Our Instructional Approach

Data Ignites the Process, the CRA Fuels the Learning

Numeracy Consultants LLC (Updated May 2025)

Introduction

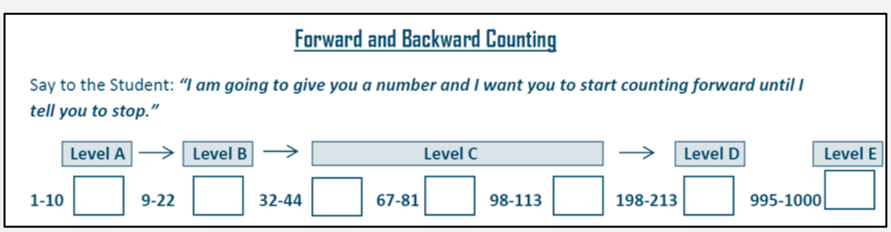
The Primary Numeracy Assessment is a groundbreaking, standards-based tool developed by Numeracy Consultants to support educators in identifying key numeracy standards where students may struggle within their state-mandated curriculum. Built on extensive research and the latest understanding of numeracy development, this assessment filters through a wide array of standards to focus on those most critical to early mathematical success.

Covering a broad spectrum of foundational numeracy domains, the Primary Numeracy Assessment targets essential skills such as forward and backward counting, skip counting by tens, the strategic use of addition and subtraction, place value understanding, numeral recognition, and the concept of part-part-whole relationships. These core competencies form the basis of a student's long-term mathematical development, supporting both academic progress and real-world problem-solving abilities.

At the heart of our instructional framework, the Primary Numeracy Assessment provides the data needed to close learning gaps and refine teaching strategies. Its insights drive a focused, evidence-based approach to instruction, empowering educators to respond to students' individual needs with accuracy and confidence.

Forward Counting: A Precursor to Addition

Forward counting serves as an essential stepping stone toward understanding numbers and performing basic mathematical operations. It is a skill that typically develops before children can confidently tackle more complex mathematical concepts such as addition. Research by Fuson and Hall (1983) highlights the importance of forward counting as a foundation for addition and subtraction. They argue that a strong ability to count forward contributes to children's early numerical sense, which is vital for understanding mathematical relationships.



Forward and Backward Counting

Say to the Student: ***"I am going to give you a number and I want you to start counting forward until I tell you to stop."***

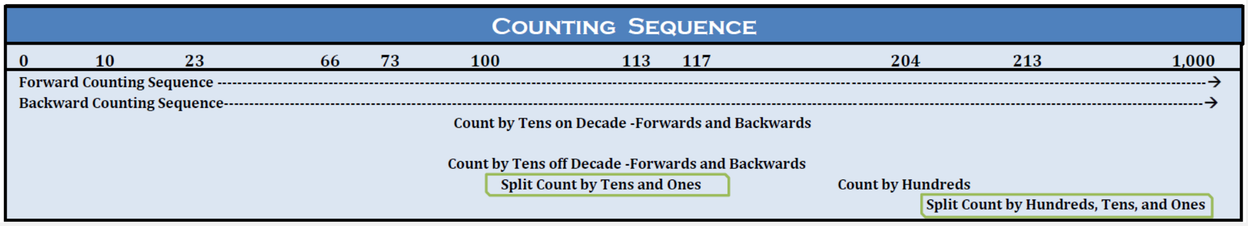
Level A → Level B → Level C → Level D Level E

1-10 ☐ 9-22 ☐ 32-44 ☐ 67-81 ☐ 98-113 ☐ 198-213 ☐ 995-1000 ☐

Primary Numeracy Assessment

Research Findings

Fuson and Hall's findings suggest a close link between forward counting fluency and a child's ability to conceptualize addition. Fluency in counting forward helps children internalize numerical sequences, an understanding that underpins the additive process, which involves moving forward along a number line (Fuson & Hall, 1983).



