Multiplicative Thinking

Skip Counting Multiplication/Division Facts
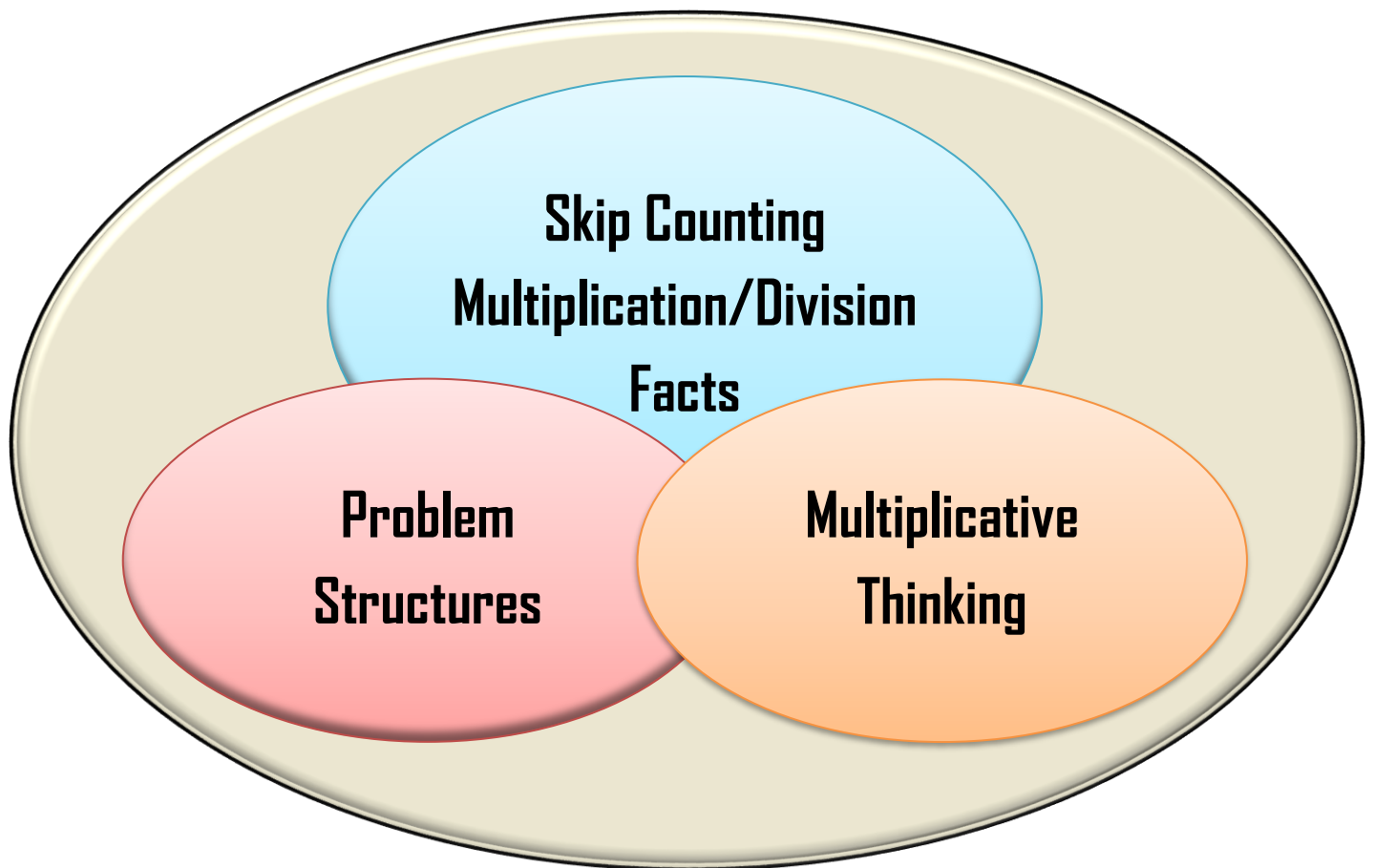
Problem Structures

