



Numeracy Specialist Program

Level 3

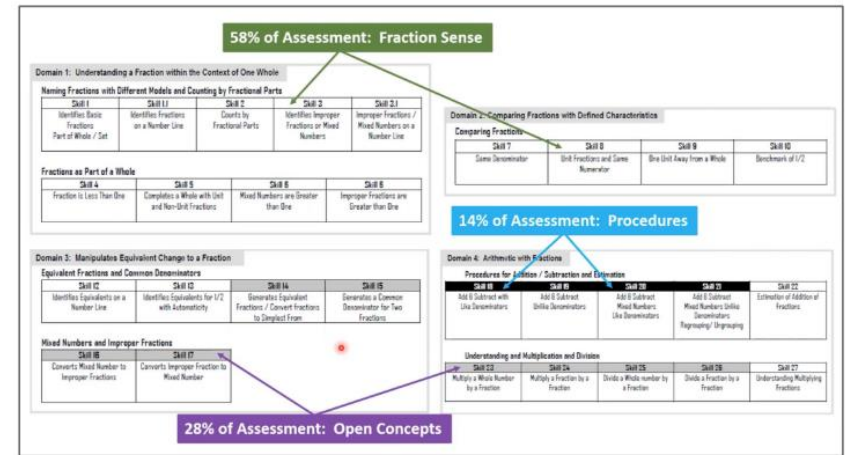
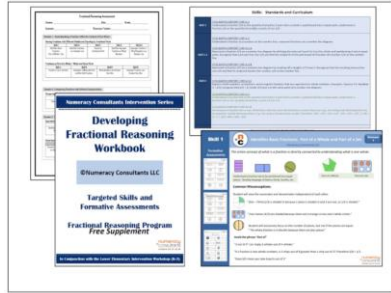
Fractional Reasoning Program

Course #4



Fractional Reasoning Overview

- There are 4 components to the program.
- The assessment is divided into 4 domains.
- The Standards & Curriculum document gives you an idea of the content of the program.
- The Developing Fractional Reasoning Workbook provides follow up practice/formative assessment.
- The Skills, Lesson Cards & Activities book contains lessons, videos & materials for each lesson.



58% of Assessment: Fraction Sense

Domain 1: Understanding a Fraction within the Context of One Whole

Naming Fractions with Different Models and Counting by Fractional Parts

Skill 1.1 Identifies Basic Fractions Part of Whole / Set	Skill 1.1.1 Identifies Fractions on a Number Line	Skill 2 Counts by Fractional Parts	Skill 2.1 Identifies Improper Fractions or Mixed Numbers	Skill 2.1.1 Improper Fractions / Mixed Numbers on a Number Line
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Fractions as Part of a Whole

Skill 4 Fraction is Less Than One	Skill 5 Completes a Whole with Unit and Non-Unit Fractions	Skill 6 Mixed Numbers are Greater Than One	Skill 6 Improper Fractions are Greater Than One
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3rd Grade/ 4th

Domain 2: Comparing Fractions with Defined Characteristics

Skill 7 Same Denominator	Skill 8 Unit Fractions and Same Numerator	Skill 9 One Unit Away from a Whole	Skill 10 Benchmark at 1/2
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3rd Grade/ 4th Grade

14% of Assessment: Procedures

Domain 3: Manipulates Equivalent Change to a Fraction

Equivalent Fractions and Common Denominators

Skill 12 Identifies Equivalents on a Number Line	Skill 12 Identifies Equivalents for 1/2 with Automaticity	Skill 14 Determines Equivalent Fractions / Convert fractions to Simplest Form	Skill 15 Determines a Common Denominator for Two Fractions
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Mixed Numbers and Improper Fractions

Skill 16 Converts Mixed Number to Improper Fractions	Skill 17 Converts Improper Fraction to Mixed Number
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4th Grade/ 5th Grade

Domain 4: Arithmetic with Fractions

Procedures for Addition / Subtraction and Estimation

Skill 18 Add / Subtract with Like Denominators	Skill 19 Add / Subtract Unlike Denominators	Skill 20 Mixed Numbers Like Denominators	Skill 21 Mixed Numbers Unlike Denominators	Skill 22 Estimation of Addition of Fractions
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Understanding and Multiplication and Division

Skill 23 Multiply a Whole Number by a Fraction	Skill 24 Multiply a Fraction by a Fraction	Skill 25 Divide a Whole number by a Fraction	Skill 26 Divide a Fraction by a Fraction	Skill 27 Understanding Multiplying Fractions
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4th Grade/ 5th Grade/ 6th Grade

28% of Assessment: Open Concepts



Assessment Skill 1 & 1.1

- Skill 1 determines if a student can identify basic fractions as part of a whole / set
- Must use correct fractional language when answering (e.g., one half **NOT** one out of two).
- Skill 1.1 determines if a student can transfer knowledge of fractions to a number line.

Domain 1: Understanding One Whole: Naming Fractions with Different Models and Counting

Say to the Student: "Name each fraction."

Say to the Student: "What fraction is the heart placed at on the number line?"

Where would 4/5 go on the number line?

Say to the Student:

"Count by eighths starting at 0 and stop at 1." Correct Incorrect

"Count backwards by sixths starting at 1 or 6/6 and stop at zero." Correct Incorrect

Say to the Student: "Name as a mixed number or an improper fraction."

