The Research Embedded into the Primary Numeracy Assessment and Framework

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Numeracy Consultants *(Updated 2022)*

Introduction

The Primary Numeracy Assessment stands as a pioneering standards-based tool methodically crafted by Numeracy Consultants to offer invaluable support to educators in pinpointing critical numeracy standards where students may face challenges within their state-mandated curriculum. A product of rigorous research into the latest insights on numeracy development, this assessment meticulously sifts through a myriad of standards, extracting the most pivotal ones to construct a comprehensive standards-based evaluation.

The Primary Numeracy Assessment delves into a comprehensive range of fundamental numeracy domains, each serving as a cornerstone for lifelong mathematical learning. This assessment meticulously explores key competencies, encompassing forward and backward counting, counting by tens, the systematic application of addition and subtraction strategies, application of place value principles, numeral identification, and the intricate understanding of part-part-whole relationships. These domains collectively lay the bedrock for not just academic achievement but also the development of essential mathematical skills that resonate throughout a student's entire educational journey and beyond.

The Primary Numeracy Assessment serves as the linchpin of our instructional approach. Without the illuminating data it provides, our efforts to address learning gaps and fine-tune our pedagogical methods would lack the precision and insight required for optimal results. ***This assessment, in essence, acts as a catalyst, igniting a targeted and data-driven instructional paradigm, enabling educators to tailor their approaches with precision, ensuring that each student's unique learning challenges are met effectively.***

Forward Counting: A Precursor to Addition

Forward counting serves as the gateway to understanding numbers and performing basic mathematical operations. It is a skill that children must master before they can confidently tackle more complex mathematical concepts. Fuson and Hall's research (1983) emphasize the significance of forward counting as a precursor to addition and subtraction. They posit that a robust ability to count forward provides children with a fundamental numerical sense, which is vital for mathematical comprehension.

Say to the Student: ***"I am going to give you a number and I want you to start counting forward until I tell you to stop."***

Level A	Level B	Level C			Level D	Level E
1-10	9-22	32-44	67-81	98-113	198-213	995-1000

Forward Counting: Levels A-E

Research Findings:

Fuson and Hall's findings suggest that a child's competence in forward counting is closely linked to their ability to understand the concept of addition. When children can count forward fluently, they establish a foundational understanding of numerical sequences. This understanding is essential for grasping the concept of adding numbers together, as addition inherently involves counting forward along the number line (Fuson & Hall, 1983).

Douglas Steffe's contributions (Steffe, 1992) further elucidate the role of forward counting in the development of number sense—a crucial aspect of numeracy. Steffe's research underscores that forward counting is not merely a mechanical skill but a cognitive process that aids children in grasping numerical relationships. According to Steffe, forward counting serves as an essential building block for numeracy. Steffe's research findings suggest that forward counting is not limited to rote memorization but plays a significant role in building a child's number sense. When children can count forward with ease, they develop an innate understanding of numerical progression, making it easier for them to recognize numerical patterns and relationships. This foundational numerical sense is a prerequisite for successfully navigating the world of mathematics (Steffe, 1992).

Backwards Counting

Backward counting, often overshadowed by its counterpart, forward counting, is a fundamental skill in numeracy development. This paper explores the significance of backward counting, emphasizing its essential role in helping children understand the inverse relationship between addition and subtraction. Drawing from a variety of researchers' work, including Baroody, Gannon, and Douglas Steffe, we shed light on how proficiency in backward counting is a key component in building subtraction skills.

Say to the Student: ***"I am going to give you a number and I want you to start counting backwards until I tell you to stop."***

Level A → **Level B** → **Level C** → **Level D**

10-1		**23-16**		**42-34**		**73-64**		**104-98**	

Backward Counting as the Key to the Inverse Relationship

Complementary to forward counting, backward counting is an invaluable skill that aids children in comprehending the inverse relationship between addition and subtraction. Baroody and Gannon's research (1984) underscored the positive correlation between proficiency in backward counting and competence in subtraction. This skill equips children with the ability to reverse the counting process, which is essential for performing subtraction operations accurately.