Multiplicative Thinking

Assessment

Multiplicative Thinking Assessment

Student Name: _____   Date: _____

Assessor: _____   Grade: _____

Basic Skills

Skill	Level A < 2 Sets	Level B 2, 5, and 10	Level C 6 Sets	Level D All Sets
Skip Counting				

Skill	Level A ≤20%	Level B 30%-50%	Level C 60%-80%	Level D 90%-100%
Multiplication Facts				
Division Facts				

Multiplicative Thinking Levels

Problem Structure	Level AA 65% or Less Correct	Level A Counts One-to-One Stress/ Rhythm Counts	Level B Additive Composition *Many-to-One Counting*	Level C Partial Multiplicative Reasoning	Level D Flexible Multiplicative Reasoning
Equal Groups Product Unknown (1-8)	3 or more incorrect				
Division (9-13)	2 or more incorrect				
Rate/Comparison/Scale (14-17)	2 or more incorrect				

If percentage correct for any problem structure is equal to 65% or less, the student is automatically a level AA for that problem structure.

If two levels are equal, the lower of the two levels is the student's tested level.

For Level C, "Partial Multiplicative Reasoning", if student shows both level B and Level D, you combine the total for a score for Level C. If the total for Level C is greater than any other level, the student's level is Level C. If it is equal to another level, the student is the lower level.

Problem Structure	Level AA Novice Understanding 0/3	Level A	Level B Developing Understanding 1/3	Level C Partial Understanding 2/3	Level D Flexible Multiplicative Reasoning 3/3
Multiplicative Connections (18-20)					

Multiplying Multiplies of Ten

Problem Structure	Level AA	Level A Single Digit X's Ten	Level B Two Digit Number X's Ten	Level C Multiple of Ten X's Single Digit	Level D Multiple of Ten X's Multiple of Ten
Multiplying Multiples of Ten					

Skip Counting

Say to the Student: ***"I am going to give you a number and I want you to start skip counting by the number until I tell you to stop."***

	Fluent		Not Fluent			
					Level A	Two or less
2-20	○		○			
5-50	○		○			
10-100	○		○		**Level B**	Two's, Five's, and Ten's
9-90	○		○			
3-30	○		○			
4-40	○		○			
7-70	○		○		**Level C**	Combination of any three levels
8-80	○		○		**Level D**	Fluent 2-10
6-60	○		○			

Multiplication/Division Fluency Assessment

Say to the Student: ***"I am going to give you some multiplication problems. I want you to solve them as quickly as you can.) DO NOT tell the student they only have 30 seconds. Repeat directions with division.***

2x4 = ☐   3x5 = ☐   4x6 = ☐   7x8 = ☐   9x7= ☐

6x4 = ☐   5x5 = ☐   7x7 = ☐   8x8 = ☐   4x4= ☐

45/5 = ☐   15/3 = ☐   9/3 = ☐   21/3 = ☐   56/7 = ☐

12/2 = ☐   18/6 = ☐   28/7 = ☐   24/6 = ☐   54/9 = ☐

Equal Groups

Show Student Image 1:   Multiplication: `Equal Groups Concrete`

Say to the student: *How many cans of soda are there total?*
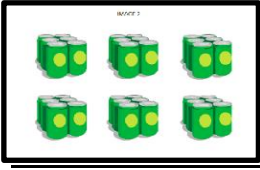
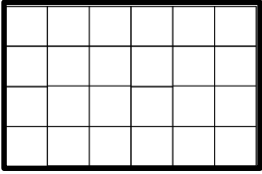|	
Flexible Multiplicative Thinking (6x6) ◯	(Stress Counts) ◯
(Skip counts) ◯	(Counts by Ones) ◯
(Repeated Addition) ◯	(Correct Answer) ◯ 36