1/2 or 1 and 2/2 1/4 or 1 and 1/4

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Assessment: Skill 2, 3 & 3.1

- Skill 2 is counting by fractional parts from 0 to 1. (Building one whole through fractional language)
- For Skill 3, the student must name the given fraction as a mixed number or improper fraction.
- For Skill 3.1, the student must identify the location of a mixed number or improper fraction on a number line

Domain 1: Understanding One Whole Naming Fractions with Different Models and Counting

Step to the Student: "Name each fraction."

Step to the Student: "What fraction is the heart placed at on the number line?"

Where would 4/5 go on the number line?

Step to the Student: "Count by fifths starting at 0 and stop at 1?"

Count backwards by eighths starting at 1 and stop at zero?

Step to the Student: "Name as a mixed number or an improper fraction."

Domain 1: Skills 1-3.1

- Explanation of why each skill was chosen and why each skill is important.
- The goal is for the student to develop a complete understanding of fractions within the context of one whole.
- Need to specifically teach Skill 1.1 as students struggle with identifying fractions on a number line.
- Counting by fractional parts for Skill 2 is the student's first experience with numbers looking numerically different but meaning the same amount.
- The skill of counting fractions helps to lay the foundation for adding fractions.

Domain 1: Understanding a Fraction within the Context of One Whole

Naming Fractions with Different Models and Counting by Fractional Parts				
Skill 1	Skill 1.1	Skill 2	Skill 3	Skill 3.1
Identifies Basic Fractions	Identifies Fractions on a Number Line	Counts by Fractional Parts	Identifies Improper Fractions or Mixed Numbers	Identifies Fractions / Mixed Numbers on a Number Line

Fractions as Part of a Whole / Whole and Some Parts

Skill 4	Skill 5	Skill 6	Skill 6
Fraction is Less Than One	Completes a Whole with One and Non-Unit Fractions	Mixed Numbers are Greater than One	Improper Fractions are Greater than One



Assessment: Skill 4, 5 & 6

- Skill 4 (Fraction is Less Than One) covers how a whole in fraction form is still more than a regular fraction.
- Checking for evidence of understanding, evidence of misconception or unable to explain.
- 50% chance of getting correct. The explanation is the most important part.
- Skill 5 has to do with completing a whole using unit or non-unit fractions.
- Keywords for Skill 6 include: more than 1 whole, only a whole, greater than a whole, one part over a whole.

Say to the Student: "Where would $\frac{5}{6}$ be on 1 and $\frac{2}{3}$ go on this number line?"

Domain 1: Understanding One Whole, Fractions as Part of a Whole

Say to the Student: "I am going to show you some fractions and numbers. I want you to tell me which is greater." Student must explain each answer. For explanations a point then the question will be marked as unable to explain.

Skill 4 Key words: One Whole, More total amount, not a whole, all the parts, not all parts.

	$\frac{3}{4}$ or 1	$\frac{3}{4}$ or $\frac{4}{5}$
<input type="checkbox"/> Evidence of understanding <input type="checkbox"/> Evidence of misconception <input type="checkbox"/> Unable to explain	<input type="checkbox"/> Evidence of understanding <input type="checkbox"/> Evidence of misconception <input type="checkbox"/> Unable to explain	<input type="checkbox"/> Evidence of understanding <input type="checkbox"/> Evidence of misconception <input type="checkbox"/> Unable to explain

Say to the Student: "I am going to show you a fraction. Tell me how much more I would need to make one whole."

Skill 5

	$\frac{1}{3}$	$\frac{4}{7}$
<input type="checkbox"/> Correct answer of $\frac{2}{3}$ <input type="checkbox"/> Incorrect Answer	<input type="checkbox"/> Correct answer of $\frac{2}{3}$ <input type="checkbox"/> Incorrect Answer	<input type="checkbox"/> Correct answer of $\frac{3}{7}$ <input type="checkbox"/> Incorrect Answer

Say to the Student: "Compare and tell me which is greater."

Skill 6

1 or $2\frac{1}{4}$	$\frac{3}{3}$ or $1\frac{1}{2}$	$\frac{5}{4}$ or 1	$\frac{7}{4}$ or $\frac{9}{9}$
<input type="checkbox"/> Evidence of understanding <input type="checkbox"/> Evidence of misconception <input type="checkbox"/> Unable to explain	<input type="checkbox"/> Evidence of understanding <input type="checkbox"/> Evidence of misconception <input type="checkbox"/> Unable to explain	<input type="checkbox"/> Evidence of understanding <input type="checkbox"/> Evidence of misconception <input type="checkbox"/> Unable to explain	<input type="checkbox"/> Evidence of understanding <input type="checkbox"/> Evidence of misconception <input type="checkbox"/> Unable to explain

Key words: More than 1 whole, only a whole, greater than a whole, one part over a whole

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Domain 1: Skills 4-6

- Do kids understand the context of a fraction by applying to one whole?
- For Skill 4 (Fraction is Less Than One), student must understand there are many ways for one whole to exist.
- Skill 5 (Completes a Whole) is an important skill to have in place in order to understand and compare fractions.
- Skill 6 (Mixed #'s/Improper Fractions are Greater Than One) develops improper fractions and mixed numbers further within the context of one whole.
- Domain 1 gives us a picture of what a student understands about fractions. Almost all of the skills are part of 3rd grade. It is important to have all of these skills in place.