Primary Numeracy Framework

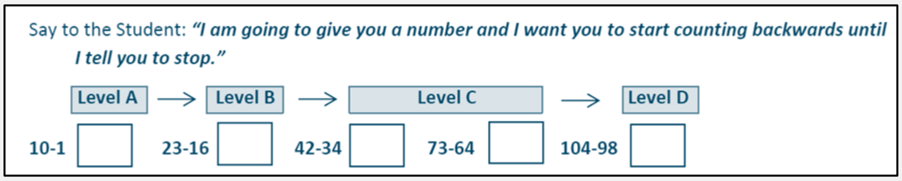
Douglas Steffe's work (1992) further supports the cognitive significance of forward counting in the development of number sense. Steffe emphasizes that forward counting is not simply a memorization task, but a mental activity that supports children in recognizing numerical relationships and patterns. As such, forward counting becomes a key building block for numeracy, laying the groundwork for later mathematical problem-solving (Steffe, 1992).

Backward Counting: A Foundational Support for Subtraction

Backward counting, often overshadowed by its more frequently practiced counterpart, forward counting, is a foundational skill in numeracy development. This paper explores the significance of backward counting, emphasizing its role in supporting children's understanding of the inverse relationship between addition and subtraction. Drawing on the work of Baroody, Gannon, and Douglas Steffe, we examine how proficiency in backward counting contributes to subtraction competence.

Backward Counting and the Inverse Relationship

Complementary to forward counting, backward counting helps children understand that subtraction is the inverse of addition. Baroody and Gannon (1984) highlighted a positive correlation between children's ability to count backward and their success in subtraction tasks. By learning to reverse the counting sequence, children begin to conceptualize subtraction as "taking away" or "moving back" on the number line, key ideas in early arithmetic.



Primary Numeracy Assessment

Research Findings

Baroody and Gannon's research suggests that children who are proficient in backward counting are more likely to understand and apply subtraction strategies effectively. Their work emphasized that backward counting is not merely a memorized routine, but a tool that enables children to navigate subtraction problems with greater confidence (Baroody & Gannon, 1984).

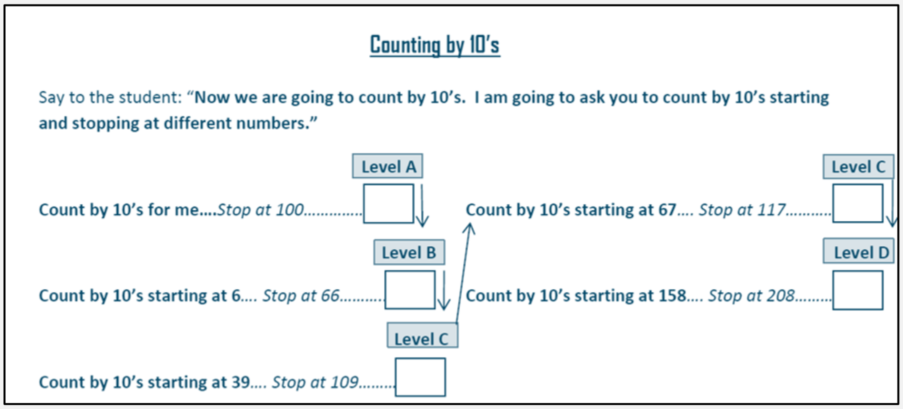
Douglas Steffe's developmental research (1994) also underscores the role of counting, including backward counting, in building number sense. Steffe found that children who construct an understanding of number sequences, both forward and backward, tend to perform better in arithmetic tasks, particularly those involving subtraction. His work emphasizes that backward counting supports the conceptual development of subtraction as an operation that undoes addition, thereby strengthening mathematical fluency.

Conclusion

Backward counting is a crucial, though sometimes underemphasized, skill in early numeracy development. Research by Baroody, Gannon, and Steffe shows that it plays a significant role in helping children understand subtraction and its inverse relationship with addition. When children can confidently count backward, they are better equipped to solve subtraction problems and reason mathematically. For this reason, both educators and parents should prioritize backward counting as a meaningful component of early math instruction.

Counting by Tens: A Foundation for Place Value Understanding

Counting by tens is a foundational numeracy skill that significantly supports children's understanding of place value, an essential concept in early mathematics. Place value refers to the value of digits in a number based on their position, and skip-counting by tens helps children intuitively grasp this structure by reinforcing grouping and numerical magnitude. Research from Fuson, Baroody, Steffe, White, and Wright consistently underscores the importance of this skill in the development of number sense.



