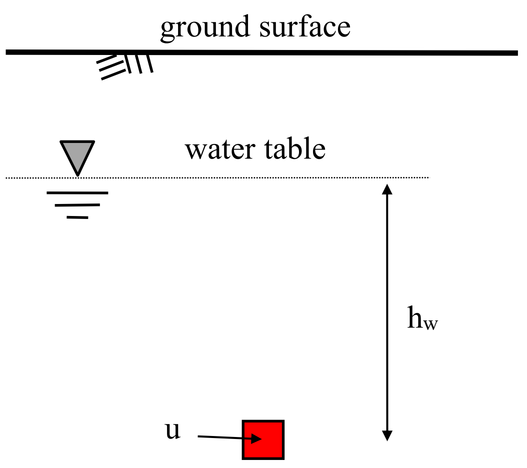


Figure 2.4. Pore water pressure.

Under hydrostatic conditions the pore pressure at a given point is given by the hydrostatic pressure.

Pore pressure,

where

h_w = the depth below the water table or overlying water surface.

2.3 Effective stress

The loads applied to soil masses are carried by either the solid grains or the water in the pores, and at times by both. With reference to Figure 2.5, as the pore water is loaded (via the applied total stress, σ) the pressure within it (u) increases causing the water to flow out of the soil, and the load is transferred to the solid grains (σ'). The flow of water under pressure out of a soil mass is called drainage. The rate of drainage depends on the permeability of the soil. Engineering behaviour involving strength and compressibility depend on the intensity of stresses passing through the solid granular fabric of the soil, i.e. *effective stresses*.

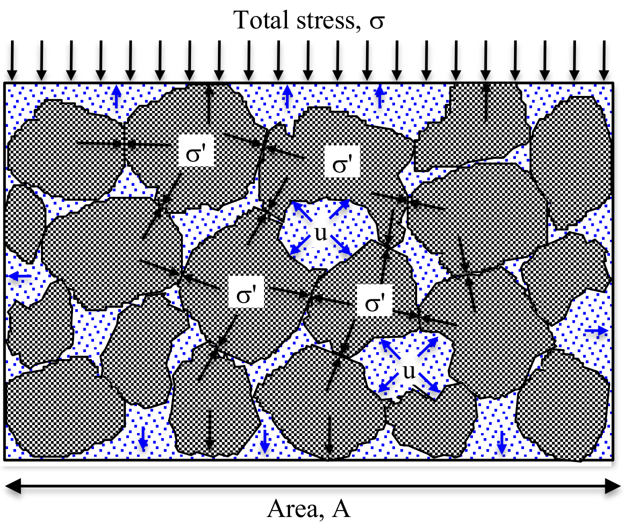


Figure 2.5. Load carried by soil particles and pore water.

Ground movements and instabilities can be caused by changes in total stress; e.g. loading due to foundations, unloading due to excavations. They can also be caused by changes in pore pressures; e.g. slope failures which occur after rainfall due to increasing pore pressures. In fact it is the combined effect of total stress and pore pressure that controls soil behaviours such as shear strength, compression and distortion.

This combined effect is called the effective stress (σ') and is given by:

Effective stress = Total stress – Pore water pressure

2.3.1 Terzaghi's principle of effective stress

Karl Terzaghi was born in Vienna and subsequently became professor of soil mechanics in the USA. In 1936 he was the first person to propose the relationship for effective stress. All measurable effects of a change of stress, such as compression, distortion and a change of shearing resistance, are due exclusively to changes in effective stress.

The adjective *effective* is particularly apt, because it is effective stress that is effective in causing important changes: changes in strength, changes in volume and changes in shape.

2.3.2 Drained and undrained soil

Throughout the rest of your career in civil engineering you will hear the expressions ***drained*** and ***undrained*** in reference to the state of soil or of loading conditions.

Drained does not mean the soil is dry of water. Similarly, undrained does not mean that every pore is full of water.

Refer again to Figure 2.5. As the normal stress is applied due to the external load, the pore pressure increases and this internal water pressure (known as the excess pore water pressure) causes the flow of water out of the voids. As the slightly pressurised water flows, the remaining water begins to experience a reduction in the excess pore water pressure. Eventually the pore water pressure returns to its initial value (i.e. the excess pore water pressure is zero). The rate at which this water flow occurs varies depending on the type of soil. Granular soils (e.g. sands) have much higher permeability than cohesive soils (e.g. clays) so the water drains much more quickly in granular soils. When the excess pore water pressure equals zero the soil is said to be *drained*. Until that point the soil is *undrained*.

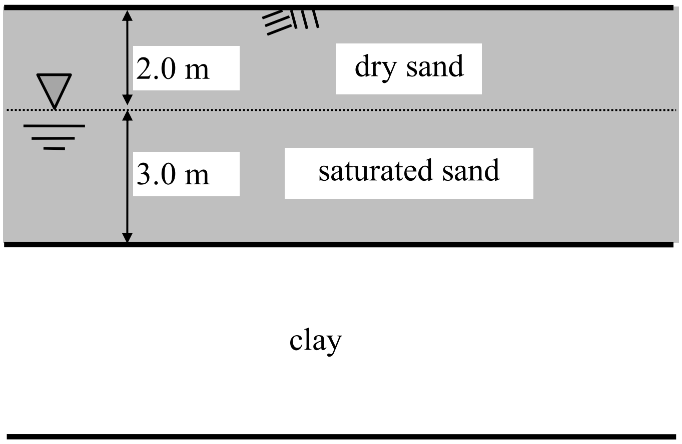
It should be fairly obvious that granular soils can become *drained* very quickly. They are said to be in a *drained state*. Consequently, when doing geotechnical analysis and design for granular soils, we will almost always do a *drained analysis*, or consider *drained conditions*.

Clays however have significantly lower permeabilities than granular soils and so drainage in clay takes much longer (years, as opposed to minutes/hours). Because of this, clays remain in an *undrained state* for a long time, as the excess pore water pressure very slowly dissipates and heads towards zero. When doing geotechnical analysis and design for cohesive soils therefore, we will consider *undrained conditions* and will do an *undrained analysis*. We will carry out a drained analysis too since the soil will after all eventually reach a drained state.

In *undrained* analyses, we consider the effect of the *total stresses* whereas in *drained* analyses we work with *effective stresses*. Thus, you will often hear the terms *total stress analysis* and *effective stress analysis* used to describe *undrained* and *drained* analyses respectively.

Example 2.1: Total and effective stresses

The figure shows 3 soil layers on a site.



Unit weights are:

dry sand, γ_d = 16 kN/m^3
saturated sand, γ_{sat} = 20 kN/m^3

Determine the vertical total stress, pore water pressure, and vertical effective stress at depths of 2.0m and 5.0m.

Solution:

Example 2.2: Total and effective stresses

A soil profile consists of 5m sand overlying 4m gravel, resting on bedrock. GWL is 2m below ground surface.

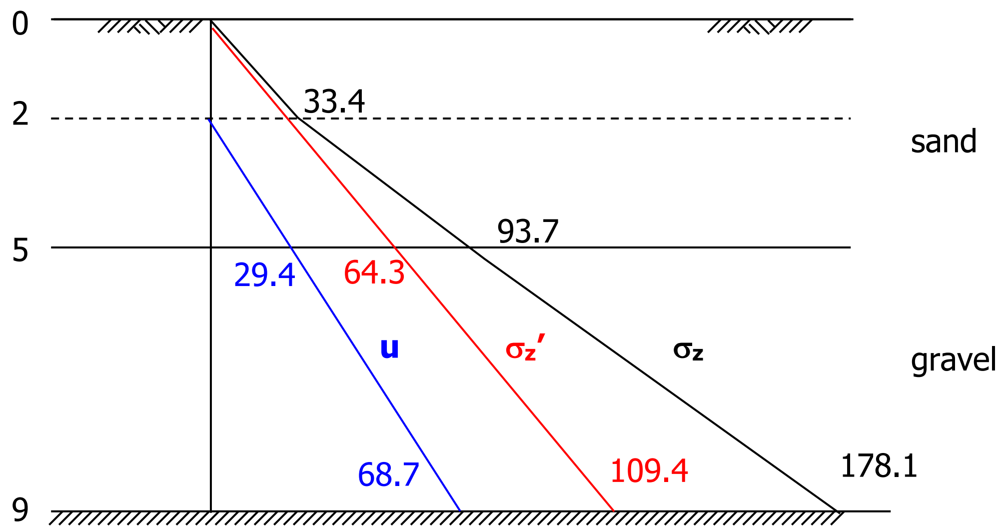
ρ_b sand above GWL = 1.7 Mg/m^3
ρ_{sat} sand below GWL = 2.05 Mg/m^3
ρ_{sat} gravel = 2.15 Mg/m^3

(a) Determine distributions of vertical total stress, pore water pressure, and vertical effective stress with depth down to bedrock.

Take ρ_w = 1.0 Mg/m^3

(b) How do the distributions change if the GWL is lowered to the sand/gravel interface?

Solution:


0
2
5
9
33.4
93.7
178.1
29.4
68.7
64.3
109.4
u
σz'
σz
sand
gravel

SECTION 3 – SHEAR STRENGTH OF SOILS

3.1 Failure mechanism of soils

The failure mechanism that occurs in soils is usually shear. Hence, knowledge of the shear strength of soils is an important element in most design calculations. (e.g. for retaining walls and slope stabilities, foundation design). Examples of failure modes are shown in Figure 3.1.