Domain 1: Understanding a Fraction within the Context of One Whole

Skill 1	Skill 2	Skill 3	Skill 4	Skill 5
Identifies Basic Fractions Part of Whole / Set	Identifies Fractions on a Number Line	Counts by Fractional Parts	Identifies Improper Fractions or Mixed Numbers	Identifies Improper Fractions or Mixed Numbers on a Number Line

Fractions as Part of a Whole / Whole and Some Parts

Skill 4	Skill 5	Skill 6	Skill 6
Fractions is Less Than One	Completes a Whole with Unit and Non-Unit Fractions	Mixed Numbers are Greater Than One	Improper Fractions are Greater Than One

or

$\frac{3}{4}$ or 1

$\frac{3}{3}$ or $\frac{4}{5}$

Domain 1: Understanding a Fraction within the Context of One Whole

Naming Fractions with Different Models and Counting by Fractional Parts			
Skill 1	Skill 2	Skill 3	Skill 4
Identifies basic parts of a fraction	Identifies fractions on a number line	Counts by fractions	Identifies improper fractions
Identifies fractions on a number line	Counts by fractions	Identifies improper fractions	Identifies fractions on a number line

Fractions as Part of a Whole / Whole and Some Parts

Skill 4	Skill 5	Skill 6	Skill 8
Explains when one whole is made up of some parts	Compares fractions with like denominators	Mixed Numbers and Fractions	Improper Fractions and Mixed Numbers

Developing Fractional Reasoning Workbook

Prerequisite Knowledge: Students should understand basic number concepts and be able to count by ones and tens.

Domain 1: Summary

- Domain 1 is the foundational piece.
- Kids must understand a fraction within the context of one whole.
- With older students, you can micro-target the skills needed.
- There is nothing wrong with reteaching the entire domain even if the assessment shows they have some skills in place. Moving skill by skill helps ensure all are in place.
- Developing more depth of student knowledge by not only naming the fraction, but also understanding within the context of one whole.



Skill 1.1 Identifies Fractions on a Number Line

98% of 3rd graders could shade $\frac{1}{4}$ of a unit whole, only 31% could find $\frac{1}{4}$ of a number line (Payne 1984)

Objective:

- To expand a student's knowledge of fractions from part of a whole and part of a set to a number line.
- To identify a fraction on a partitioned number line.
- To place a fraction on a number line that is not partitioned.

Common Misconceptions:

Student will count hashes instead of viewing it as a measurement between hashes. " $\frac{1}{4}$ is shaded"

When students partition a number line into fourths, they will draw four lines instead of three lines.

When students locate fractions on a number line they use their knowledge of whole numbers.

Draw a dot where $\frac{2}{3}$ would go.

Developing Fractional Reasoning Workbook

Prerequisite Knowledge: Students should understand basic number concepts and be able to count by ones and tens.

Skill 1.1: Activity 2

Fourths

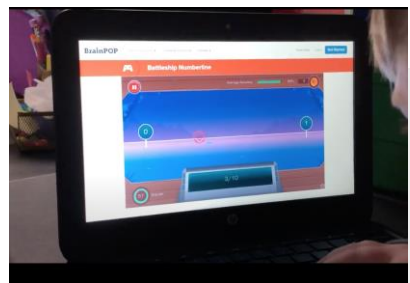
Sixths

Eighths

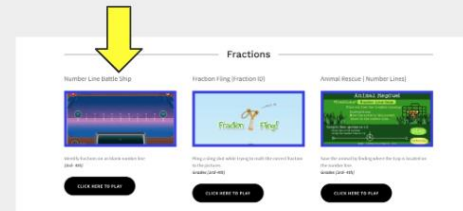
Fifths

Domain 1: Skill 1.1 Activity 2 - Lesson

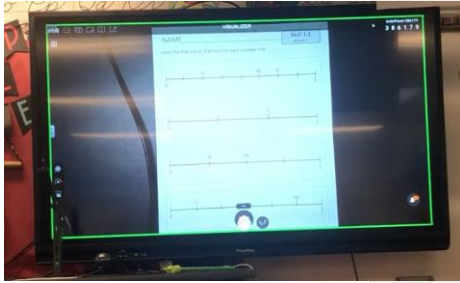
- What is the number line partitioned into?
- Use fraction language. Be sure to count the pieces, not the lines.
- Move to number lines without partitions. You must visualize what the number line is cut into.
- Teacher-led modeling of online practice. Located under "GAMES" on numeracyconsultants.net
- Number Line Battleship - student practice matches today's lesson.



Skill 1.1: Activity 2



Skill 1.1: Activity 1 & 2



NAME _____ Kerry Foster 7799 Skill 1.1 Activity 1

Label the fraction or fractions on each number line.

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NAME _____ Kerry Foster 7799 Skill 1.1 Activity 2

Draw a dot where the fraction would go.

Draw a dot on the line where $\frac{1}{2}$ would go.

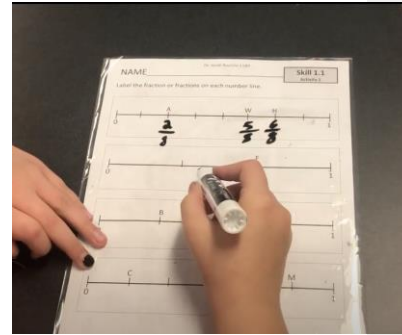
Draw a dot on the line where $\frac{1}{3}$ would go.

Draw a dot on the line where $\frac{2}{3}$ would go.

Draw a dot on the line where $\frac{3}{4}$ would go.