Primary Numeracy Assessment

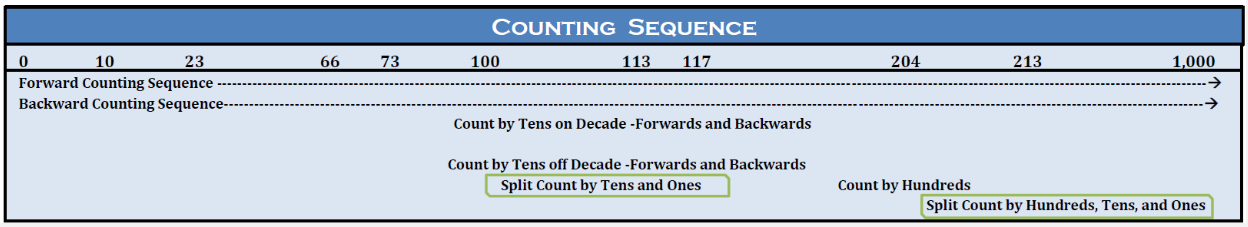
Research Findings

Fuson and Briars (1990) emphasize that counting by tens provides children with a structured method for managing larger quantities, while also introducing them to the concept that each digit in a number represents a specific power of ten. This repeated exposure lays a critical foundation for understanding how numbers are composed and decomposed in our base-ten system.

Building on this, Baroody and Ginsburg (1983) highlight how skip-counting supports children's understanding of grouping, particularly the grouping of quantities into sets of ten. This understanding is central to grasping place value, where each digit's position corresponds to a multiple of ten. By counting in tens, children begin to view numbers as composed of units, tens, hundreds, and beyond.

Steffe (1992) extends this perspective by arguing that counting by tens is not merely a rote procedure, but a cognitive process that helps children build deeper numerical relationships. As they count in tens, children begin to see how the placement of digits contributes to the overall value of numbers, enhancing their conceptual understanding of place value and its role in arithmetic operations.

Similarly, Wright (1984) stresses that counting by tens serves as a bridge between basic counting strategies and more structured mathematical reasoning. His research shows that students who can skip-count effectively are better prepared to grasp base-ten representations and apply regrouping strategies in addition and subtraction.



Primary Numeracy Framework

Finally, White (1982) and colleagues demonstrated that counting by tens helps children connect symbolic numerals with real quantities. This ability to link the abstract and the concrete is essential for transitioning from counting physical objects to understanding numerical magnitude and the structure of the base-ten system at a more advanced level.

Conclusion

Counting by tens plays a central role in numeracy development and serves as a critical stepping stone to understanding place value. Research across decades affirms its value in fostering grouping, symbol-quantity correspondence, and structured numerical thinking. Educators and caregivers should intentionally include skip-counting activities in early math instruction to support children's deeper understanding of the base-ten system and prepare them for more complex operations.

Arithmetic Thinking Strategies: From Counting to Conceptual Fluency

Early arithmetic, particularly addition and subtraction, forms the bedrock of a child’s mathematical development. As children build fluency, they adopt various strategies to solve problems, ranging from simple counting to more efficient mental computation. These strategies evolve along a developmental continuum, gradually shifting from concrete to abstract thinking (Baroody & Dowker, 2003). Understanding this progression allows educators to better support children’s mathematical growth and guide them toward fluency and flexibility in problem-solving.

Common Strategies in Arithmetic Development

Recounting or "Counting All"

This is one of the earliest, and least efficient, strategies children use when learning addition. In this approach, they begin counting from one and continue all the way to the total. For example, when solving 6 + 4, a child might count, “1, 2, 3, 4, 5, 6,” then “7, 8, 9, 10.” Although typical for beginners, this method is slow and can impede the development of mental math skills if relied on for too long (Fuson & Briars, 1990).

Early	Addition and Subtraction Strategies		
Counting Emergent Perceptual Figurative	**Basic Addition and Subtraction Strategies** Counting On, Counting Back, Counting Up +0 +1, +2, -0, -1, -2 Early Doubles 1+1, 2+2, 3+3, 4+4, 5+5	**Addition and Subtraction Strategies to 20** Make 10 Doubles +1-+2, -1-2 Back Through 10 Add Through 10	**Addition and Subtraction to 100** Decade Combinations Jumping and Splitting Method Expanded Notation Addition/Subtraction Shortcut/ Algorithm

Primary Numeracy Framework

Counting On

As children progress, they often adopt the more efficient strategy of counting on. Instead of starting from one, they begin with the larger addend and count up by the value of the smaller one. For instance, to solve 6 + 4, a child starts at 6 and counts "7, 8, 9, 10." This shift represents a significant developmental milestone—from counting individual objects to performing mental addition—and supports the growth of number sense (Fuson & Briars, 1990; Fuson, 1992).