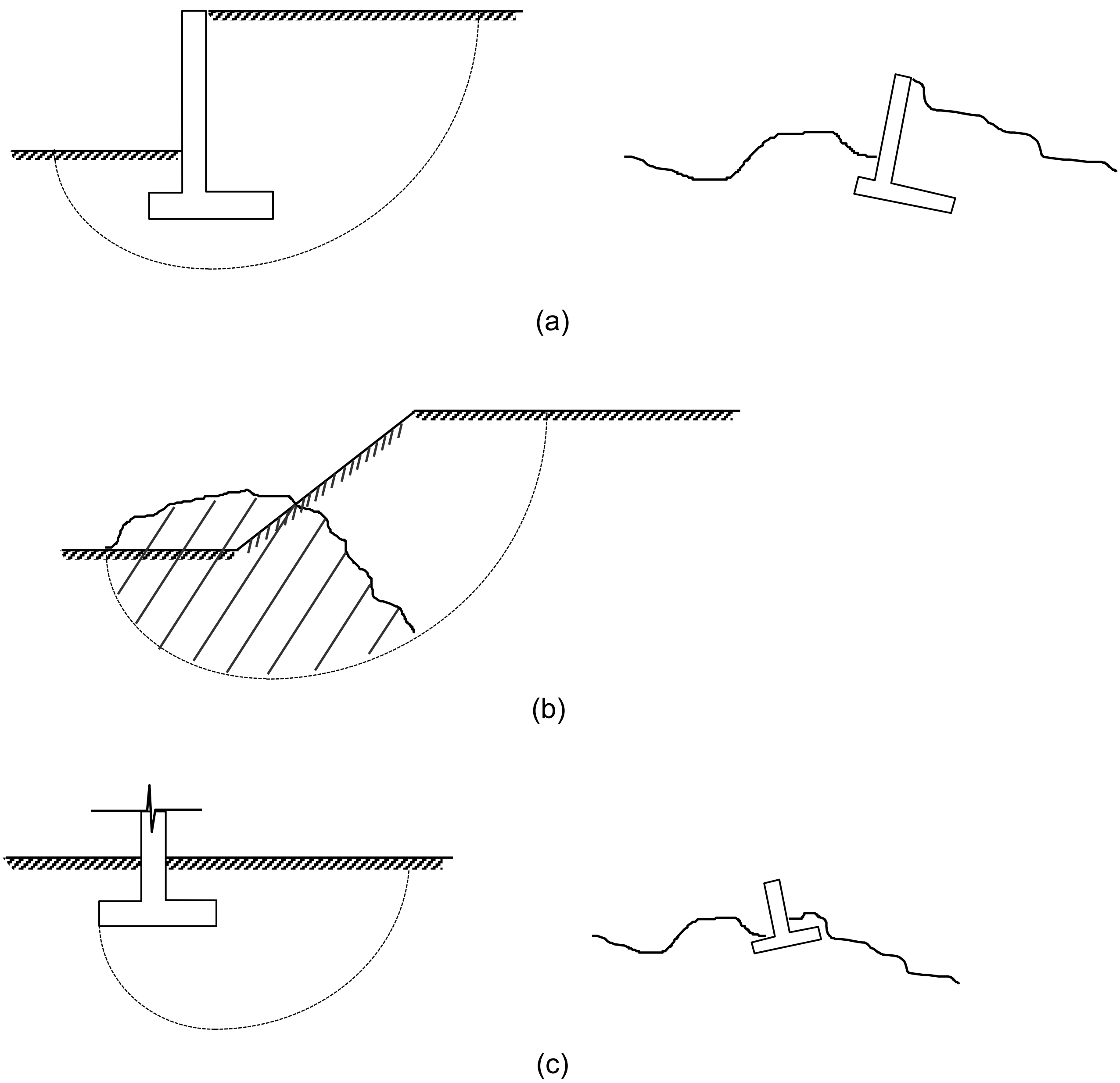


Figure 3.1. Modes of failure: (a) retaining wall, (b) slope, (c) shallow foundation.

3.2 Analysis of shear strength

The shear strength of a soil in any direction is the maximum shear stress that can be applied to the soil in that direction. When this maximum has been reached, the soil is regarded as having failed.

3.2.1 Friction

Consider a block of weight *W* resting on a rough surface, or plane (Figure 3.2).

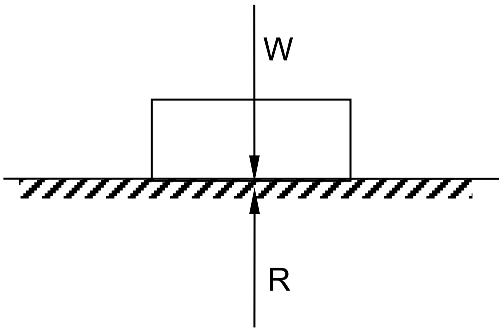


Figure 3.2. Friction on horizontal plane.

The vertical reaction force *R* is equal to *W*. If a horizontal force *H* is now applied, the direction of *R* is no longer vertical.

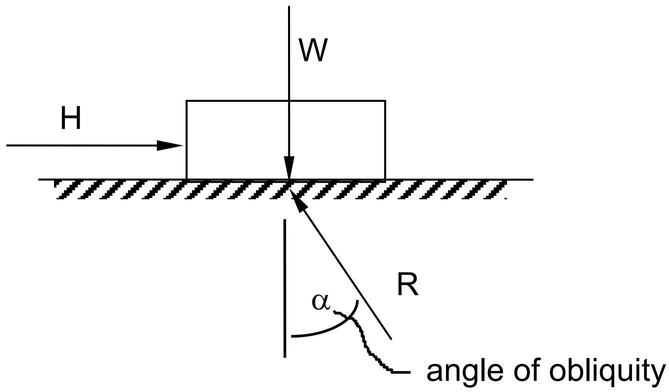
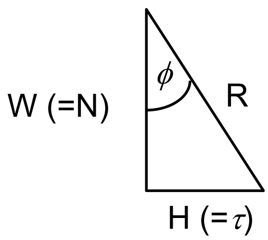


Figure 3.3. Vertical and horizontal forces acting on horizontal plane.

As friction is overcome, the block starts to slide and α equals ϕ (the angle of friction).

The reaction force *R* can be considered through the triangle of forces (Figure 3.4).



$H = R \sin \phi$
$W = R \cos \phi$

Figure 3.4. Triangle of forces.

The frictional resistance to sliding is the horizontal component of *R*, i.e. τ

tan ϕ is the coefficient of friction.

If we consider W acting on unit area, we get

where
τ = shearing resistance
σ_n = normal stress on failure plane (= load/area)

From the relationship, we can see that $\tau \propto \sigma_n$. Therefore, we can plot a straight line graph of τ against σ_n (Figure 3.5).

Figure 3.5. Plot of shear stress, τ against normal stress, σ_n.

In soils work, we refer to τ as the *shear strength* of the soil.

The plane on which the block rests is analogous to a failure surface within a soil mass as represented in Figure 3.6.

e.g.

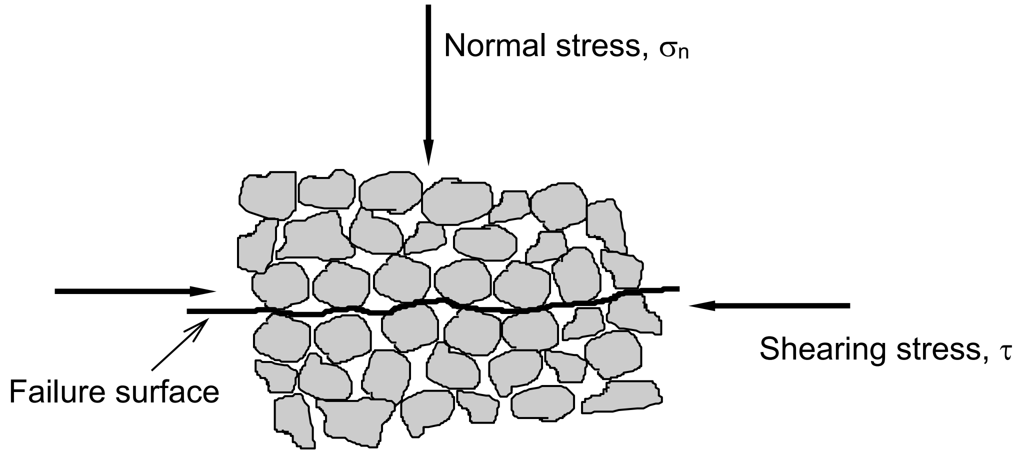


Figure 3.6. Failure surface in a soil mass.

So, if we can obtain ϕ for a granular soil, we can establish the shear strength mobilised for a given normal stress. If, at any point in a soil mass, the shear stress becomes equal to the shear strength, failure of the soil will occur at that point.

3.2.2 Complex stress

When a body is acted upon by external forces then any plane within the body will be subjected to a stress that is generally inclined to the normal to the plane. Such a stress has both a normal and a tangential component and is known as a *compound*, or *complex*, *stress*.

Principal plane

A plane that is acted upon by a normal stress only is known as a principal plane, there is no tangential, or shear, stress present. As is seen in the next section dealing with principal stress, only three principal planes can exist in a stressed mass.