Show Student Image 2: Multiplication: `Equal Groups Concrete`

Say to the student: *How many squares are there total?*

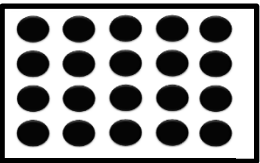|	
Flexible Multiplicative Thinking (4x6) or (6x4) ◯	(Stress Counts) ◯
(Skip counts) ◯	(Counts by Ones) ◯
(Repeated Addition) ◯	(Correct Answer) ◯ 24

Show Student Image 3: Multiplication: `Equal Groups Concrete`

Say to the student: *How many dots are there in the array?*

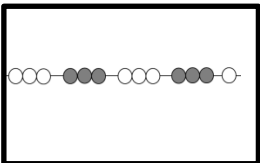|	
Flexible Multiplicative Thinking (4x5) or (5x4) ◯	(Stress Counts) ◯
(Skip counts) ◯	(Counts by Ones) ◯
(Repeated Addition) ◯	(Correct Answer) ◯ 20

Show Student Image 4: Multiplication: `Equal Groups Concrete  +1`

Say to the student: *How many beads are there?*

Flexible Multiplicative Thinking (4x3+1) ◯	(Stress Counts) ◯
(Skip counts) ◯	(Counts by Ones) ◯
(Repeated Addition) ◯	(Correct Answer) ◯ 13

Show Student Image 5: Multiplication: `Equal Groups Semi-Concrete`

Say to the student: *All of the cards are fours, what is the total?*

Flexible Multiplicative Thinking (4x4) ◯	(Stress Counts) ◯
(Skip counts) ◯	(Counts by Ones) ◯
(Repeated Addition) ◯	(Correct Answer) ◯ 16

Show Student Image 6: Multiplication: `Equal Groups Semi-Concrete`

Say to the student: *If there were 4 tables in a room that looked like this, how many people would there be?*

Flexible Multiplicative Thinking (5x4) or (4x5) ◯   (Stress Counts) ◯

(Skip counts) ◯   (Counts by Ones) ◯

(Repeated Addition) ◯   (Correct Answer) ◯ 20

Show Student Image 7: Multiplication: `Equal Groups Semi-Concrete`

Say to the student: *The rectangle is covering up some of the dots in the array? How many dots are there total?*

Flexible Multiplicative Thinking (3x5) or (5x3) ◯   (Stress Counts) ◯

(Skip counts) ◯   (Counts by Ones) ◯

(Repeated Addition) ◯   (Correct Answer) ◯ 15

Show Student Image 8: Multiplication: `Equal Groups Semi-Concrete`

Say to the student: *There are 4 shirts in each drawer in this dresser. How many shirts are there total?*
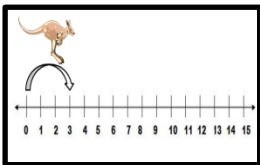
Flexible Multiplicative Thinking (4x6) or (6x4) ◯   (Stress Counts) ◯

(Skip counts) ◯   (Counts by Ones) ◯

(Repeated Addition) ◯   (Correct Answer) ◯ 24

Problem Structure	Incorrect	Level A Counts by One Stress/ Rhythm Counts	Level B Additive Composition *Many-to-One Counting*	Level C Partial Multiplicative Reasoning	Level D Flexible Multiplicative Reasoning
Equal Groups Product Unknown (1-8)				⬅ ➡	

Division

Show Student Image 9: `Division Measurement`

Say to the student: *Kangaroo jumps 3 feet per jump. How many jumps will it take the Kangaroo to jump 15 feet?*

Flexible Multiplicative Thinking (3x_=15) or (15/3) ◯	(Stress Counts) ◯
(Skip counts) ◯	(Counts by Ones) ◯
(Repeated Addition/Subtraction) ◯	(Correct Answer) ◯ 5

Show Student Image 10: `Division Measurement`

Say to the student: *The man has 24 cookies. He puts 6 cookies a bag. How many bags does he need?*