Make Ten (Decomposing and Recomposing)

This strategy builds on the understanding of ten as a key benchmark number. Students break apart one of the addends to create a sum of ten with the other, then add what's left. For example, in 7 + 8, a child might decompose 8 into 3 and 5, add 7 + 3 to make 10, and then add the remaining 5 to reach 15. Often referred to as "friends of ten," this strategy promotes flexible thinking and strengthens mental calculation (Carpenter & Moser, 1984).

Addition and Subtraction Conceptual Strategies

Say to the Student: ***"I am going to ask you to solve some addition and subtraction problems."***

7+6=13 DB - CO – M- F	**8+4=12** DB - CO – M- F	**9+6=15** DB - CO – M- F	**8+7=15** DB - CO – M- F
10-7=3 DB - CB/CO – M – F	**7-6=1** DB – CB/CO – M- F	**15-8= 7** DB – CB/CO – M- F	**14-6=8** DB – CB/CO – M- F

Drops Back to 1- DB	Counts On- CO	Counts Back- CB	Memory- M	Flexible Thinking- F
Uses fingers or objects to represent all numbers included. May start counting from one.	Counts on by ones *May use fingers to track counts*	Counts back by ones *May use fingers to track counts*	Known Immediate, Explanation *"I just knew It"*	Uses 10 structure Doubles + or- Addition/Subtraction Relationship

Primary Numeracy Assessment

Doubles and Doubles Plus or Minus

Recognizing doubles (e.g., 6 + 6) provides a useful anchor for related addition facts. To solve 6 + 7, a child might recall that 6 + 6 = 12, then add 1 more to reach 13. This approach not only enhances arithmetic fluency but also deepens understanding of number relationships and fact families (Baroody & Dowker, 2003).

Instructional Implications

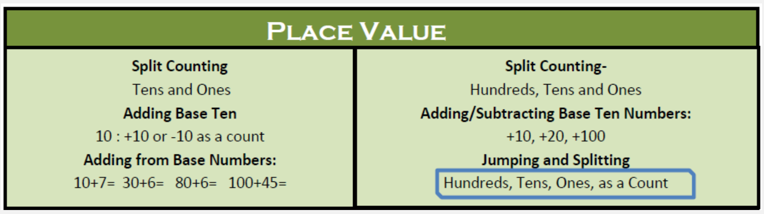
As children mature mathematically, they should be encouraged to transition from inefficient strategies such as "counting all" to more advanced and flexible ones like "make ten" or "doubles plus one." According to Baroody and Dowker (2003), over-reliance on early-stage strategies can lead to slower calculation, reduced fluency, and difficulty with more complex mathematical tasks. Educators play a vital role in guiding this transition, using targeted instruction and practice to foster strategic thinking and conceptual understanding.

Conclusion

The progression from basic counting to sophisticated mental strategies is central to arithmetic learning. While early strategies like recounting may serve as initial stepping stones, it is critical for educators to promote more efficient and conceptually rich strategies. Doing so not only enhances fluency but also deepens children's understanding of numbers, operations, and the structure of our number system.

Understanding Place Value: A Cornerstone of Early Numeracy

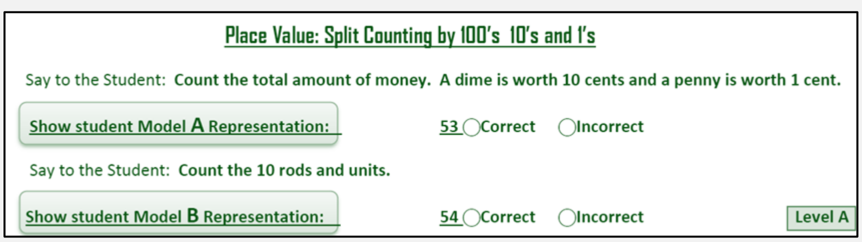
Place value is a foundational concept in early mathematics education, especially during the primary grades. It refers to the understanding that the value of a digit is determined by its position within a number. This understanding is essential for developing number sense, performing arithmetic operations, and progressing toward more advanced mathematical ideas. The significance of place value in early numeracy development is well-documented in both research and educational standards.