Principal stress

The normal stress acting on a principal plane is referred to as a principal stress. At every point in a soil mass, the applied stress system that exists can be resolved into three principal stresses that are mutually orthogonal. The principal planes corresponding to these principal stresses are called the major, intermediate and minor principal planes and are so named from a consideration of the principal stresses that act upon them. The largest principal stress, σ_1, is known as the major principal stress and acts on the major principal plane. Similarly the intermediate principal stress, σ_2, acts on the intermediate principal plane whilst the smallest principal stress, σ_3, called the minor principal stress, acts on the minor principal plane. Critical stress values and obliquities generally occur on the two planes normal to the intermediate plane so that the effects of σ_2 can be ignored and a two-dimensional solution is possible.

3.3 The Mohr circle diagram

Figure 3.7a shows a major principal plane, acted upon by a major principal stress, σ_1, and a minor principal plane, acted upon by a minor principal stress, σ_3. By considering the equilibrium of an element within the stressed mass (Figure 3.7b) it can be shown that on any plane, inclined at angle θ to the direction of the major principal plane, there is a shear stress, τ, and a normal stress, σ_n. The magnitudes of these stresses are:

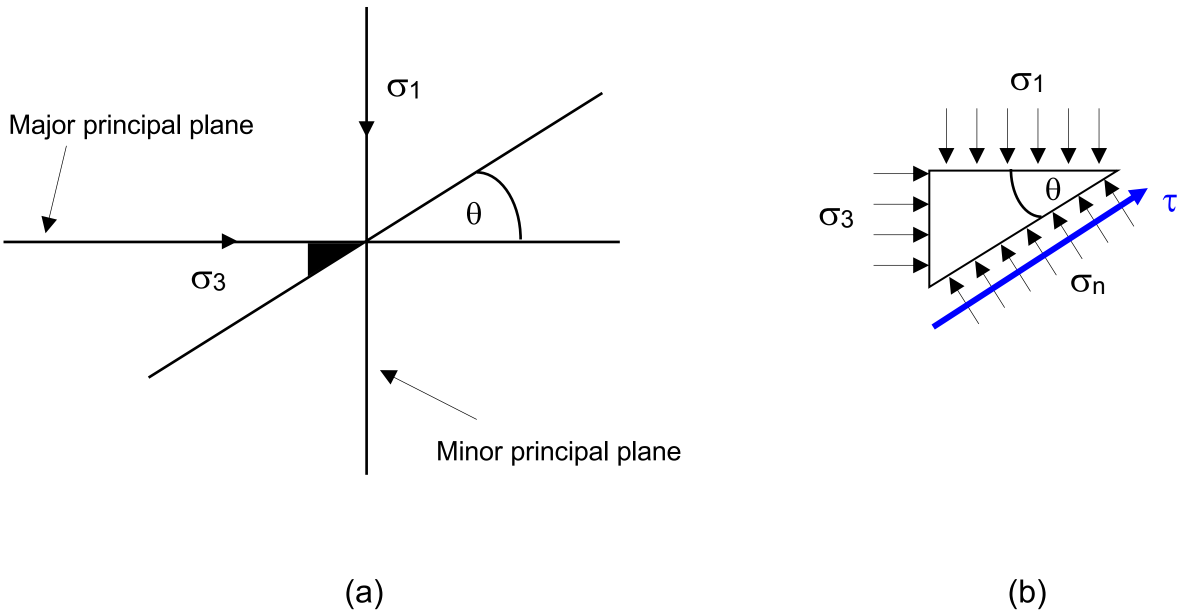


Figure 3.7. Stress induced by two principal stresses, σ_1 and σ_3, on a plane inclined at θ to σ_3.

These formulae lend themselves to graphical representation, and it can be shown that the locus of stress conditions for all planes through a point is a circle (generally called a Mohr circle). In order to draw a Mohr circle diagram a specific convention must be followed, all normal stresses (including principal stresses) being plotted along the axis OX while shear stresses are plotted along the axis OY. For most cases the axis OX is horizontal and OY is vertical, but the diagram is sometimes rotated to give correct orientation. The convention also assumes that the direction of the major principal stress is parallel to axis OY, i.e. the direction of the major principal plane is parallel to axis OX.

To draw the diagram, first lay down the axes OX and OY, then set off OA and OB along the OX axis to represent the magnitudes of the minor and major principal stresses respectively, and finally construct the circle with diameter AB. This circle is the locus of stress conditions for all planes passing through the point A, i.e. a plane passing through A and inclined to the major principal plane at angle θ cuts the circle at D. The coordinates of the point D are the normal and shear stresses on the plane (Figure 3.8).

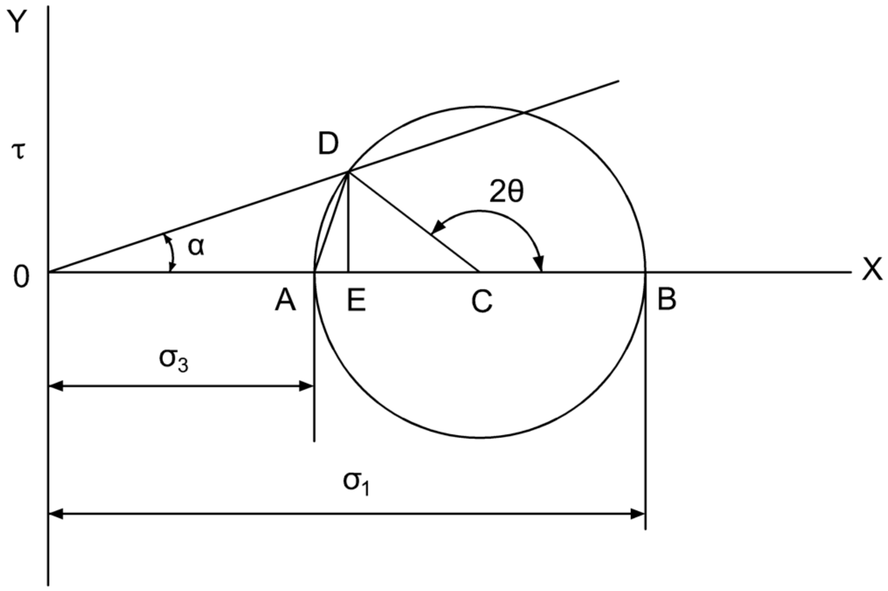


Figure 3.8. Mohr circle diagram.

In Figure 3.8, OE and DE represent the normal and shear stress components of the complex stress acting on plane AD. From the triangle of forces ODE it can be seen that this complex stress is represented in the diagram by the line OD, whilst the angle DOB represents the angle of obliquity, α, of the resultant stress on plane AD.

3.3.1 Limit conditions

It has been stated that the maximum shearing resistance is developed when the angle of obliquity equals its limiting value, ϕ. For this condition the line OD becomes a tangent to the stress circle, inclined at angle ϕ to axis OX (Figure 3.9).

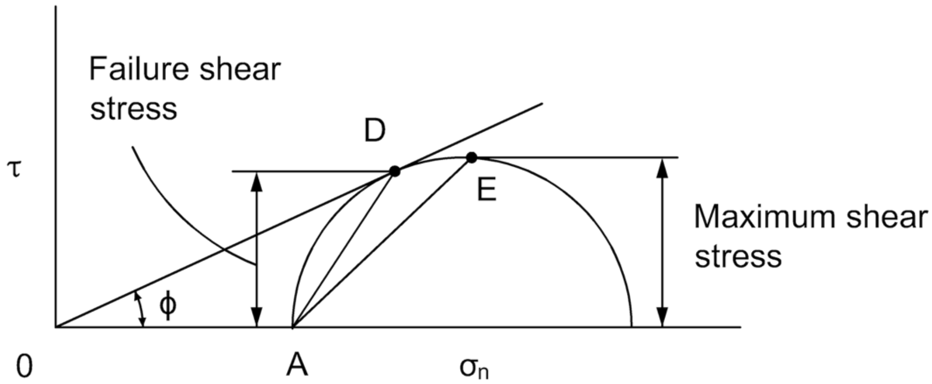


Figure 3.9. Mohr circle diagram for limit shear resistance.

An interesting point that arises from Figure 3.9 is that the failure plane is not the plane subjected to the maximum value of shear stress. The criterion of failure is maximum obliquity, not maximum shear stress. Hence, although the plane AE in Figure 3.9 is subjected to a greater shear stress than the plane AD, it is also subjected to a larger norm al stress and therefore the angle of obliquity is less than on AD, which is the plane of failure.

Example 3.1: Angle of shearing resistance and angle of failure plane

On a failure plane in a purely frictional mass of dry sand the total stresses at failure were: shear = 3.5 kPa; normal = 10.0 kPa.

Determine (a) by calculation and (b) graphically the resultant stress on the plane of failure, the angle of shearing resistance of the soil, and the angle of inclination of the failure plane to the major principal plane.

Solution:

(a) By calculation

The soil is frictional, therefore the strength envelope must go through the origin. The failure point is represented by point D in Figure 3.10a with coordinates (10, 3.5).

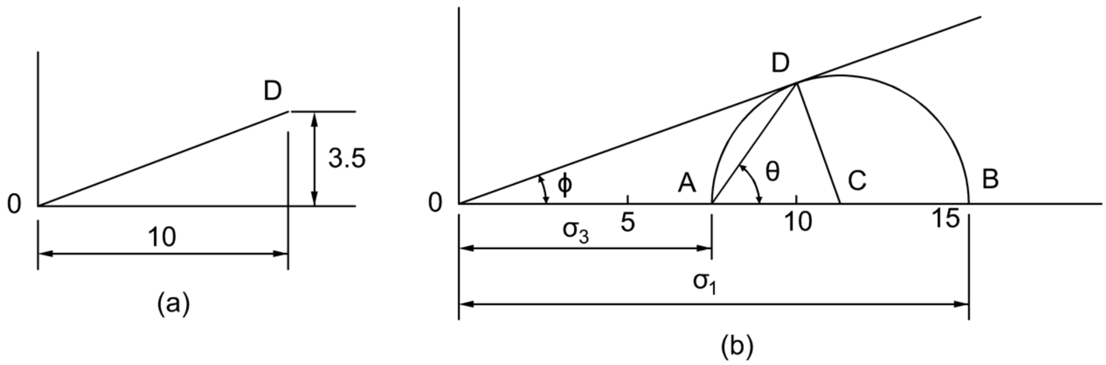


Figure 3.10. Example 3.1.