Research Findings:

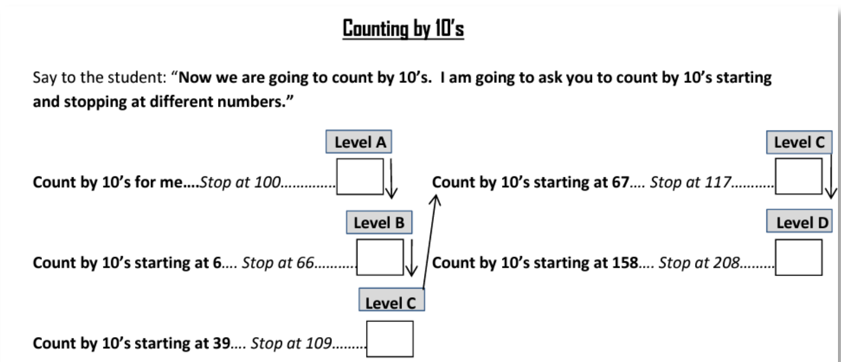
Baroody and Gannon's research findings suggest that children who are proficient in backward counting are better equipped to grasp the concept of subtraction. They are more likely to recognize that subtraction involves "taking away" or "counting backward" from a given quantity. This understanding forms the foundation for successful subtraction operations and problem-solving skills (Baroody & Gannon, 1984).

Douglas Steffe's studies (Steffe, 1994) delve deeper into the relationship between backward counting and subtraction performance. His work highlights that children who excel in backward counting tend to perform better in tasks involving subtraction. This reinforces the critical role of backward counting in preparing children for more advanced mathematical concepts. Steffe's research findings emphasize that proficiency in backward counting is not only associated with better subtraction performance but also with a deeper understanding of subtraction as an inverse operation to addition. Children who can effectively count backward are more likely to grasp that subtraction "undoes" addition, forming the basis for mathematical fluency and problem-solving in numeracy (Steffe, 1994).

In conclusion, backward counting stands as a crucial skill in numeracy development, particularly for mastering subtraction. Research findings from scholars like Baroody, Gannon, and Douglas Steffe highlight the significant role of backward counting in helping children understand the inverse relationship between addition and subtraction. Proficiency in backward counting equips children with the ability to perform subtraction operations accurately and comprehend subtraction as an essential mathematical concept. Thus, educators and parents should recognize the importance of fostering this skill to ensure children's success in numeracy.

Counting by Tens

Counting by tens is a fundamental numeracy skill that lays a solid foundation for comprehending the concept of place value in mathematics. Place value, a cornerstone concept, governs the value of each digit within a number based on its position. Counting by tens is a crucial precursor to grasping this concept, as it enables children to discern the significance of each digit's position within a number. Multiple esteemed researchers, including Fuson, Baroody, Steffe, White, and Robert Wright, have conducted studies that underscore the paramount importance of counting by tens in fostering a profound understanding of place value.



Research Findings:

Research by Fuson and Briars (1990) underscores the pivotal role of counting by tens in early mathematical development. They assert that counting by tens not only provides a structured approach to counting larger quantities but also reinforces the concept of place value. When children engage in counting by tens, they naturally develop an intuitive sense that each position in a multi-digit number signifies a distinct order of magnitude, such as tens, hundreds, thousands, and so forth. This foundational understanding forms an essential basis for tackling more advanced mathematical operations and problem-solving tasks.

Baroody and Ginsburg's work (1983) emphasizes that counting by tens fosters a sense of grouping within numbers. It enables children to recognize that numbers can be organized into sets of ten, a fundamental principle within our base-ten number system. This grouping concept lies at the heart of place value, where each digit's position represents a specific power of ten. By grasping the notion of grouping and mastering counting by tens, children naturally progress toward making sense of place value and are better equipped to navigate mathematical concepts involving larger numbers.

Furthermore, Douglas Steffe's research (Steffe, 1992) delves into the development of children's numerical understanding. Steffe's work highlights that counting by tens is not merely a mechanical exercise but rather a cognitive process that aids children in comprehending numerical relationships. When children count by tens, they not only become proficient in numerical sequencing but also develop a profound sense of the role each digit plays in determining the overall value of a number. Additionally, Robert Wright's contributions (Wright, 1984) emphasize the importance of counting by tens in the context of early mathematical development. Wright's research underscores that counting by tens offers a structured approach to counting and provides a foundation for understanding the grouping of numbers in tens, a crucial concept for place value comprehension.

Research conducted by White and colleagues (White, 1982) has demonstrated that counting by tens supports children's ability to connect numerical symbols with concrete quantities. This connection between abstract symbols and tangible quantities is a pivotal step in the development of numerical fluency and place value comprehension.

To conclude, counting by tens is a critical skill in numeracy development, playing a vital role in understanding place value in mathematics. Researchers such as Fuson, Baroody, Steffe, White, and Robert Wright have illuminated its importance in providing structure to counting, fostering an understanding of grouping, and preparing children for more advanced mathematical concepts. Recognizing the significance of counting by tens, educators and parents should actively encourage its development, thereby enhancing children's mathematical proficiency.