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Skill 1.1: Activity 1 & 2



NAME _____ Kerry Foster 7799 Skill 1.1 Activity 1

Label the fraction or fractions on each number line.

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NAME _____ Kerry Foster 7799 Skill 1.1 Activity 2

Draw a dot where the fraction would go.

Draw a dot on the line where $\frac{1}{2}$ would go.

Draw a dot on the line where $\frac{1}{3}$ would go.

Draw a dot on the line where $\frac{2}{3}$ would go.

Draw a dot on the line where $\frac{3}{4}$ would go.

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Domain 2: Comparing Fractions with Defined Characteristics

Key to the Student: "I am going to show you how fractions and I want you to compare and tell me which fraction is greater and explain why"

Skill 7

2 OF 8

3 OF 8

NOTES:

Evidence of understanding
 Use of procedure
 Evidence of misconception

Skill 8

1 OF 4

2 OF 3

NOTES:

Evidence of understanding
 Use of procedure
 Evidence of misconception

Skill 7 Question 2: Both fractions are the same size from being one whole, use that size to explain which fraction is greater.

5 OF 6

3 OF 4

NOTES:

Evidence of understanding
 Use of procedure
 Evidence of misconception

Skill 8 Question 1 and 2: Turn the benchmark of 1/2 to explain to me which fraction is greater.

4 OF 10

6 OF 10

NOTES:

Evidence of understanding
 Use of procedure
 Evidence of misconception

Assessment: Skill 7 & 8



- Domain 2 has to do with comparing fractions with defined characteristics.
- Now you are looking for evidence of understanding, use of procedure or evidence of misconception.
- If use a procedure, must still give an explanation of their reasoning.
- Skill 7 compares fractions with the same/like denominator.
- Skill 8 compares fractions with the same numerator.


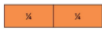
Domain 2: Skills 7 & 8



Domain 2: Comparing Fractions with Defined Characteristics

Comparing Fractions			
Skill 7	Skill 8	Skill 9	Skill 10
Same Denominator	Not 1 Fraction and Same Numerator	One Not Key From a Whole	Benchmark of 1/2

Teach as a strategy not a rule.
This strategy will not always work.....
This strategy works because.....


 $\frac{3}{4} > \frac{2}{4}$


- Skills 7-10 involve comparisons where the student uses fractional reasoning to determine the answer.
- It is important to use manipulatives to explain why comparing the numerator of fractions with the same denominator works.
- For Skill 7, students must understand why the strategy works in this situation.
- For Skill 8, must look at the denominators to know what we are comparing. Important to have physical & visual experiences of fractions with like numerators and different denominators.
- Plan to spend more time teaching Skill 8 ("less" is "more" concept).

Domain 2: Comparing Fractions with Defined Characteristics

Say to the Student: "I am going to show you two fractions and I want you to compare and tell me which fraction is greater and explain why"

SKILL 7

Evidence of understanding
 Use of Procedure
 Evidence of misconception

$\frac{2}{8}$ OF $\frac{6}{8}$

NOTES: Numerator denominator confusion. Treats Numerator as A denominator.

SKILL 8

Evidence of understanding
 Use of Procedure
 Evidence of misconception

$\frac{1}{4}$ OF $\frac{1}{8}$

Evidence of understanding
 Use of Procedure
 Evidence of misconception

$\frac{3}{7}$ OF $\frac{3}{9}$

NOTES: Nonsense visual. Inverse number relationship for reciprocals.

SKILL 9 Question 2: Both fractions are one piece away from being one whole. Use that piece to explain which fraction is greater.

Evidence of understanding
 Use of Procedure
 Evidence of misconception

$\frac{5}{6}$ OF $\frac{3}{4}$

NOTES:

SKILL 10 Question 1 and 2: Use the benchmark of $\frac{1}{2}$ to explain to me which fraction is greater.

Evidence of understanding
 Use of Procedure
 Evidence of misconception

$\frac{6}{10}$ OF $\frac{4}{9}$

NOTES:

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Assessment: Skill 9 & 10

- For Skill 9, both fractions are one piece away from a whole.
- For the 2nd question, both fractions are one piece away from being one whole. The student needs to use that piece to explain which fraction is greater.
- For Skill 10, the student must use the benchmark of $\frac{1}{2}$ to explain which fraction is greater.
- Repeat the directions if the student does not use " $\frac{1}{2}$ " in the explanation.
- There is not a Skill 11 on the assessment.**

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Domain 2: Comparing Fractions with Defined Characteristics

Comparing Fractions	Skill 7	Skill 8	Skill 9	Skill 10
	Same Denominator	Unit Fractions and Same Numerator	One Unit Away from a Whole	Benchmark of $\frac{1}{2}$

Teach as a strategy not a rule. Inverse relationship between the denominator and the size of the piece. Now less is more because you need less to complete a whole.

This strategy will not always work.....

This strategy works because.....


$\frac{7}{8} < \frac{4}{5}$

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Domain 2: Skills 9 & 10

- The goal of Skill 9 is to develop fraction sense when comparing fractions one unit away from a whole.
- You will want to remind students of previous skills involving counting fractional parts.
- Lessons use visuals to help with understanding "less is more."
- Skill 9 applies a situational strategy utilizing what was learned previously.
- Skill 10 lessons help to slowly develop the benchmark of $\frac{1}{2}$. Begin with even denominators and then move to odd.
- For a student to be successful in Domain 2, must be 100% fluent in Domain 1.