(b) Graphically
The procedure (Figure 3.10b) is first to draw the axes OX and OY and then, to a suitable scale, set off point D with coordinates (10, 3.5); join OD (this is the strength envelope). The stress circle is tangential to OD at the point D; draw line DC perpendicular to OD to cut OX in C, C being the centre of the circle.
With centre C and radius CD draw the circle establishing the points A and B on the x-axis.

3.3.2 Strength envelopes

If ϕ is assumed constant for a certain material, then the shear strength of the material can be represented by a pair of lines passing through the origin, O, at angles $+\phi$ and $-\phi$ to the axis OX (Figure 3.11). These lines comprise the Mohr strength envelope for the material.

In Figure 3.11 a state of stress represented by circle A is quite stable as the circle lies completely within the strength envelope. Circle B is tangential to the strength envelope and represents the condition of incipient failure, since a slight increase in stress values will push the circle over the strength envelope and failure will occur. Circle C cannot exist as it is beyond the strength envelope.

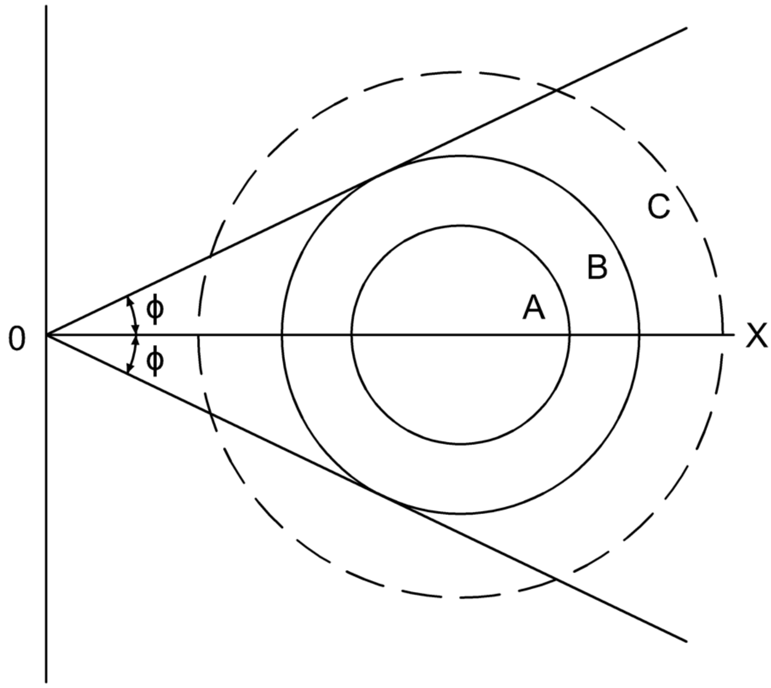


Figure 3.11. Mohr strength envelope.

3.3.3 Relationship between ϕ and θ

In Figure 3.12, $\angle DCO = 180° - 2\theta$.
In triangle ODC: $\angle DOC = \phi$, $\angle ODC = 90°$, $\angle OCD = 180° - 2\theta$. These angles summate to 180°, i.e.

$$\phi + 90° + 180° - 2\theta = 180°$$

hence

$$\theta = \phi/2 + 45°$$

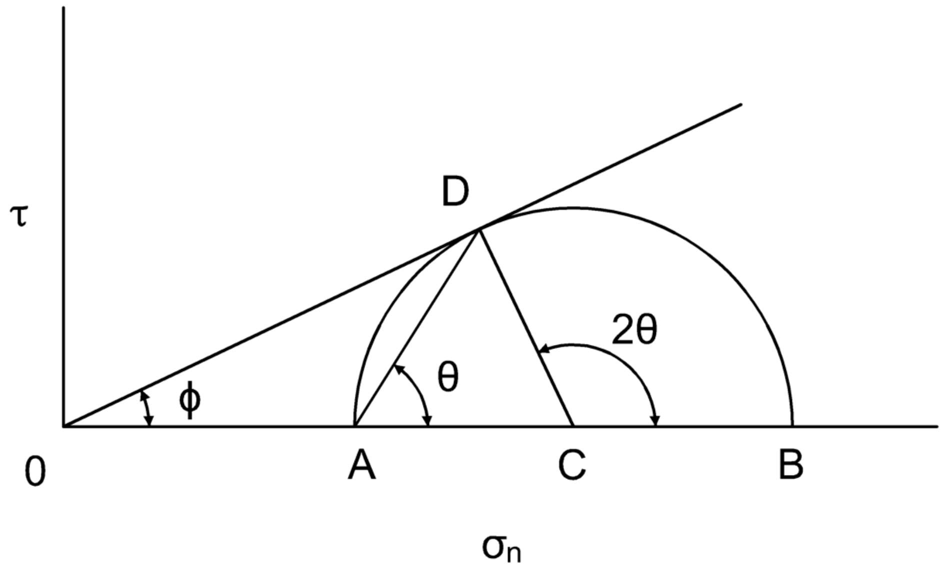


Figure 3.12. Relationship between ϕ and θ.

3.4 Cohesion

It is possible to make a vertical cut in silts and clays and for this cut to remain standing, unsupported, for some time. This cannot be done with a dry sand which, on removal of the cutting implement, will slump until its slope is equal to an angle known as the *angle of repose*. In silts and clays, therefore, some other factor must contribute to shear strength. This factor is called cohesion and results from the state of drainage within the soil mass: water in a cohesive soil cannot drain quickly and so the soil is claimed to be in an undrained state, and the undrained cohesion is providing strength to the soil. In terms of the Mohr diagram this means that the strength envelope for the soil, for undrained conditions, no longer goes through the origin but intercepts the shear stress axis (see Figure 3.13). The value of the intercept, to the same scale as σ_n, gives a measure of the unit cohesion available and is given the symbol c_u. In Figure 3.13, as the soil is in an *undrained* state, the angle of shearing resistance, ϕ_u is equal to zero.

Figure 3.13. A cohesive soil subjected to undrained conditions and zero total normal stress will still exhibit a shear stress, c_u.

3.5 Coulomb's law of soil shear strength

It can be seen that the shear resistance offered by a particular soil is made of the two components of friction and cohesion. Frictional resistance does not have a constant value but varies with the value of normal stress acting on the shear plane whereas cohesive resistance has a constant value which is independent of the value of σ_n. In 1776 Coulomb suggested that the equation of the strength envelope of a soil could be expressed by the straight line equation

where

τ_f = shear stress at failure, i.e. the shear strength
c = unit cohesion
σ = total normal stress on failure plane
ϕ = angle of shearing resistance.

The equation gave satisfactory predictions for sands and gravels, for which it was originally intended, but it was not so successful when applied to silts and clays. The reasons for this are now well known and are that the drainage conditions under which the soil is operating together with the rate of the applied loading have a considerable effect on the amount of shearing resistance the soil will exhibit. None of this was appreciated in the 18th century, and this lack of understanding continued more or less until 1925 when Terzaghi published his theory of effective stress.

Note: It should be noted that there are other factors that affect the value of the angle of shearing resistance of a particular soil. They include the effects of such items as the amount of friction between the soil particles, the shape of the particles and the degree of interlock between them, the density of the soil and its previous stress history.

Effective stress, σ'

If a soil mass is subjected to the action of compressive forces applied at its boundaries then the stresses induced within the soil at any point can be estimated by the theory of elasticity, described in Section 1. For most soil problems, estimations of the values of the principal stresses σ_1, σ_2 and σ_3 acting at a particular point are required. Once these values have been obtained, the values of the normal and shear stresses acting on any plane through the point can be computed.
At any point in a saturated soil each of the three principal stresses consists of two parts:

(1) u, the neutral pressure acting in both the water *and* in the solid skeleton in every direction with equal intensity;
(2) the balancing pressures $(\sigma_1 - u)$, $(\sigma_2 - u)$ and $(\sigma_3 - u)$.

As explained in Section 1.3, Terzaghi's theory is that only the balancing pressures, i.e. the effective principal stresses, influence volume and strength changes in saturated soils:

Principal effective stress = Principal normal stress – Pore water pressure

i.e.

$$\sigma'_1 = \sigma_1 - u, \text{ etc.}$$

3.6 Modified Coulomb's law

Shear strength depends upon effective stress and not total stress. Coulomb's equation must therefore be modified in terms of effective stress and becomes:

where

c' = unit cohesion, with respect to effective stresses
σ' = effective normal stress acting on failure plane
ϕ' = angle of shearing resistance, with respect to effective stresses.

It is seen that, dependent upon the loading and drainage conditions, it is possible for a clay soil to exhibit purely frictional shear strength (i.e. to act as a '$c' = 0$' or 'ϕ'' soil), when it is loaded under drained conditions or to exhibit only cohesive strength (i.e. to act as a '$\phi = 0$' or 'c_u' soil) when it is loaded under undrained conditions. Obviously, at an interim stage the clay can exhibit both cohesion and frictional resistance (i.e. to act as a '$c' - \phi'$' soil).