Arithmetic Thinking Strategies:

Arithmetic skills, particularly addition and subtraction, lay the foundation for a child's mathematical development. As students progress through their early education, they develop an array of strategies for tackling these essential mathematical operations (Baroody & Dowker, 2003). This section delves into the most prevalent strategies employed by children, emphasizing their effectiveness and implications for optimal pedagogy.

Addition and Subtraction Conceptual Strategies

Say to the Student: ***"I am going to ask you to solve some addition and subtraction problems."***

7+6=13	**8+4=12**	**9+6=15**	**8+7=15**
DB - CO – M- F	**DB - CO – M- F**	**DB - CO – M- F**	**DB - CO – M- F**
10-7=3	**7-6=1**	**15-8= 7**	**14-6=8**
DB - CB/CO – M – F	**DB – CB/CO – M- F**	**DB – CB/CO – M- F**	**DB – CB/CO – M- F**

Drops Back to 1- DB	**Counts On- CO**	**Counts Back- CB**	**Memory- M**	**Flexible Thinking- F**
Uses fingers or objects to represent all numbers included. May start counting from one.	Counts on by ones *May use fingers to track counts*	Counts back by ones *May use fingers to track counts*	Known Immediate, Explanation *"I just knew It"*	Uses 10 structure Doubles + or- Addition/Subtraction Relationship

Dropping back to one to and or recounting is a rudimentary and notably low-level strategy where children commence addition with the larger number and subsequently count back to one while adding the smaller number (Fuson & Briars, 1990). To illustrate, in solving 6 + 4, a child starts with 1 and counts to 6 then then starts counts from 6 culminating in the answer, which is 10. However, this strategy should be discouraged as children progress in their mathematical development, as it can impede fluency and hinder efficient calculation.

Counting on strategy is a foundational mathematical concept that plays a crucial role in early childhood education. It involves the process of starting with a given number and incrementing it by one or more, step by step, to reach a desired total. This strategy not only aids in the development of basic addition skills but also enhances a child's number sense and understanding of numerical relationships. Research by Fuson and Briars (1990) highlights the significance of the counting on strategy in children's mathematical development, emphasizing its role in bridging the gap between counting and addition.

By using this strategy, children gradually transition from relying on counting objects one by one to mentally adding numbers, paving the way for more efficient and advanced mathematical thinking as they progress in their education. In the classroom, educators often incorporate counting on activities to help young learners build a strong foundation in arithmetic and mathematical problem-solving skills (Fuson, 1992). Thus, the counting on strategy is a vital component of early math education, fostering the development of essential mathematical competencies in children.

The make ten strategy, often referred to as the friends of ten or decomposing and recomposing, is a pivotal approach to addition. This strategy involves breaking numbers down into pairs that sum to ten or other benchmark numbers (Carpenter & Moser, 1984). For instance, when confronted with 7 + 8, a child may decompose 8 into 3 + 5, enabling them to solve 7 + 3 first, arriving at 10, and then simply adding the remaining 5 to obtain the correct answer of 15. This strategy not only cultivates number sense but also enhances mental calculation fluency.

The doubles plus or minus strategy centers on recognizing that a number, when added to or subtracted from its double, results in a benchmark number (Baroody & Dowker, 2003). For example, to solve 6 + 7, a student may discern that 7 is merely one more than 6. Consequently, they can compute 6 + 6, arriving at 12, and then simply add 1 to obtain the correct answer of 13. This approach not only hones mental arithmetic skills but also fosters fluency in manipulating numbers.

The Importance of Effective Strategies

Children traverse a developmental continuum in refining their arithmetic strategies as they accumulate experience and deepen their grasp of numerical concepts (Baroody & Dowker, 2003). While dropping back to one and recounting might serve as an initial stepping stone, it is imperative to guide students toward more efficient strategies like the make ten and doubles plus or minus to develop fluency and flexibility with their thinking. These strategies not only promote mathematical fluency but also reinforce a profound conceptual understanding of numbers.

Low-level strategies, such as dropping back to one and counting back to one and permanently counting on, can inadvertently hinder a child's mathematical advancement (Baroody & Dowker, 2003). Over-reliance on such approaches can lead to sluggish calculation speed, restricted fluency, and difficulties in grappling with more intricate mathematical concepts. Therefore, educators must actively steer students away from these low-level strategies as they mature mathematically.