Flexible Multiplicative Thinking (6x_=24) or (24/6) ◯	(Stress Counts) ◯
(Skip counts) ◯	(Counts by Ones) ◯
(Repeated Addition/Subtraction) ◯	(Correct Answer) ◯ 4

Show Student Image 11: `Division Measurement`

**Say to the student:** *The tree has 20 apples. 5 apples will go in each bucket. How many buckets are needed.*

Flexible Multiplicative Thinking (__x 5=20) or (20/5) ◯	(Stress Counts) ◯
(Skip counts) ◯	(Counts by Ones) ◯
(Repeated Addition/Subtraction) ◯	(Correct Answer) ◯ 4

Show Student Image 12: `Division Partitive`

**Say to the student:** *There are 21 total apples. I want you to evenly divide them into 3 boxes.*
How many apples are in each box?
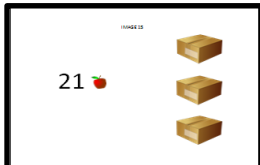
Flexible Multiplicative Thinking (3x_=21) or (21/3) ◯	(Stress Counts) ◯
(Skip counts) ◯	(Counts by Ones) ◯
(Repeated Addition/Subtraction) ◯	(Correct Answer) ◯ 7

Show Student Image 13: `Division Partitive`

**Say to the student:** *The 18 doughnuts are equally put into 6 boxes. How many doughnuts will go in each box?*
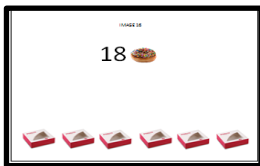
Flexible Multiplicative Thinking (6x_=18) or (18/6) ◯	(Stress Counts) ◯
(Skip counts) ◯	(Counts by Ones) ◯
(Repeated Addition/Subtraction) ◯	(Correct Answer) ◯ 3

Problem Structure	Incorrect	Level A Counts by One Stress/ Rhythm Counts	Level B Additive Composition *Many-to-One Counting*	Level C Partial Multiplicative Reasoning	Level D Flexible Multiplicative Reasoning
Division(9-13)				⬅ ➡	

Extending Multiplication: Rate/ Comparison

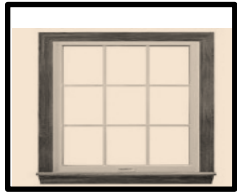***Show Student Image 14: Multiplication: Rate***

Say to the student: *The man can walk 5 miles in one hour. How many miles will he walk if he walks for 6 hours?*
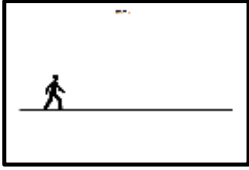
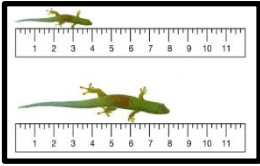|	
Flexible Multiplicative Thinking (5x6) or (6x5) ◯	(Stress Counts) ◯
(Skip counts) ◯	(Counts by Ones) ◯
(Repeated Addition) ◯	(Correct Answer) ◯ 30

Show Student Image 15: Multiplication: Rate

Say to the student: *It takes 3 minutes to wash each window pane section. How long would it take for someone to wash all of the window panes?*

Flexible Multiplicative Thinking (3x9) or (9x3) ◯	(Stress Counts) ◯
(Skip counts) ◯	(Counts by Ones) ◯
(Repeated Addition) ◯	(Correct Answer) ◯ 27

Show Student Image 16: Multiplicative Comparison

Say to the student: *Is the second lizard twice as long as the first. Explain why.*

Flexible Multiplicative Thinking (2x4) or (4x2) ◯	(Stress Counts) ◯
(Skip counts) ◯	(Counts by Ones) ◯
(Repeated Addition) ◯	(Correct Answer) ◯ **YES** *"2x is same as twice as much"*