3.7 The Mohr–Coulomb yield theory

Over the years various yield theories have been proposed for soils. The best known ones are: the Tresca theory, the von Mises theory, the Mohr–Coulomb theory and the critical state theory.

The Mohr–Coulomb theory does not consider the effect of strains or volume changes that a soil experiences on its way to failure; nor does it consider the effect of the intermediate principal stress, σ_2. Nevertheless satisfactory predictions of soil strength are obtained and, as it is simple to apply, the Mohr–Coulomb theory is widely used in the analysis of most practical problems which involve soil strength.

The Mohr strength theory is really an extension of the Tresca theory, which in turn was probably based on Coulomb's work – hence the title. The theory assumes that the difference between the major and minor principal stresses is a function of their sum, i.e. $(\sigma_1 - \sigma_3) = f(\sigma_1 + \sigma_3)$. Any effect due to σ_2 is ignored.
The Mohr circle has been discussed earlier in this section and a typical example of a Mohr circle diagram is shown in Figure 3.14. The intercept on the shear stress axis of the strength envelope is the intrinsic pressure, i.e. the strength of the material when under zero normal stress. As we know, this intercept is called cohesion in soil mechanics.

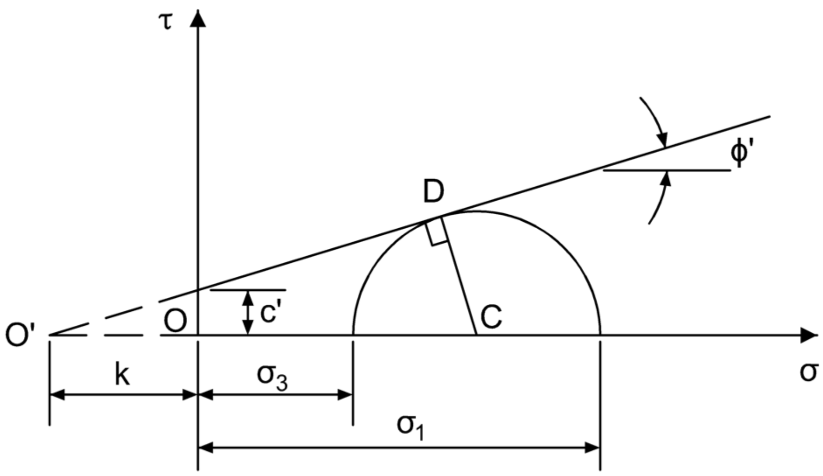


Figure 3.14. Mohr circle diagram.

In Figure 3.14:

$$sin\phi = \frac{DC}{O'C} = \frac{\frac{1}{2}(\sigma_1 - \sigma_3)}{k + \frac{1}{2}(\sigma_1 + \sigma_3)} = \frac{\sigma_1 - \sigma_3}{2k + \sigma_1 + \sigma_3}$$

Hence

$$\sigma_1 - \sigma_3 = 2k \sin\phi + (\sigma_1 + \sigma_3) \sin\phi$$

Now

$$k = c \cot\phi$$

$$\Rightarrow \quad (\sigma_1 - \sigma_3) = 2c \cos\phi + (\sigma_1 + \sigma_3) \sin\phi$$

which is the general form of the Mohr–Coulomb theory.

The equation can be expressed in terms of either total stress (as shown) or effective stress:

$$\sigma'_1 - \sigma'_3 = 2c' \cos\phi' + (\sigma'_1 + \sigma'_3) \sin\phi'$$

3.8 Determination of the shear strength parameters

The shear strength of a soil is controlled by the effective stress that acts upon it and it is therefore obvious that a geotechnical analysis involving the operative strength of a soil should be carried out in terms of the effective stress parameters ϕ' and c′. This is the general rule and, as you might expect, there is at least one exception. The case of a fully saturated clay subjected to undrained loading is more appropriately analysed using total stress values and c_u than by an effective stress approach.

It is seen therefore that both the values of the undrained parameter c_u, and of the drained parameters, ϕ' and c' are generally required. They are obtained from the results of laboratory tests carried out on representative samples of the soil with loading and drainage conditions approximating to those in the field where possible. The tests in general use are the direct shear box test, the triaxial test and the unconfined compression test, an adaptation of the triaxial test.

3.8.1 The direct shear box test

The apparatus consists of a brass box, split horizontally at the centre of the soil specimen. The soil is gripped by perforated metal grilles, behind which porous discs can be placed if required to allow the sample to drain (see Figure 3.15).

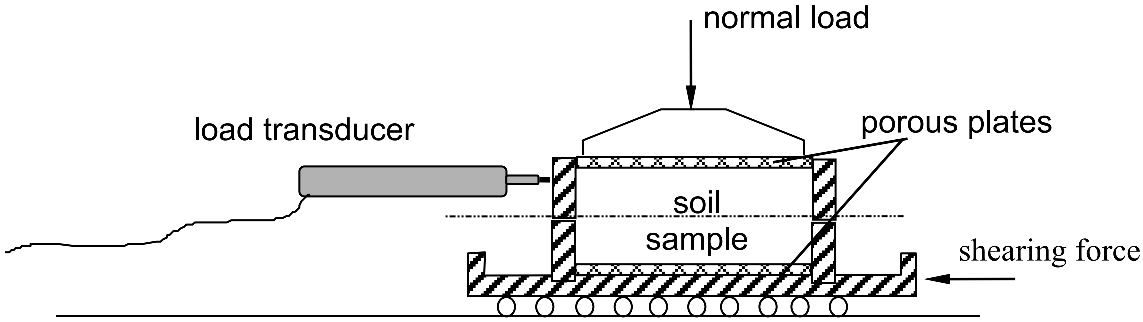


Figure 3.15. Shear box assembly.

The usual plan size of the sample is 60 × 60 mm^2, but for testing granular materials such as gravel or stony clay it is necessary to use a larger box, generally 300 × 300 mm^2, although even greater dimensions are sometimes used.

A vertical load is applied to the top of the sample by means of weights. As the shear plane is predetermined in the horizontal direction the vertical load is also the normal load on the plane of failure. Having applied the required vertical load a shearing force is gradually exerted on the box from an electrically driven screw jack. The shear force is measured by means of a load transducer connected to a computer.

By means of additional transducers (fixed to the shear box) it is possible to determine both the horizontal and the vertical strains of the test sample at any point during shear:

$$\text{Horizontal strain (\%)} = \frac{\text{Movement of box}}{\text{Length of sample}}$$

The load reading is taken at fixed horizontal displacements, and failure of the soil specimen is indicated by a sudden drop in the magnitude of the reading or a levelling off in successive readings. In most cases the computer plots a graph of the shearing force against horizontal strain as the test progresses. Failure of the soil is visually apparent from a turning point in the graph (for dense soils) or a levelling off of the graph (for loose soils).

The apparatus can be used for both drained and undrained tests. Although undrained tests on silts and sands are not possible (because drainage will occur) the test procedure can be modified to maintain constant volume conditions during shear by adjusting the hanger weights. This procedure, in effect, gives an undrained state.

A sand can be tested either dry or saturated. If dry there will be no pore water pressures and the intergranular pressure will equal the applied stress. If the sand is saturated, the pore water pressure will be zero due to the quick drainage, and the intergranular pressure will again equal the applied stress.

From the results obtained, a graph is plotted of the shear stress against displacement of the lower half of the box (Figure 3.16).

Figure 3.16. Shear box: Shear stress against displacement.

The maximum of the graph occurs at the point of failure. The shear stress at this stage (τ_f) is the shear stress at failure, for the particular normal stress applied.

With a series of tests with different applied normal loads, we can establish ϕ.

To calculate τ_f:

$$\tau_f = \frac{\text{maximum shear force at failure (kN)}}{\text{cross-sectional area of shear box (m}^2\text{)}}$$

To calculate σ_n:

$$\sigma_n = \frac{\text{normal load applied (kN)}}{\text{cross-sectional area of shear box (m}^2\text{)}}$$

Then, using each pair of results, we can plot a graph of τ_f against σ_n (Figure 3.17).

Figure 3.17. Shear box: Shear stress against normal stress.

If both axes are drawn to the same scale we can measure ϕ using a protractor.

Example 3.2: Shear box test (i)

Drained shear box tests were carried out on a series of soil samples with the following results:

Test no.	Total normal Stress (kPa)	Total shear stress at failure (kPa)
1	100	98
2	200	139
3	300	180
4	400	222

Determine the cohesion and the angle of friction of the soil, with respect to effective stress.

Solution:

In this case both the normal and the shear stresses at failure are known, so there is no need to draw stress circles and the four failure points may simply be plotted. These points must lie on the strength envelope and the best straight line through the points will establish it (Figure 3.18).

Figure 3.18. Example 3.2

Example 3.3: Shear box test (ii)

The following results were obtained from a drained shear box test carried out on a set of undisturbed soil samples:

Normal load (kN)	0.2	0.4	0.8
Strain (%)	Shearing force (N)		
0	0	0	0
1	21	33	45
2	46	72	101
3	70	110	158
4	89	139	203
5	107	164	248
6	121	180	276
7	131	192	304
8	136	201	330
9	138	210	351
10	138	217	370

11	137	224	391
12	136	230	402
13		234	410
14		237	414
15		236	416
16			417
17			417
18			415

The cross-sectional area of the box was 3600 mm^2 and the test was carried out in a fully instrumented shear box apparatus.