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Skill 8  **Compares Unit Fractions and Same Numerators** Domain 2

Formative Assessments:

Compare fractions, unit fractions, and fractions with the same numerator.

When comparing fractions with the same numerator focus on the inverse relationship between the size of the denominator and the size of the parts. As the denominator increases, the size of the parts decreases. The smaller the denominator, the larger the parts. Emphasize this only works when the numerators are the same. When the numerators is the same, the denominator will assess which one is greater.

$\frac{1}{8} < \frac{1}{5}$
 $\frac{3}{7} > \frac{3}{9}$

Same Numerator:


When comparing $\frac{2}{3}$ and $\frac{2}{6}$, students can conclude that $\frac{2}{3}$ is the larger fraction because thirds are larger than sixths, and two of the larger pieces must be more than two of the smaller pieces. This analogy involves understanding that an inverse relationship exists between the number of parts a unit is partitioned into and the size of the parts (Carnegie-Bethel-Park, Lead RMP Project, 1997).

$\frac{2}{3} > \frac{2}{6}$

Common Misconceptions:

$\frac{1}{8} > \frac{1}{6}$ Student will use whole number reasoning and ignore the inverse relationship between the size of the denominator and the size of the parts. " $\frac{1}{8}$ is more than $\frac{1}{6}$ is because 8 is larger than 6.

$\frac{5}{8} > \frac{5}{6}$ Student will use whole number reasoning and ignore the inverse relationship between the size of the denominator and the size of the parts. " $\frac{5}{8}$ is more than $\frac{5}{6}$ is because 8 is larger than 6.



Domain 2
Skill 8: Activity 1

Skill 8: Activity 1

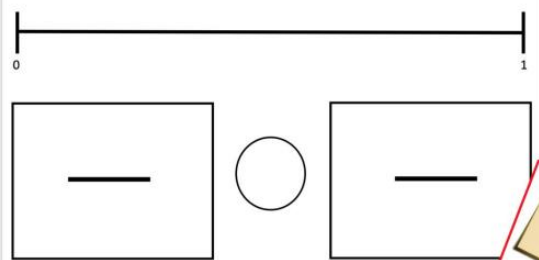
Objective:

- To understand when comparing fractions with the same numerator, focus on the denominator and the size of the pieces.

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Skill 8 : Activity 1

Domain 2
Skill 8: Activity 1



Write down testing points:

Write down:
 $\frac{1}{6} + \frac{1}{6} =$
 $\frac{2}{6} = \frac{1}{3}$

Kerry Kester ONLY 7799 

Domain 3: Manipulates Change to a Fraction, Equivalent Fractions and Common Denominators

Say to the Student: "What are all the fractions that are equivalent to $\frac{1}{2}$?" Correct Incorrect
 "What are all the fractions that are equivalent to $\frac{1}{3}$?" Correct Incorrect
 "What are all the fractions that are equivalent to $\frac{1}{4}$?" Correct Incorrect

Skill 12

Say to the Student: "Which fractions are equivalent or the same as $\frac{1}{2}$?" No paper permit.

Skill 13

$\frac{1}{3}$	$\frac{2}{4}$	$\frac{3}{5}$	$\frac{4}{8}$	$\frac{6}{10}$	$\frac{3}{6}$	$\frac{7}{7}$
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Say to the Student: "Write an equivalent fraction for $\frac{2}{3}$." "Write an equivalent fraction for $\frac{5}{15}$ that is in simplest form."

Skill 14

$\frac{3}{5}$	$\frac{5}{15}$
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Say to the Student: "Find a common denominator for the fraction sets below."

Skill 15

$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{6}$
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Assessment: Skill 12, 13, 14 & 15



- For Skill 12, looking for the student to make the connection between the number lines to name the equivalent fractions.
- Students don't always view the number lines as lining up with each other.
- For Skill 13, the equivalent fractions for $\frac{1}{2}$ must be named with automaticity.
- For Skill 14, the student must write an equivalent fraction through multiplying and then dividing for simplest form.
- Skill 14 & 15 are both open concept. The student must determine which procedure to apply.

Domain 3: Skills 12-15



Domain 3: Manipulates Equivalent Change to a Fraction

Equivalent Fractions and Common Denominators

Skill 12 Identifies Equivalents on a Number Line	Skill 13 Identifies Equivalents for $\frac{1}{2}$ with Automaticity	Skill 14 Generates Equivalent Fractions / Convert Fractions to Simplest Form	Skill 15 Generates a Common Denominator for Two Fractions
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Mixed Numbers and Improper Fractions

Skill 16 Converts a Mixed Number to an Improper Fraction	Skill 17 Converts an Improper Fraction to a Mixed Number
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3. What fraction is equivalent to $\frac{1}{2}$?

- For Skill 12, students need to understand equivalent fractions when given visuals or physical tools.
- Skill 13 requires students to focus on the shaded area. They must be able to determine even #'s (up to 20) divisible by 2 ($\frac{1}{2}$ of the denominator).
- Skill 14 helps students to understand they're just changing the pieces, not the total amount. (e.g., "x" by $\frac{3}{3} = "x" \cdot 1$)
- Teach equivalence through partitioning of fractions using visual to mathematical approach.
- For Skill 15, many other skills are involved. Be sure to give tools needed to help with determining common denominators.

Assessment: Skill 16 & 17



- Skill 16 & 17 are part of the written portion of the assessment.
- Student completes on his/her own before or after the interview portion of the assessment.
- Both skills are considered open concept as the student must apply a known procedure in order to answer.