Primary Numeracy Framework

The Role of Place Value in Building Number Sense

A strong grasp of place value is critical to developing **number sense**, which includes understanding numerical relationships, estimating, and recognizing patterns. The National Research Council (2001), in its landmark report *Adding It Up: Helping Children Learn Mathematics*, identifies number sense as a core strand of mathematical proficiency and highlights place value as a key component in cultivating it. When students comprehend that digits represent different values depending on their position, they become more adept at making sense of large numbers and identifying numerical patterns, thus laying the groundwork for mental math and flexible thinking.



Primary Numeracy Assessment

Place Value and Operational Fluency

Understanding place value is also vital for developing **mathematical fluency**. Fuson (1990) describes how conceptual structures for multi-unit numbers, such as tens, hundreds, and thousands, enable students to perform addition and subtraction more efficiently. With a firm grasp of place value, children are better equipped to regroup numbers, carry or borrow in multi-digit operations, and solve problems with confidence. This fluency translates into greater flexibility when applying arithmetic strategies and solving word problems.

A Prerequisite for Decimals and Fractions

Place value knowledge is not only essential for whole number operations but also forms the basis for understanding **decimals and fractions**. According to the Common Core State Standards for Mathematics (2010), place value concepts extend naturally into decimal notation and support students' ability to compare and compute with fractional values. Without a solid foundation in place value, students may struggle to comprehend how numbers smaller than one fit into our base-ten system.

Conclusion

Place value is a cornerstone of numeracy that supports the development of number sense, fluency, and problem-solving skills. It is also a critical stepping stone to understanding more complex mathematical concepts, such as decimals and fractions. Both research literature and national standards recognize the central role place value plays in shaping students’ mathematical competence. As such, intentional instruction in place value should remain a priority in early mathematics education.

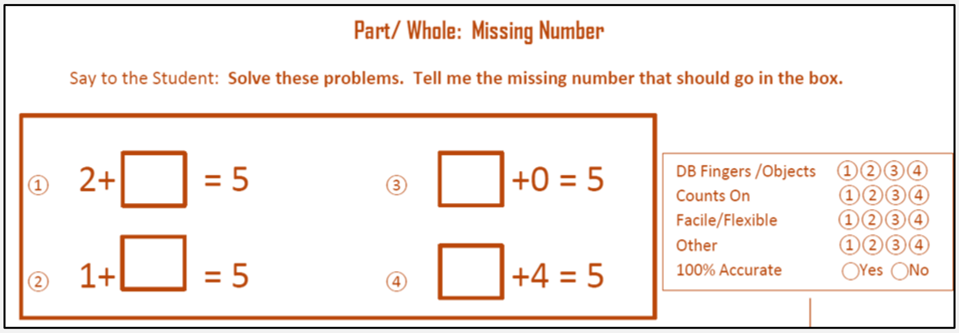
Part-Whole Relationships and the Development of Algebraic Thinking

Understanding part-whole relationships is fundamental in early numeracy development. This concept involves recognizing that numbers can be decomposed into smaller parts and recombined to form a whole, laying the groundwork for arithmetic operations and algebraic reasoning.

Foundational Research on Part-Whole Understanding

Leslie P. Steffe's research emphasizes that children naturally engage in part-whole thinking from an early age, even before formal education begins. This intuitive grasp forms the foundation for more advanced mathematical concepts, such as addition and subtraction, by understanding how numbers can be broken down and recombined.

Douglas H. Clements and Julie Sarama have also highlighted the importance of part-whole relationships in early childhood mathematics. Their work on learning trajectories demonstrates that activities focusing on number composition and decomposition, such as number bonds, significantly enhance children's numerical understanding.



Primary Numeracy Assessment

Transitioning to Algebraic Thinking

Part-whole reasoning is instrumental in the development of algebraic thinking. Kaput (2008) identifies generalization and the use of symbolic representations as core aspects of algebraic reasoning, both of which are rooted in part-whole understanding.