Determine the strength parameters of the soil in terms of effective stress.

Solution:
The plot of load transducer readings against strain is shown in Figure 3.19.

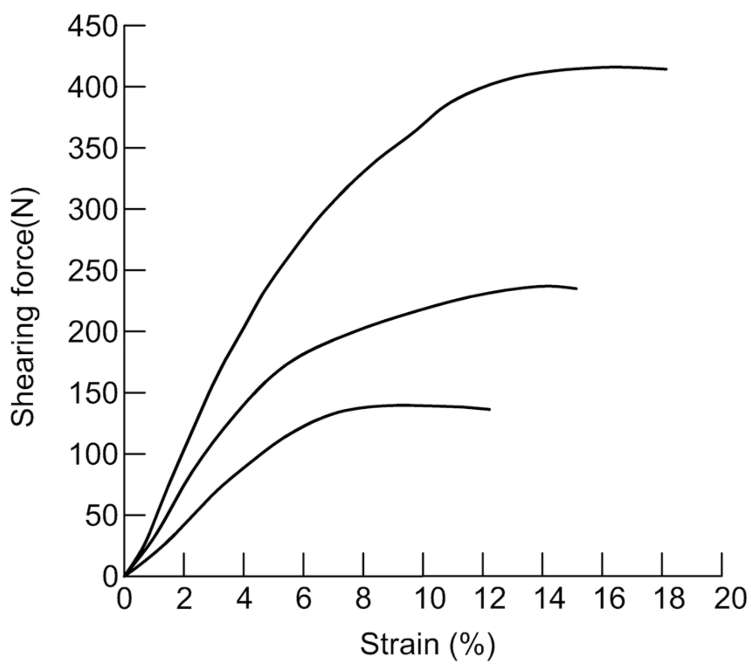


Figure 3.19. Example 3.3: Shearing force against strain.

From this plot the maximum readings for normal loads of 0.2, 0.4 and 0.8 kN were 138, 237 and 417 N.

For this particular case the maximum readings could obviously have been obtained directly from the tabulated results, but viewing the plots is sometimes useful to demonstrate whether one of the sets of readings differs from the other two.

The shear stress at each maximum load reading is calculated.

Normal load (kN)	Normal stress (kPa)	Shear force (N)	Shear stress (kPa)
0.2	$\frac{0.2 \times 10^6}{3600} = 56$	138	$\frac{0.138 \times 10^6}{3600} = 38$
0.4	111	237	66
0.8	222	417	116

The plot of shear stress to normal stress is given in Figure 3.20.

Figure 3.20. Example 3.3: shear stress against normal stress.

3.8.2 The effect of density on shear strength

The initial density of a soil tested in a shear box determines the way in which the soil will behave during shearing. If two samples of the same soil are tested in the shear box, one sample placed in a loose state and the other compacted to a higher density, the plots of both measured shear stress and vertical displacement to horizontal displacement will be of the forms shown in Figures 3.21a and c. The strength envelopes (shear stress against normal stress) are shown in Figure 3.21b.

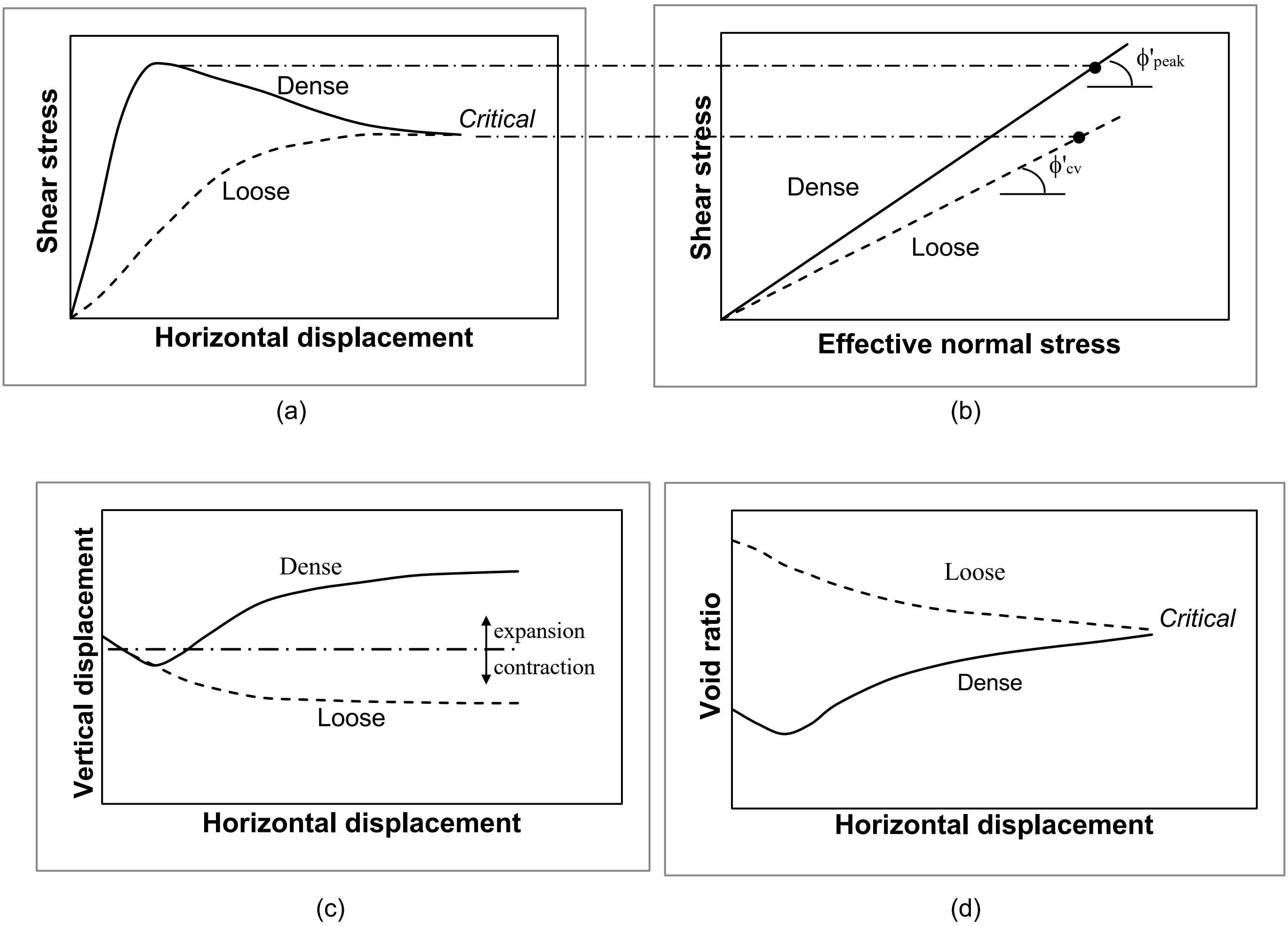


Figure 3.21. Shear box results on loose and dense samples of the same soil: (a) shear stress v's horizontal displacement; (b) shear stress v's normal stress; (c) vertical displacement v's horizontal displacement; (d) void ratio v's horizontal displacement.

If the movement of pore water is restricted, the shear strength of the sand will be affected: the dense sand will have negative pore pressures induced in it, causing an increase in shear strength, while a loose sand will have positive pore pressures induced with a corresponding reduction in strength (Figures 3.21a and b). A practical application of this effect occurs when a pile is driven into sand, the load on the sand being applied so suddenly that, for a moment, the water it contains has no time to drain away.

It is seen from Figure 3.21c that as the shear box test progresses, the initially dense soil expands in volume (dilates) – indicted by the increase in vertical displacement of the sample – until a steady volume is reached. In contrast, the initially loose soil contracts until some steady state of volume is reached. This steady state of volume (referred to as the critical volume) is found to be the same value for both samples and, as shearing continues, the volume remains constant.

The corresponding density of the soil at this volume is known as the critical density.

The critical volume is not evident in Figure 3.21c, but if we consider the changes in void ratio that are happening during shearing, we can identify the critical volume (Figure 3.21d). Here the loose sample starts with a high void ratio, and the dense sample starts with a lower one. As the test progresses, the void ratios converge thus indicating the presence of the critical volume, and the existence of conditions which create a state known as the critical state.

From Figure 3.21 we see that both the ultimate shear strength and the ultimate void ratio are the same for both samples. Thus, it is seen that soils will reach the critical state when undergoing continuous shearing at constant stress and constant volume.

3.8.3 The triaxial test

As its name implies this test (Figure 3.22) subjects the soil specimen to three compressive stresses at right angles to each other, one of the three stresses being increased until the sample fails in shear. Its great advantage is that the plane of shear failure is not predetermined as in the shear box test.