Domain 3: Manipulates Change to a Fraction: Mixed Numbers and Improper Fractions
Distribute to the Student the *Arithmetic Procedures 16-21 sheet*:

Skill 16: $2\frac{1}{5} \times \frac{11}{12}$ **Skill 17:** $2\frac{1}{4} \times \frac{1}{2}$

Domain 4: Arithmetic with Fractions: Addition and Subtraction
Distribute *Arithmetic Procedures 16-21 WORKSHEET*:

Skill 18: $\frac{1}{4} + \frac{4}{8} = \frac{5}{8}$ $\frac{5}{10} - \frac{3}{10} = \frac{2}{10}$ **Skill 19:** $\frac{1}{4} + \frac{3}{8} = \frac{5}{8}$ $\frac{4}{5} - \frac{1}{2} = \frac{3}{10}$

Skill 20: $3\frac{1}{5} + 2\frac{1}{5} = 4\frac{2}{5}$ $5\frac{2}{10} - 2\frac{1}{10} = 3\frac{1}{10}$ **Skill 21:** $3\frac{2}{6} + 2\frac{1}{6} = 5\frac{3}{6}$ $8\frac{1}{2} - 5\frac{1}{2} = 2\frac{0}{15}$

Say to the Student: "Estimate where $2/8 + 1/2$ would go on the number line?" **Skill 22**
Say to the Student: "Estimate where 1 and $1/12 + 8/12$ would go on the number line?" **Skill 23**

Domain 4: Arithmetic with Fractions: Multiplication and Division
Distribute *Applying Arithmetic Procedures WORKSHEET (Skill 23-26)*

What is $1/4$ of 24? **Skill 23** Answer: 6
What is $2/3$ of $1/2$? **Skill 24** Answer: $1/3$ or $1/6$

How many $1/9$'s will go into $1/3$? **Skill 25** Answer: 3
How many $1/6$'s will go into $1/3$? **Skill 26** Answer: 2

If 5 is multiplied by $1/6$ will the product be larger or smaller than 5? Explain **Skill 27** Smaller
If 5 is multiplied by $2/3$ will the product be larger or smaller than 5? Explain **Skill 27** Larger

©Numeracy Counts/Factorial Assessments

Domain 3: Skills 16 & 17 Plus Summary



- Mixed numbers and improper fractions were taught back in Domain 1.
- They may be able to determine the answer using alternative methods, but the numbers are getting too large.
- For Skills 16 & 17, you want the students to be able to determine the answer procedurally plus be able to explain their thinking.
- Domain 3 is a very important domain. Many students have difficulty with these skills.
- Everything builds to get to the next skill by ensuring prior skills are in place.

Domain 3: Manipulates Equivalent Change to a Fraction
Equivalent Fractions and Common Denominators

Skill 12	Skill 13	Skill 14	Skill 15
Identifies Equivalents on a Number Line	Identifies Equivalents for $1/2$ with Automaticity	Generates Equivalent Fractions / Converts Fractions to Simplest Form	Generates a Common Denominator for Two Fractions

Mixed Numbers and Improper Fractions

Skill 16	Skill 17
Converts a Mixed Number to an Improper Fraction	Converts an Improper Fraction to a Mixed Number

$\frac{9}{5}$ $1\frac{4}{5}$

Skill 13

Formative Assessments

Identify Equivalents for $\frac{1}{2}$ with Automaticity

Domain 3

Recognize equivalent fractions for $\frac{1}{2}$ with automaticity.

Recognize equivalent fractions using visual models. Be able to recognize equivalent fractions for $\frac{1}{2}$ without models without using a process or procedure.

$\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$

Why Automaticity with $\frac{1}{2}$ is so important:

Automaticity with one half is a very important skill. Having this skill in place will enable students to quickly use estimation to find out the reasonableness of an answer. It will also help the student keep perspective when they are working with larger numerators and denominators in fractions. Equivalent fractions for $\frac{1}{2}$ are usually one of the first equivalents that are taught to a student.

Common Misconceptions:

Double of 2 is 4 Student does not understand the concept that the double of a number is directly connect to half of a number.

Half of 4 is 2 Student may not understand conceptually what an equivalent fraction is because they inappropriately use whole number reasoning.

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Domain 3: Skill 13 Lesson

- In previous lesson used fractional pieces to find equivalent fractions.
- Today learning how to find equivalent fractions using a picture.
- “Whatever you do to one piece, you have to do to the other piece.” (Equal partitioning)
- Extend practice to thirds & fourths using blank sheet.

Independent Practice

Domain 3

Skill 13: Activity 2

Skill 13: Activity 2

Objective:

- To generate equivalent fractions for $\frac{1}{2}$ through partitioning.

$\frac{1}{2}$

=

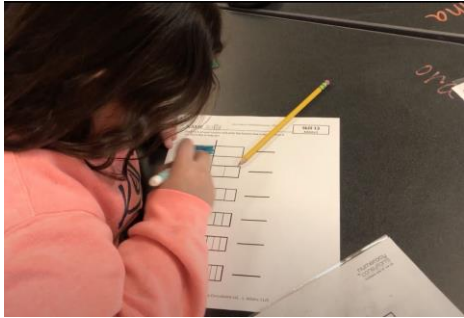
26


Skill 13: Activity 2

$\frac{1}{2}$

=

27





Domain 3 Skill 13: Activity 2

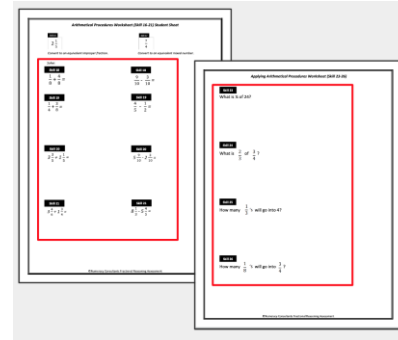
NAME _____
Maths Master 2209
Skill 13
Activity 2


Shade in $\frac{1}{3}$ of each fraction and write the fraction that is equivalent to $\frac{1}{3}$. Use the $\frac{1}{3}$ line to help you.