Mason (2008) further explains that recognizing patterns and relationships through part-whole analysis enables students to transition from arithmetic to algebra. This progression allows learners to connect real-world situations with abstract mathematical representations, fostering a deeper understanding of algebraic concepts.

Part/Whole Relationship			
Part/Whole 0-5 Partition 5 Combinations of less than 5 Combinations of 5 Missing Parts ≤ 5	**Part/Whole 6-10** Partition 10 Combinations less than 10 Combinations of 10 Missing Parts ≤ 10	**Part/Whole 10-20** Partition 20 Combinations less than 20 Combinations of 20 Missing Parts ≤ 20	**Part/Whole 20-100** Partition 100 Combinations of 100 Missing Parts ≤ 100

Primary Numeracy Framework

Implications for Instruction

Research underscores the effectiveness of explicit instruction and playful learning experiences that involve part-whole relationships. Clements and Sarama advocate for integrating such activities into early childhood education to build a strong conceptual foundation in mathematics.

Educators are encouraged to design strategies that promote part-whole understanding, such as using manipulatives, visual representations, and interactive games. These approaches not only support arithmetic proficiency but also prepare students for the abstract reasoning required in algebra.

Numeral Identification

Research by Jordan et al. (2006) demonstrated that preschoolers' numeral identification skills are strong predictors of their mathematics performance in kindergarten. This longitudinal study found that children who could accurately identify numerals before entering kindergarten showed better number sense growth and overall math achievement during their early school years.

Numeral Identification			
To 10	**To 100**	**To 1000**	**1000+**

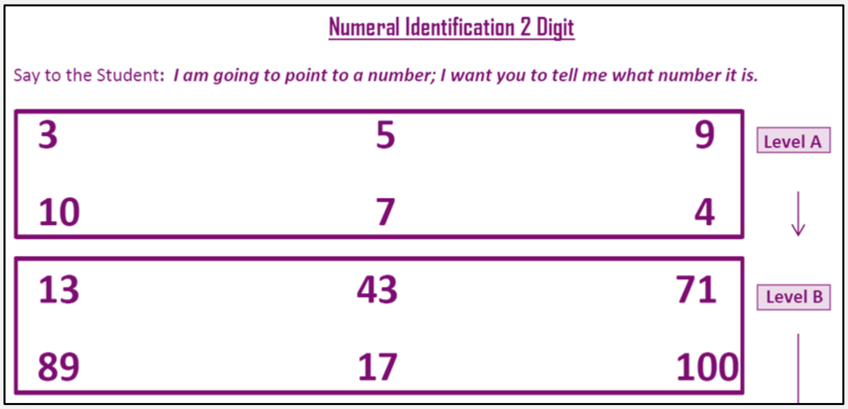
Primary Numeracy Framework

Similarly, Duncan et al. (2007) analyzed data from six longitudinal studies and concluded that early mathematics skills, including numeral identification, are significant predictors of later academic achievement. Their findings emphasized the importance of early numeracy competencies in forecasting future success in mathematics.

Association with Number Sense and Mathematical Learning

Mazzocco et al. (2011) investigated the relationship between preschoolers' precision in the Approximate Number System (ANS) and later school mathematics performance. Their study found that children with higher ANS precision, which relates to an intuitive sense of number, performed better in formal mathematics tasks at age six.

This suggests that early numerical abilities, including numeral identification, are linked to the development of number sense and subsequent mathematical learning.



Primary Numeracy Assessment

Importance in Everyday Contexts

Beyond academic settings, numeral identification is crucial for everyday activities such as telling time, understanding prices, measuring quantities, and managing finances. A study highlighted that early numeracy skills, including numeral identification, are powerful predictors of school-age mathematical learning and performance, underscoring their relevance in daily life.

Implications for Education

Given the strong evidence linking early numeral identification to later mathematical achievement, educators and parents should prioritize activities that promote numeral recognition in early childhood. Incorporating games, visual aids, and interactive exercises that focus on identifying and using numerals can build a solid foundation for future mathematical learning.

In summary, proficiency in numeral identification is a critical early skill that supports the development of number sense, mathematical reasoning, and practical numeracy needed for everyday tasks. Early interventions and educational strategies that enhance numeral recognition can have lasting positive effects on a child's academic trajectory and daily life competencies.