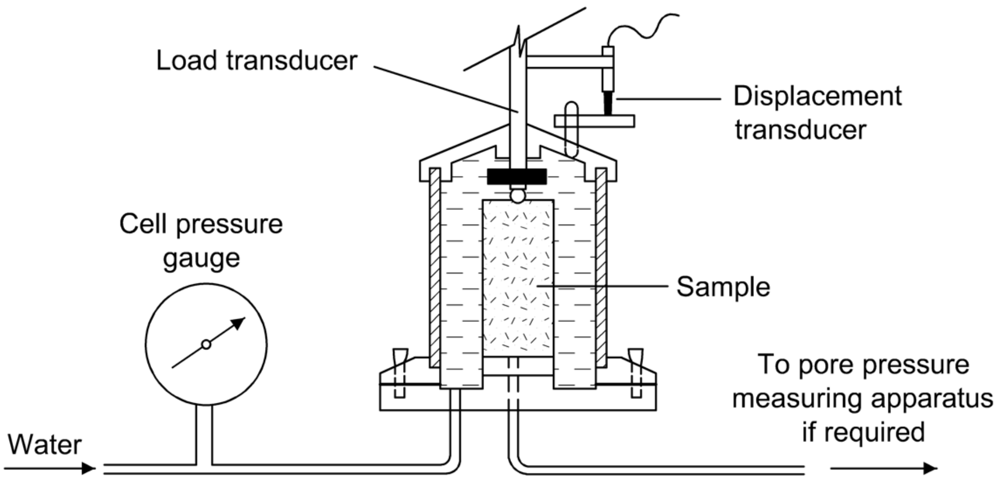


Figure 3.22. The triaxial apparatus.

The soil sample test is cylindrical with a height equal to twice its diameter. In the UK the usual sizes are 76 mm high by 38 mm diameter and 200 mm high by 100 mm diameter.

The test sample is first placed on the pedestal of the base of the triaxial cell and a loading cap is placed on its top. A thin rubber membrane is then placed over the sample, including the pedestal and the loading cap, and made watertight by the application of tight rubber ring seals, known as 'O' rings, around the pedestal and the loading cap.

The upper part of the cell, which is cylindrical and generally made of Perspex, is next fixed to the base and the assembled cell is filled with water. The water is then subjected to a predetermined value of pressure, known as the cell pressure, which is kept constant throughout the length of the test. It is this water pressure that subjects the sample to an all-round pressure.

The additional axial stress is created by an axial load applied through a load transducer, in a similar way to that in which the horizontal shear force is applied in the shear box apparatus. By the action of an electric motor the axial load is gradually increased at a constant rate of strain and as the axial load is applied the sample suffers continuous compressive deformation. The amount of this vertical deformation is obtained from a deformation transducer.

Throughout the test, until the sample fails, readings of the deformation transducer and corresponding readings of axial load are taken. With this data the computer plots the variation of the axial load on the sample against its vertical strain.

Determination of the additional axial stress

From the load transducer it is possible at any time during the test to determine the additional axial load that is being applied to the sample.

During the application of this load the sample experiences shortening in the vertical direction with a corresponding expansion in the horizontal direction.

This means that the cross-sectional area of the sample varies, and it has been found that very little error is introduced if the cross-sectional area is evaluated on the assumption that the volume of the sample remains unchanged during the test. In other words the cross-sectional area is found from:

$$\text{Cross-sectional area} = \frac{\text{Volume of sample}}{\text{Original length} - \text{Vertical deformation}}$$

Principal stresses

The intermediate principal stress, σ_2, and the minor principal stress, σ_3, are equal and are the radial stresses caused by the cell pressure, p_c. The major principal stress, σ_1, consists of two parts: the cell water pressure acting on the ends of the sample and the additional axial stress from the load transducer, q. To ensure that the cell pressure acts over the whole area of the end cap, the bottom of the plunger is drilled so that the pressure can act on the ball seating.

From this we see that the triaxial test can be considered as happening in two stages (Figure 3.23), the first being the application of the cell water pressure (p_c, i.e. σ_3), while the second is the application of a deviator stress (q, i.e. $[\sigma_1 - \sigma_3]$).

A set of at least three samples is tested. The deviator stress is plotted against vertical strain and the point of failure of each sample is obtained. The Mohr circles for each sample are then drawn and the best common tangent to the circles is taken as the strength envelope (Figure 3.24). A small curvature occurs in the strength envelope of most soils, but this effect is slight and for all practical work the envelope can be taken as a straight line.

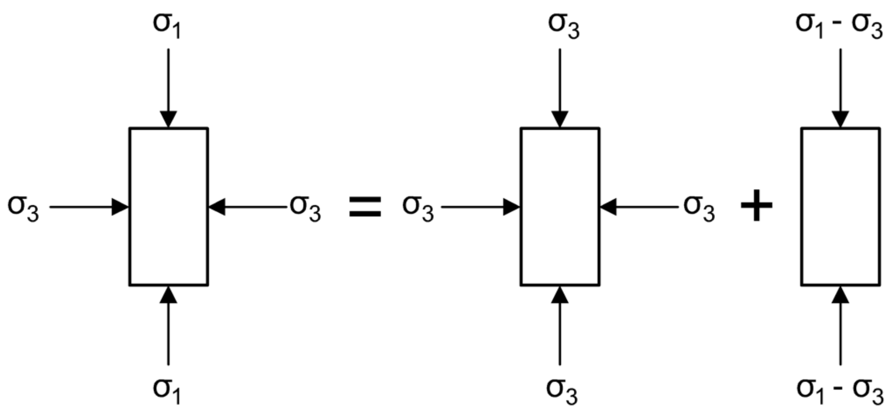


Figure 3.23. Stresses in the triaxial test.

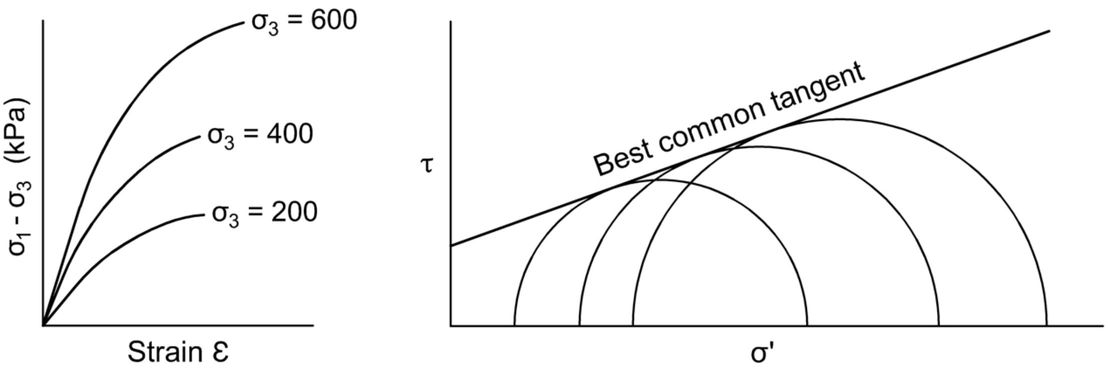


Figure 3.24. Typical triaxial test results.

Types of failure

Not all soil samples will fail in pure shear; there are generally some barrelling effects as well. In a sample that fails completely by barrelling there is no definite failure point, the deviator stress simply increasing slightly with strain. In this case an arbitrary value of the failure stress is taken as the stress value at 20 % strain (see Figure 3.25).

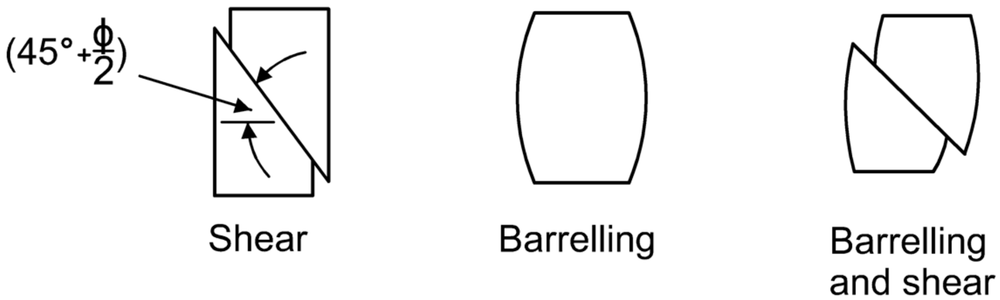


Figure 3.25. Types of failure in the triaxial test.

3.8.4 The unconfined compression test

In this test (Figure 3.26) no all-round pressure is applied to the soil specimen and the results obtained give a measure of the *unconfined* compressive strength of the soil. The test is only applicable to cohesive soils and, although not as popular as the triaxial test, it is used where a rapid result is required. An electric motor within the base unit drives the platen supporting the specimen upwards and the load carried by the soil is recorded by the load transducer. The vertical strain is recorded by a displacement transducer and the load–displacement curve is plotted on a PC connected to the system. The load and strain readings at failure are used to give a direct measure of the unconfined compressive strength of the soil.

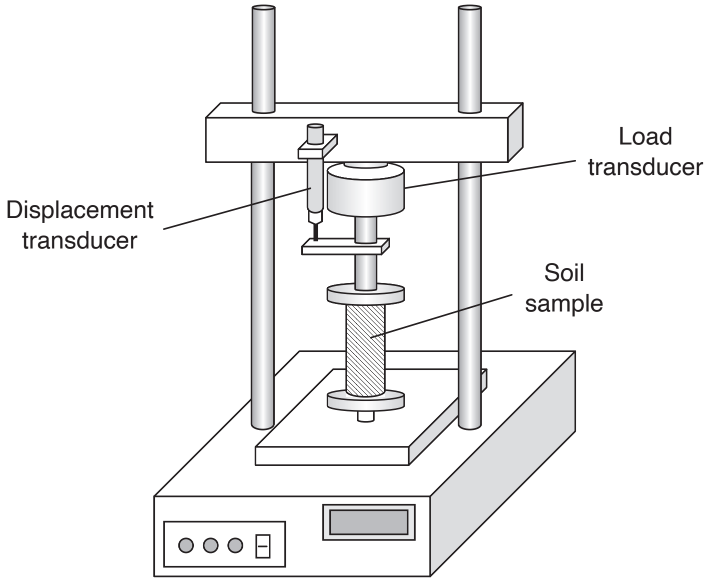


Figure 3.26. The unconfined compression test.