Place Value:

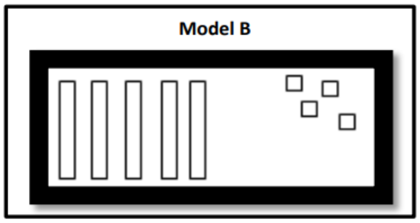
Place value is a pivotal concept in numeracy development, particularly during the primary grades, and it serves as a critical foundation for comprehending our base-ten number system. This concept entails recognizing that the value of a digit within a number is determined by its positional value within that number. The significance of place value in early mathematics education cannot be overstated, with numerous scholars highlighting its crucial role.

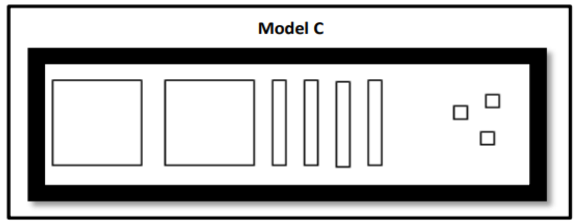
Say to the Student: **I want you to count by tens and ones. For example, if we were going to count to 23, it would be done like this, 10, 20, 21, 22, 23.** ***NO REPRESENTATION***

Say to the student: **Count to 42 by tens and ones:**	◯**Correct**	◯**Incorrect**	
Say to the student: **Count to 36 by tens and ones:**	◯**Correct**	◯**Incorrect**	
Say to the student: **Count to 104 by tens and ones:**	◯**Correct**	◯**Incorrect**	**Level B**

Research underscores the significance of place value in fostering number sense, a foundational aspect of mathematical proficiency. The National Research Council's influential report, "Adding It Up: Helping Children Learn Mathematics," emphasizes the importance of number sense and its impact on mathematical achievement (National Research Council, 2001). Place value plays a central role in developing this number sense, as it allows students to not only understand numbers but also discern patterns and perform mental calculations more efficiently.

Moreover, place value is instrumental in nurturing mathematical fluency and problem-solving skills. A study by Fuson (1990) delves into the concept of conceptual structures for multiunit numbers, highlighting how understanding place value aids in carrying out addition, subtraction, and other mathematical operations more effectively. It enables students to manipulate numbers confidently, a skill that is indispensable for tackling both elementary arithmetic problems and the more intricate mathematical challenges encountered in later grades.





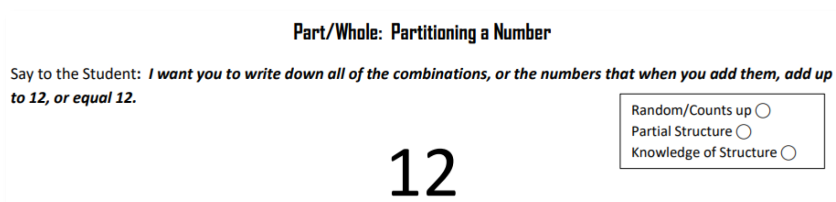
Furthermore, a solid grasp of place value is a prerequisite for comprehending advanced mathematical concepts such as decimals and fractions. The Common Core State Standards for Mathematics (2010) explicitly reference the role of place value in understanding decimals and fractions. These concepts build upon the foundational understanding of the relationship between digits and their positions in numbers, making place value a critical stepping stone in a student's mathematical journey.

In summary, place value holds a central position in numeracy development during the primary grades. It fosters number sense, mathematical fluency, and problem-solving skills while serving as a prerequisite for comprehending advanced mathematical concepts. The scholarly literature and educational standards converge in underscoring the indispensable role of place value in shaping students' mathematical competence and confidence.

Part Whole Relationship and Algebraic Thinking

Part-part-whole relationships play a fundamental role in the development of numeracy skills in young learners. This concept, extensively researched by Douglas H. Clements, Julie Sarama, and other experts in the field of mathematics education, highlights the importance of understanding how numbers can be decomposed into smaller components and how these components relate to the whole. One prominent researcher in this area is Leslie P. Steffe, who has made significant contributions to our understanding of early mathematical cognition. Steffe's work emphasizes the notion that children's natural development of numeracy begins with the recognition and manipulation of part-part-whole relationships.

Steffe's research has shown that children naturally engage in part-whole thinking from an early age, even before formal education begins. This intuitive grasp of part-part-whole relationships forms the foundation for more advanced mathematical concepts. For example, understanding that a number can be broken down into smaller components is crucial for performing addition and subtraction operations. As children progress in their mathematical development, this foundational knowledge allows them to solve increasingly complex problems involving multiple components.