Show Student Image 17: Multiplicative Comparison

Say to the student: *Does the second dog weigh 3 times as much as the first? Explain why.*
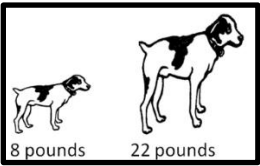
Flexible Multiplicative Thinking (8x3) or (3x8) ◯	(Stress Counts) ◯
(Skip counts) ◯	(Counts by Ones) ◯
(Repeated Addition) ◯	(Correct Answer) ◯ **NO** *"3x as much is 24"*

Problem Structure	Incorrect	**Level A** Counts by One Stress/ Rhythm Counts	**Level B** Additive Composition *Many-to-One Counting*	**Level C** Partial Multiplicative Reasoning	**Level D** Flexible Multiplicative Reasoning
Rate/Multiplicative Comparison (14-17)				⬅ ➡	

Multiplicative Connections: Inverse Relationship

Show Student Image 18: `Multiplicative Connections:  Using a known Fact to Solve`

Say to the student: *Three times eight equals twenty-four. Can you use this problem to help you figure out 4 x 8?*

IMAGE 17

3 x 8 = 24

4 x 8 =

Multiplicative Thinking (24 +8=32) ◯

Unable to connect or explain ◯

Show Student Image 19: `Multiplication Connections:  Inverse Relationship`

Say to the student: *The top problem says that 6 x 4=24. How can you use 6 x 4 to help you solve 24/6=?*

IMAGE 17

6 x 4 = 24

24 ÷ 6=
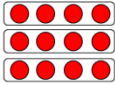
Multiplicative Thinking Inverse Relationship (6x4 =24 to 24/6=4) ◯

Unable to connect or explain ◯

Show Student Image 20: `Multiplication Connections:  Inverse Relationship`

Say to the student: *Can you give me a multiplication problem for this array? After answer, "Can you give me a division problem?"*

IMAGE 18

Multiplicative Thinking Inverse Relationship (3x4 =12 and 12/3=4) ◯

Unable to connect or explain ◯

Problem Structure	Level AA Novice Understanding 0/3	Level A	Level B Developing Understanding 1/3	Level C Partial Understanding 2/3	Level D Flexible Multiplicative Reasoning 3/3
Multiplicative Connections					

Multiplying by Ten or Multiples of Ten

Say to the Student: ***"Now I am going to ask you some more multiplication problems. Answer the best you can.***

Level A

8 x 10 = ⬜   **10 x 6=** ⬜

Level B

14 x 10= ⬜   **38 x 10=** ⬜   **10 x 45=** ⬜

Level C

20 x 6 = ⬜   **40 x 3 =** ⬜   **60 x 6 =** ⬜

Level D

30 x 50 = ⬜   **50 x 90=** ⬜

Student Progress Monitoring

Student Name: _____   Date: _____

Focus Area: _____   Grade: _____

Date: Skill Set	Level	Level	Level	Level	Level	Level
Basic Skills						
Skip Counting						
Multiplication Facts						
Division Facts						
Problem Structures						
Equal Groups						
Division						
Rate/Comparison/Scale						
Multiplicative Connections						
Multiplying by Ten						
Multiplying by Ten Or Multiplies of Ten						

Documentation of Activities

Basic Skills	Problem Structures	Multiplying by Ten or Multiples of Ten

NOTES: _____

_____

_____

_____

2x4 = ☐   3x5 = ☐   4x6 = ☐   7x8 = ☐   9x7= ☐

6x4 = ☐   5x5 = ☐   7x7 = ☐   8x8 = ☐   4x4= ☐

Score: _____ **/10**

Cut -

45/5 = ☐   15/3 = ☐   9/3 = ☐   21/3 = ☐   56/7 = ☐

12/2 = ☐   18/6 = ☐   28/7 = ☐   24/6 = ☐   54/9 = ☐

Score: _____ **/10**