1	2	—
1	2	—
1	2	—
1	2	—
1	2	—
1	2	—

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Assessment: Written Procedures Skill 18-21 & Skill 23-26

- Skill 18-21 are a separate worksheet of procedures.
- Skill 23-26 are also a procedures worksheet. You can run the two sheets front to back.
- Refer to Free Webinar (1 hour 40 min. mark) for tips on scoring written procedures)
- Skill 22 & 27 are interview questions.

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Domain 3: Manipulating Change to a Fraction: Mixed Numbers and Improper Fractions
 Distribute to the Student the *Arithmetic Procedures 16-21 sheet*

Skill 16 $2\frac{1}{2} + 1\frac{1}{3}$ **Skill 17** $2\frac{1}{4} + 3\frac{3}{4}$

Domain 4: Arithmetic with Fractions: Addition and Subtraction
 Distribute *Arithmetic Procedures 16-21 WORKSHEET*

Skill 18 $\frac{1}{8} + \frac{4}{8} = \frac{5}{8}$ $\frac{9}{10} - \frac{3}{10} = \frac{6}{10}$ **Skill 19** $1\frac{3}{4} - \frac{5}{8}$ $\frac{4}{5} - \frac{1}{2} = \frac{3}{10}$

Skill 20 $3\frac{2}{5} + 2\frac{1}{5} = 4\frac{3}{5}$ $5\frac{7}{10} - 2\frac{3}{10} = 3\frac{4}{10}$ **Skill 21** $3\frac{2}{3} + 2\frac{1}{4} = 5\frac{11}{12}$ $8\frac{1}{5} - 5\frac{2}{5} = 2\frac{3}{5}$

Say to the Student: "Estimate where $2\frac{1}{8} + 3\frac{3}{8}$ would go on the number line!" **Skill 22**
 Say to the Student: "Estimate where 1 and $2\frac{1}{2} + 8\frac{1}{8}$ would go on the number line?" **Skill 23**

Domain 4: Arithmetic with Fractions: Multiplication and Division
 Distribute *Applying Arithmetic Procedures WORKSHEET (Skill 23-26)*

What is $\frac{1}{2}$ of 24? **Skill 23** Answer: 6
 What is $\frac{2}{3}$ of 36? **Skill 24** Answer: 6/22 or 36
 How many $\frac{1}{2}$'s will go into 4? **Skill 25** Answer: 22
 How many $\frac{1}{6}$'s will go into $\frac{1}{4}$? **Skill 26** Answer: 6

If 5 is multiplied by $\frac{1}{5}$ will the product be larger or smaller than 5? Explain. **Skill 27** smaller
 If 5 is multiplied by 2 will the product be larger or smaller than 5? Explain. **Skill 27** larger

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Assessment: Skill 22 & 27 (Interview)



- For Skill 22, the estimate of where the answer would go on the number line should be within a $\frac{1}{2}$ to be given credit.
- For Skill 27, again the student should not solve to find the answer.
- They need to be able to explain their thinking.

Domain 4: Arithmetic with Fractions
 Procedures for Addition / Subtraction and Estimation

Skill 18 Add & Subtract with Like Denominators **Skill 19** Add & Subtract Unlike Denominators **Skill 20** Add & Subtract Mixed Numbers Like Denominators **Skill 21** Add & Subtract Mixed Numbers Unlike Denominators **Skill 22** Estimation of Addition of Fractions

Understanding Multiplication and Division

Skill 23 Multiply a Whole Number by a Fraction **Skill 24** Multiply a Fraction by a Fraction **Skill 25** Divide a Whole Number by a Fraction **Skill 26** Divide a Fraction by a Fraction **Skill 27** Understanding Multiplication of Fractions

Distribute *Arithmetic Procedures 16-21 WORKSHEET*

Skill 16 $2\frac{1}{2} + 1\frac{1}{3}$ **Skill 17** $2\frac{1}{4} + 3\frac{3}{4}$

Skill 18 $\frac{1}{8} + \frac{4}{8} = \frac{5}{8}$ $\frac{9}{10} - \frac{3}{10} = \frac{6}{10}$ **Skill 19** $1\frac{3}{4} - \frac{5}{8}$ $\frac{4}{5} - \frac{1}{2} = \frac{3}{10}$

Skill 20 $3\frac{2}{5} + 2\frac{1}{5} = 4\frac{3}{5}$ $5\frac{7}{10} - 2\frac{3}{10} = 3\frac{4}{10}$ **Skill 21** $3\frac{2}{3} + 2\frac{1}{4} = 5\frac{11}{12}$ $8\frac{1}{5} - 5\frac{2}{5} = 2\frac{3}{5}$

Say to the Student: "Estimate where $2\frac{1}{8} + 3\frac{3}{8}$ would go on the number line!" **Skill 22**
 Say to the Student: "Estimate where 1 and $2\frac{1}{2} + 8\frac{1}{8}$ would go on the number line?" **Skill 23**

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Domain 4: Skills 18-27



- Skills 18-21 are all procedural, indicated by the black heading.
- Skill 22 measures the raw understanding (fraction sense) of estimating fractions.
- The gray heading for Skills 23-26 means the problems can be solved procedurally or with conceptual understanding.
- It is a plus if a student can solve these problems conceptually. Then just need to learn procedure.
- Skills 27 is based on knowledge of understanding of fractions.
- Prior skills set the stage for Domain 4. Understanding the skills before & after each skill will help your instruction.