References

Baroody, A. J., & Dowker, A. (2003). *The development of arithmetic concepts and skills: Constructive adaptive expertise*. Lawrence Erlbaum Associates.

Baroody, A. J., & Gannon, K. E. (1984). The development of the commutativity principle and efficient strategies for solving basic addition and subtraction combinations. *Journal for Research in Mathematics Education*, 15(2), 132–150. https://doi.org/10.2307/748682

Baroody, A. J., & Ginsburg, H. P. (1983). The effects of instruction on children's understanding of the "equals" sign. *The Elementary School Journal*, 84(2), 199–212. https://doi.org/10.1086/461356

Carpenter, T. P., & Moser, J. M. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education*, 15(3), 179–202. https://doi.org/10.5951/jresematheduc.15.3.0179NCTM Publications+1NCTM Publications+1

Clements, D. H., & Sarama, J. (2023). Learning trajectories in early mathematics – Sequences of acquisition and teaching. In R. E. Tremblay, M. Boivin, & R. DeV. Peters (Eds.), *Encyclopedia on Early Childhood Development*. Retrieved from https://www.child-encyclopedia.com/learning-trajectories-early-mathematics-sequences-acquisition-and-teaching

Common Core State Standards Initiative. (2010). *Common Core State Standards for Mathematics*. National Governors Association Center for Best Practices & Council of Chief State School Officers. https://www.corestandards.org/Math/

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., ... & Japel, C. (2007). School readiness and later achievement. *Developmental Psychology*, 43(6), 1428–1446. https://doi.org/10.1037/0012-1649.43.6.1428

Fuson, K. C. (1990). Conceptual structures for multiunit numbers: Implications for learning and teaching multidigit addition, subtraction, and place value. *Cognition and Instruction*, 7(4), 343–403. https://doi.org/10.1207/s1532690xci0704_4

Fuson, K. C. (1992). *Children's counting and concepts of number*. Springer-Verlag.

Fuson, K. C., & Briars, D. J. (1990). Using a base-ten blocks learning/teaching approach for first- and second-grade place-value and multidigit addition and subtraction. *Journal for Research in Mathematics Education*, 21(3), 180–206. https://doi.org/10.5951/jresematheduc.21.3.0180NCTM Publications+1NCTM Publications+1

Fuson, K. C., & Hall, J. W. (1983). The acquisition of early number word meanings: A conceptual analysis and review. In H. P. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 49–107). Academic Press.

Kaput, J. J. (2008). What is algebra? What is algebraic reasoning? In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 5–17). Routledge.

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2006). Development of number combination skill in the early school years: When do fingers help? *Developmental Science*, 10(5), 607–614. https://doi.org/10.1111/j.1467-7687.2007.00617.x

Mason, J. (2008). Representing generalising. In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 65–86). Routledge.

Mazzocco, M. M. M., Feigenson, L., & Halberda, J. (2011). Preschoolers' precision of the approximate number system predicts later school mathematics performance. *PLoS ONE*, 6(9), e23749. https://doi.org/10.1371/journal.pone.0023749

Mason, J. (2008). Representing generalising. In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 65–86). Routledge.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. National Academy Press. https://doi.org/10.17226/9822

Steffe, L. P. (1992). Learning stages in the construction of the number sequence. In J. Bideaud, C. Meljac, & J. Fischer (Eds.), *Pathways to number: Children's developing numerical abilities* (pp. 83–98). Lawrence Erlbaum Associates.

Steffe, L. P. (1994). Children's multiplying schemes. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 3–39). State University of New York Press.

Steffe, L. P. (2002). A new hypothesis concerning children's fractional knowledge. *Journal of Mathematical Behavior*, 21(2), 179–194.

White, J. W. (1982). The impact of number concept training on the place value understanding of first-grade children. *Journal for Research in Mathematics Education*, 13(2), 98–102.

Wright, R. J. (1984). A study of the numerical knowledge of young children. *Mathematics Education Research Journal*, 1(1), 1–17.

Steffe, L. P. (2002). A new hypothesis concerning children's fractional knowledge. *Journal of Mathematical Behavior*, 21(2), 179–194.