Part/Whole: Partitioning a Number

Say to the Student: ***I want you to write down all of the combinations, or the numbers that when you add them, add up to 12, or equal 12.***

Random/Counts up ○
Partial Structure ○
Knowledge of Structure ○

12

Part-whole thinking is a fundamental cognitive process that lays the groundwork for the development of algebraic thinking (Sfard, 2002). When individuals engage in part-whole thinking, they break down complex problems or quantities into smaller, more manageable components, fostering the ability to recognize patterns and relationships

(Kaput, 2008). This process is instrumental in the transition from arithmetic to algebra, as it enables students to see the connection between real-world situations and abstract mathematical representations (Mason, 2001). As students manipulate and relate these parts, they gradually develop the ability to work with variables, construct equations, and solve algebraic problems (Kieran, 2004). Part-whole thinking, thus, acts as a bridge that helps learners transition from concrete to abstract mathematical concepts, facilitating a deeper understanding of algebra (Falkner et al., 2017).

① $2+\boxed{} = 5$	③ $\boxed{}+0 = 5$
② $1+\boxed{} = 5$	④ $\boxed{}+4 = 5$

① $7+\boxed{}=10$	③ $\boxed{}+4 = 10$
② $1+\boxed{}=10$	④ $5+\boxed{} = 10$

Furthermore, the work of Douglas H. Clements and Julie Sarama has highlighted the importance of explicit instruction and playful learning experiences that involve part-part-whole relationships in early childhood education. Their research has demonstrated that activities such as number bonds, where students explore how numbers can be decomposed and recomposed, can significantly enhance numeracy skills. These activities help children build a strong conceptual understanding of numbers, paving the way for more advanced mathematical concepts in later grades.

In conclusion, research by Steffe and others underscores the critical role of part-part-whole relationships in the development of numeracy skills. Understanding how numbers can be decomposed and recombined is essential for building a solid foundation in mathematics. Educators should leverage this research to design effective teaching strategies that promote a deep understanding of part-part-whole relationships among young learners.

Numeral Identification

Numerical identification holds a crucial role in nurturing basic numeracy skills, and its significance is underscored by an array of research studies across different fields. Numeracy, defined as the capability to comprehend and manipulate numbers, serves as a fundamental skill with far-reaching implications in various facets of daily life, from personal finance management to informed decision-making in our increasingly data-driven world. Proficiency in numeral identification, which involves recognizing and interpreting numerical symbols, serves as the cornerstone upon which more advanced mathematical competencies are built.

13	**43**	**71**
89	**17**	**100**

A plethora of studies have consistently established the pivotal role of early numeral identification in predicting subsequent mathematical achievement. For instance, Jordan et al. (2006) conducted a notable study published in the journal "Child Development," highlighting a strong correlation between preschoolers' numeral identification skills and their performance in kindergarten-level mathematics. This evidence underscores the critical nature of early exposure to numerals and the importance of fostering the skill to recognize and differentiate them as a crucial first step in establishing a solid mathematical foundation.

Furthermore, the importance of numeral identification extends to everyday tasks such as time-telling, quantity interpretation, measurement, and budgeting. Without a firm grasp of numerals, individuals may encounter challenges when making accurate calculations or comprehending numerical information encountered in diverse contexts. In educational settings, educators consistently emphasize the significance of numeral identification as the initial stage for introducing more complex mathematical concepts, aligning with empirical research emphasizing the role of numeracy as a key determinant of academic success.

In summary, the critical role of numeral identification in shaping basic numeracy is well-supported by a multitude of research studies. Proficiency in recognizing and interpreting numerical symbols serves as the foundation for understanding and effectively working with numbers, empowering individuals to navigate mathematical challenges and make informed decisions in their daily lives (Jordan et al., 2006). Consequently, early exposure to numeral recognition remains a cornerstone of effective education, given its strong association with subsequent mathematical achievement. This perspective is reinforced by various other studies in the field of education and cognitive development, collectively emphasizing the pivotal role of numeral identification in the development of numeracy skills (e.g., Duncan et al., 2007; Mazzocco et al., 2011).

In conclusion, the development of the Primary Numeracy assessment underscores the invaluable role of Numeracy Consultants in shaping effective educational tools. Through a meticulous process that involved a comprehensive analysis of state standards and extensive research to identify the most critical standards, these consultants have contributed significantly to the advancement of numeracy education. By aligning the assessment with the essential standards, they have ensured that it accurately reflects the core competencies students need to acquire. This approach not only enhances the assessment's validity and reliability but also ensures that it effectively serves as a valuable tool for educators striving to promote numeracy skills among primary students.

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