Skill 22 **Estimating for Reasonableness with Addition / Subtraction** **Domain 4**

Formative Assessments

The most important reference mark for fractions are 0, $\frac{1}{2}$, and 1 (Van de Walle 2006).

Traditionally estimation and mental math have been thought of as supplemental skills. However, based on a survey of adults, Carlson (1985) found that most of the mathematics used in everyday life relies far more on estimation and mental computation than on traditional computation...

Traditionally mental math and estimation have been taught after student's master pencil and paper computation. However, Kilpatrick (2001) found not only that children can learn to compute mentally and to estimate before leaving formal pencil and paper computational procedures, but also that mental math and estimation activities prior to formal work with computation actually enhance learning of computation (Spangler 2013).

Research Item (Reys & Young, 1998)

Without Calculating an exact answer, circle the best estimate for $32 \frac{1}{2} - 7 \frac{1}{2}$.

	6 th Grade	8 th Grade
A. 1	37%	20%
B. 27	25%	38%
C. 19	36%	14%
D. 21	58%	13%
E. DK	13%	18%

$1 \frac{5}{11} + 4 \frac{11}{12} =$
 $1 \frac{1}{7} + 5 = 6 \frac{1}{2}$

$3 \frac{2}{11} + 6 \frac{11}{12} =$
 $3 + 7 = 10$

Common Misconceptions

Students are unable to use reference points and need to compute the problem before they are able to make an estimate.

$1 \frac{7}{8} + 2 \frac{5}{6} = ?$

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Domain 4

Skill 22: Activity 1

Skill 22: Activity 1

Objective:

- Understand how to estimate for reasonableness when adding or subtracting two fractions that are less than one.

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Skill 22: Activity 1

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numbers are all we do

Domain 4

Skill 22: Activity 1

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Numeracy Consultants Fractional Reasoning Intervention Webinar

(Webinar time frames are listed below as a quick reference for follow up viewing.)

Section	Time Frame
Introduction	0 - 6:50
What is Fraction Sense?	6:51 - 9:34
Fractions are Part of the Curriculum	9:37 - 11:51
Data Trends	11:54 - 25:22
Fraction Connections to Primary Numeracy	25:23 - 32:41
4 Domains of the Assessment	32:42 - 37:46
Administration of Fractional Reasoning Assessment	37:56 - 1:40:00
Scoring and Lessons	1:40:00 - 2:03:00

PROGRESS MONITORING

Fractional Reasoning Assessment

Student: _____ Date: _____ Grade: _____

Assessor: _____ Classroom Teacher: _____

Domain 1: Understanding a Fraction within the Context of One Whole

Measuring Fractions with Different Models and Counting by Fractional Parts

Skill 1	Skill 11	Skill 2	Skill 3	Skill 31
Identifies basic fractions as a number line.	Counts by fractional parts.	Identifies improper fractions on a number line.	Identifies improper fractions on a number line.	Identifies improper fractions on a number line.

Fractions as Part of a Whole / Whole and Given Parts

Skill 4	Skill 5	Skill 6	Skill 6
Fraction is less than one.	Composites a whole with unit and non-unit fractions.	Model numbers are greater than one.	Improper fractions are greater than one.

Domain 2: Comparing Fractions with Defined Characteristics

Comparing Fractions

Skill 7	Skill 8	Skill 9	Skill 10
Same denominators.	Unit fractions and same numerators.	See skill flow from a whole.	Benchmark of 1/2.

Domain 3: Manipulates Equivalent Change to a Fraction

Equivalent Fractions and Common Denominators

Skill 12	Skill 13	Skill 14	Skill 15
Identifies equivalents on a number line.	Identifies equivalents to 1/2 with benchmark.	Generates equivalent fractions / common fractions in shaded strip.	Generates a common denominator for two fractions.

Mixed Numbers and Improper Fractions

Skill 16	Skill 17
Converts a mixed number to an improper fraction.	Converts an improper fraction to a mixed number.

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Domain 1: Progress Monitoring

Date	Skill 1	Skill 11	Skill 2	Skill 3	Skill 31	Skill 4	Skill 5	Skill 6	Skill 6	%
5/2/21	+	+	+	+	+	+	+	+	+	100%

Domain 2: Progress Monitoring

Date	Skill 7	Skill 8	Skill 9	Skill 10	%
5/2/21	x	x	x	x	0%

Domain 3: Progress Monitoring

Date	Skill 12	Skill 13	Skill 14	Skill 15	Skill 16	Skill 17	%
5/2/21	x	+	x	x	x	x	16%

Domain 4: Progress Monitoring

Date	Skill 16	Skill 17	Skill 20	Skill 21	Skill 22	Skill 23	Skill 24	Skill 25	Skill 26	Skill 27	%
5/2/21	+	x	+	x	x	x	x	x	x	x	20%