3.9 Determination of the shear strength parameters from triaxial testing

3.9.1 Determination of the total stress parameter c_u

The undrained shear test

The simplest method to determine values for the total stress parameter c_u is to subject suitable samples of the soil to this test. In the test the soil sample is prevented from draining during shear and is therefore sheared immediately after the application of the normal load (in the shear box) or immediately after the application of the cell pressure (in the triaxial apparatus). A sample can be tested in 15 minutes or less, so that there is no time for any pore pressures developed to dissipate or to distribute themselves evenly throughout the sample.

Measurements of pore water pressure are therefore not possible and the results of the test can only be expressed in terms of total stress.

The unconfined compression apparatus is only capable of carrying out an undrained test on a clay sample with no radial pressure applied. The test takes about a minute. Undrained tests on silts and sands are not possible in the shear box.

Example 3.4: Quick undrained triaxial test

A sample of clay was subjected to an undrained triaxial test with a cell pressure of 100 kPa and the additional axial stress necessary to cause failure was found to be 188 kPa. Assuming that $\phi_u = 0°$, determine the value of additional axial stress that would be required to cause failure of a further sample of the soil if it was tested undrained with a cell pressure of 200 kPa.

Solution:

The first step is to draw the stress circle that represents the conditions for the first test, i.e. $\sigma_3 = 100$ kPa and $\sigma_1 = 188 + 100 = 288$ kPa. The circle is shown in Figure 3.28 and the strength envelope representing the condition that $\phi_u = 0°$ is now drawn as a horizontal line tangential to the stress circle. The next step is to draw the stress circle with $\sigma_3 = 200$ kPa and tangential to the strength envelope. Where this circle cuts the normal stress axis it gives the value of σ_1, which is seen to be 388 kPa.

Figure 3.28. Example 3.4.

3.9.2 Determination of the effective stress parameters c' and ϕ'

There are two relevant triaxial tests.

i) The drained test

A porous disc is placed on the pedestal before the test sample is placed in position so that water can drain out from the soil. The triaxial cell is then assembled, filled with water and pressurised. The cell pressure creates a pore water pressure within the soil sample and the apparatus is left until the sample has consolidated, i.e. until the pore water pressure has been dissipated by water seeping out through the porous disc into the pore pressure measuring device (see Figure 3.29). This process usually takes about a day but is quicker if a porous disc is installed beneath the loading cap and joined to the pedestal disc by connecting strips of vertical filter paper placed on the outside of the sample but within the rubber membrane. During this consolidation stage the pore pressure is monitored so that the point when full consolidation has been reached can be identified.

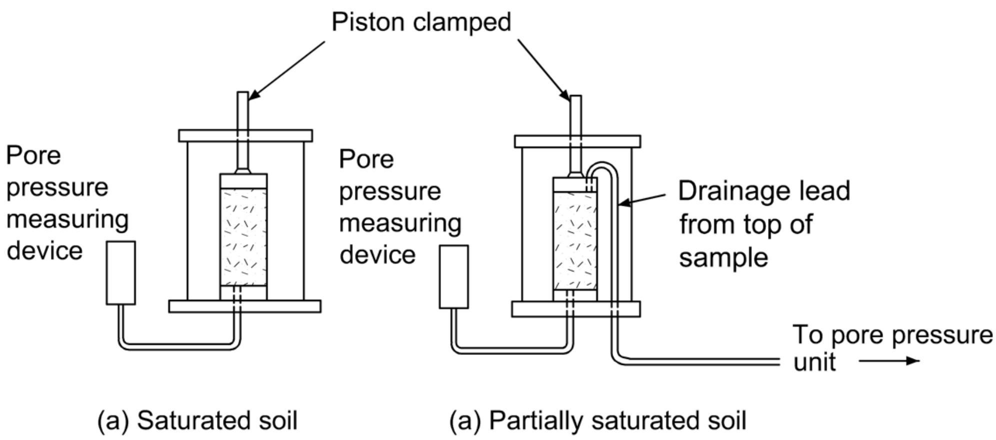


Figure 3.29. Alternative arrangements for consolidation of test samples.

An alternative method (sometimes preferable with a partially saturated soil) is to allow drainage from one end of the sample and to connect a second pore pressure measuring device to the other. When the pore water pressure reaches zero the sample is consolidated.

When consolidation has been completed the sample is sheared by applying a deviator stress at such a low rate of strain that any pore water pressures induced in the sample have time to dissipate through the porous discs. In this test the pore water pressure is therefore always zero and the effective stresses are consequently equal to the applied stresses.

The main drawback of the drained test is the length of time it takes, with the attendant risk of testing errors: an average test time for a clay sample is about three days but with some soils a test may last as long as two weeks.

Example 3.5: Drained triaxial test

A series of drained triaxial tests were performed on a soil. Each test was continued until failure and the effective principal stresses for the tests were:

Test no.	σ'_3 (kPa)	σ'_1 (kPa)
1	200	570
2	300	875
3	400	1162

Plot the relevant Mohr stress circles and hence determine the strength envelope of the soil with respect to effective stress.

Solution:

The Mohr circle diagram is shown in Figure 3.30. The circles are drawn first and then, by constructing the best common tangent to these circles, the strength envelope is obtained.

In this case it is seen that the soil is cohesionless as there is no cohesive intercept.

By measurement, $\phi' = 29°$.

Figure 3.30. Example 3.5.

ii) The consolidated undrained test

This is the most common form of triaxial test used in soils laboratories to determine c' and ϕ'. It has the advantage that the shear part of the test can be carried out in only two to three hours.

The sample is consolidated exactly as for the drained test, but at this stage the drainage connection is shut off and the sample is sheared under undrained conditions. The application of the deviator stress induces pore water pressures (which are measured), and the effective deviator stress is then simply the total deviator stress less the pore water pressure.

Although the sample is sheared undrained, the rate of shear must be slow enough to allow the induced pore water pressures to distribute themselves evenly throughout the sample. For most soils a strain rate of 0.05 mm/min is satisfactory, which means that the majority of samples can be sheared in under three hours.

Note: With respect to total stress, the strength parameter is c_u (because $\phi = 0$) while with respect to effective stresses the strength parameters are c' and ϕ'.

Testing with back pressures

It should be noted that, with some soils, the reduction of the pore water pressure to atmospheric during the consolidation stage of a triaxial test on a saturated soil sample can cause air dissolved in the water to come out of solution. If this happens, the sample is no longer fully saturated and this can affect the results obtained during the shearing part of the test.

To maintain a state of occlusion in the pore water, i.e. the state where air can no longer exist in a free state but only in the form of bubbles, its pressure can be increased by applying a pressure (known as a back pressure) to the water in the drainage line. (The soil water can still drain from the sample.) The back pressure ensures that air does not come out of solution and, by applying the same increase in pressure to the value of the cell pressure, the effective stress situation is unaltered.

The technique can also be used to create full saturation during the consolidation and shearing of partially saturated natural or remoulded soils for both the drained and consolidated undrained triaxial tests. In these cases, back pressure values often as high as 650 kPa are necessary in order to achieve full saturation.

Example 3.6: Consolidated undrained triaxial test (i)

The following results were obtained from a series of consolidated undrained triaxial tests carried out on undisturbed samples of a compacted soil:

Cell pressure (kPa)	Additional axial load at failure (N)
200	342
400	388
600	465

Each sample, originally 76 mm long and 38 mm in diameter, experienced a vertical deformation of 5.1 mm.

Draw the strength envelope and determine the Coulomb equation for the shear strength of the soil.

Solution:

Cell pressure σ_3 (kPa)	Deviator stress $(\sigma_1 - \sigma_3)$ (kPa)	Major principal stress σ_1 (kPa)
200		
400		
600		

Figure 3.31. Example 3.6.

Example 3.7: Consolidated undrained triaxial test (ii)

A series of undisturbed samples from a normally consolidated clay was subjected to consolidated undrained tests.

The results were:

Cell pressure (kPa)	Deviator stress at failure (kPa)	Pore water pressure at failure (kPa)
200	118	110
400	240	220
600	352	320

Plot the shear strength envelope of the soil in effective stress terms.

Solution:

Figure 3.32. Example 3.7.

3.9.3 Behaviour of soils under shear

We saw in Section 3.8.2 that the behaviour of granular soil under shear depends on the initial density of the soil. Before looking at the application of shear strength in practice, it is useful to introduce the following definitions.

- *Overburden*: The overburden pressure at a point in a soil mass is simply the weight of the material above it. The effective overburden is the pressure from this material less the pore water pressure due to the height of water extending from the point up to the water table.

- *Normally consolidated clay*: Clay which, at no time in its history, has been subjected to pressures greater than its existing overburden pressure.

- *Overconsolidated clay*: Clay which, during its history, has been subjected to pressures greater than its existing overburden pressure. One cause of overconsolidation is the erosion of material that once existed above the clay layer. Boulder clays are overconsolidated, as the many tons of pressure exerted by the mass of ice above them has been removed.

- *Preconsolidation pressure*: The maximum value of pressure exerted on an overconsolidated clay before the pressure was relieved.

- *Overconsolidation ratio*: The ratio of the value of the effective preconsolidation pressure to the value of the presently existing effective overburden pressure. A normally consolidated clay has an OCR = 1.0 whilst an overconsolidated clay has an OCR > 1.0.

SECTION 4 – LATERAL EARTH PRESSURE

- - end of free sample - -