Multiplicative Thinking Framework (3-5)

Skip Counting

Two or Less (A)	2's 5's 10's (B)	6 Total Sets (C)	All Sets (D)

Multiplication & Division Facts

Multiplication

0 — 10 — 20 — 30 — 40 — 50 — 60 — 70 — 80 — 90 — 100

Division

Multiplication: Equal Groups

(AA)
Student is unable to consistently compute the total when presented with a variety of equal group models. Student earned a score of 65% or less. Student's basic understanding of multiplication needs to be addressed.

(A)
Student counts one-to–one when presented with equal group problems. To progress, they need to begin to identity the number of groups, how many are in each group, and without being prompted.

(B)
Student uses the additive composition or counts many to one strategy when calculating the total for equal group problems. To progress, they need to learn how to use the multiplier and multiplicand as an operation.

(C)
Student alternates between additive composition and flexible reasoning. To progress, the student may need to practice facts. They may also need instruction that focuses on identifying the structure that exists within each problem.

(D)
Student consistently and efficiently uses the multiplier and multiplicand simultaneously as an operation. To progress, student will need to understand the inverse relationship between multiplication and division.

Division: Measurement and Partitive

(AA)
Student is unable to consistently calculate the quotient. Student earned a score of 65% or less. To progress, student needs to understand the basics: that division is finding how many groups or how many in each group.

(A)
Student solves division problems by using one-to-one counting either perceptually or figuratively. Student may need to draw out problem. To progress, they need to connect division to multiplication.

(B)
Student solves division problems by use of additive composition/ skip counting. To progress student needs to understand the direct connection between multiplication and division, and understand the inverse relationship.

(C)
Student solves division problems by alternating between additive composition and flexible reasoning. To progress, student needs to focus on understanding the relationship between the multiplier, multiplicand, and product.

(D)
Student solves division problems by consistently applying flexible multiplicative reasoning. Student has a proficient understanding of the inverse relationship between multiplication and division.

Extending Multiplication: Rate/Multiplicative Comparison

(AA)
Student is unable to consistently compute the answer to rate and multiplicative comparison problems. Student earned a score of 65% or less. To progress, the student needs to connect rate and multiplicative comparison to equal groups.

(A)
The student counts one-to–one to solve the problems. Student may need to connect their current understanding of rate/multiplicative comparison to their basic understanding of equal groups and multiplication.

(B)
Student solves the problems by using additive composition in one form or another or counts many to one. To progress, student needs to connect to their knowledge of equal groups.

(C)
Student solves the problems by alternating between flexible thinking and additive composition strategies. Student may need to focus on the connection of facts to the structure inherent within each problem.

(D)
Student solves the problems by consistently applying flexible multiplicative reasoning. Student has a proficient understanding of the connection between multiplication and different problem structures.

Multiplicative Connections: Inverse Relationship

(AA)
Student is unable to conceptually **use a known fact** to help them solve another fact and **explain the inverse relationship between multiplication and division.**

(B) 1/3 (C) 2/3 3/3 (D)
Student is able to **use a known fact** to help solve another multiplication problem within the same multiplication family. For example, a student is able to understand and explain how 3 x 8 can be used to solve 4 x 8. Student is also able to use **the inverse relationship** between multiplication and division to solve a division problem with a satisfactory explanation. Finally, the student is able to **produce both a multiplication problem and a division problem** for a multiplicative situation.

Multiplying by Multiplies of Ten

(AA)
Student cannot calculate a single digit number X ten with automaticity. They may be able to calculate the correct answer by counting by tens, but they do not have automaticity.

(A)
Student can multiply a single digit number by ten and know the answer with automaticity.

(B)
Student can multiply a two digit number by ten and know the answer with automaticity.

(C)
Student can multiply a two digit multiple of ten by a single digit with automaticity.

(D)
Student can multiply two digit multiples of ten by each other with automaticity.

Multi-Digit Multiplication & Division

Place Value Method

23x36=

	20	**+ 3**
30	600	90
+	120	18
6		

600
120
90
+18
828

Expanded Notation

23x36
20 + 3
30 + 6
6x3= 18
6x20= 120
30x3= 90
30x20= 600
828

Shortcut/Algorithm

23
x 36
138
+690
828

Place Value Method

834/6=

	100	**+ 30**	**+ 9**
6	834	234	54
	600	180	54
	234	54	0

Expanded Notation

9 **=139**
30
100
6 834
-600
234
-180
54
-54
0