2210 Syllabus Content:

1.1 Data Representation

Candidates should be able to:

- Understand how and why computers use binary to represent all forms of data
- Understand the denary, binary and hexadecimal number systems
- Convert between
 - positive denary and positive binary
 - positive denary and positive hexadecimal
 - positive hexadecimal and positive binary
- Understand how and why hexadecimal is used as a beneficial method of data representation
- Add two positive 8-bit binary integers
- Understand the concept of overflow and why it occurs in binary addition
- Perform a logical binary shift on a positive 8-bit binary integer and understand the effect this has on the positive binary integer
- Use two's complement to represent positive and negative 8-bit binary integers
 - Convert a positive binary or denary integer to a two's complement 8-bit integer and vice versa
 - Convert a negative binary or denary integer to a two's complement 8-bit integer and vice versa
- Understand how and why a computer represents text and the use of character sets, including American standard code for information interchange (ASCII) and Unicode

Denary Number System:

We know decimal or denary number system has (base 10). This uses digits 0 to 9 and has place values below

10 00 0	1000	100	10	units
3	1	4	2	1

Binary number system:

The **binary** system on computers uses combinations of 0s and 1s and has (base 2).

128	64	32	16	8	4	2	1
(27)	(26)	(25)	(24)	(23)	(22)	(21)	(20)





A typical binary number would be:

1 1 1 0 1 1 1 0

Binary place values

You can also break a **binary** number down into place-value columns, but each column is a power of two instead of a power of ten. For oxample, take a binary number like 1001

-or example, t	ake a Dinary H	under like 10	UI.
Eights 8s (2³)	Fours 4s (2ຶ)	Twos 2s (2΄)	Ones 1s (2 [°])
1	0	0	1

By looking at the place values, we can calculate the equivalent denary number.

That is:

 $(1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) = 8 + 0 + 0 + 1$ $(\mathbf{1} \times 8) + (\mathbf{0} \times 4) + (\mathbf{0} \times 2) + (\mathbf{1} \times 1) = 8 + 1$ = 9

Converting binary to denary

notes of Sir Ma To calculate a large **binary** number like **10101000** we need more place values of multiples of 2.

- $2^7 = 128$
- $2^6 = 64$
- $2^5 = 32$
- $2^4 = 16$ •
- $2^3 = 8$
- $2^2 = 4$
- $2^1 = 2$
- $2^0 = 1$

In **denary** the sum is calculated as:

 $(1x2^{7}) + (0 x 2^{6}) + (1 x 2^{5}) + (0 x 2^{4}) + (1 x 2^{3}) + (0 x 2^{2}) + (0 x 2^{1}) +$ $(0x2^{0}) = 168$ $(\mathbf{1} \times 128) + (\mathbf{0} \times 64) + (\mathbf{1} \times 32) + (\mathbf{0} \times 16) + (\mathbf{1} \times 8) + (\mathbf{0} \times 4) + (\mathbf{0} \times 2) + (\mathbf{0} \times 6)$ 1) = 128 + 32 + 8 = 168

The table below shows denary numbers down the left with their equivalent binary numbers marked out below the base 2 columns. Each individual column in the table represents a different **place value** equivalent to the base 2 powers



Convert between denary

					Binary	pattern			
		Place value 128	Place value 64	Place value 32	Place value 16	Place value 8	Place value 4	Place value 2	Place value 1
	0								0
	1								1
	2							1	0
	3							1	1
Denary number	4						1	0	0
m	5						1	0	1
y n	6						1	1	0
nar	7						1	1	1
De	8					1	0	0	0
	9					1	0	0	1
	10					1	0	1	0
	255	1	1	1	1	1	1	1	1
							10		

Converting denary to binary: Method 1

There are two methods for converting a **denary** (base 10) number to **binary** (base 2). This is method one.

Divide by two and use the remainder

Divide the starting number by 2. If it divides evenly, the binary digit is 0. If it does not - if there is a remainder - the binary digit is 1.

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Worked example: Denary number 83
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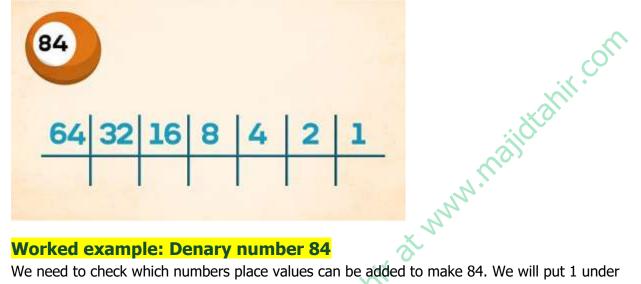
 83 ÷ 2 = 41 41 ÷ 2 = 20 20 ÷ 2 = 10 10 ÷ 2 = 5 5 ÷ 2 = 2 2 ÷ 2 = 1 	remainder remainder remainder remainder remainder remainder	- T
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	remainder remainder	0 1

Put the remainders in **reverse** order to get the final number: **1010011**.



Converting denary to binary: Method 2

There are two methods for converting a **denary** (base 10) number to binary (base 2). This method uses Place Values



Worked example: Denary number 84

We need to check which numbers place values can be added to make 84. We will put 1 under the numbers to e added and 0 under the numbers which are not added.

- 1. We select Place value 64 so we put **1** under it.
- 2. We select place value 16 and put 1 under it
- 3. We selected place value 4 and put **1** under it.
- 4. Adding 64+16+4 gives us 84 so our number becomes:

64	32	16	8	4	2	1
1	0	1	0	1	0	0

Adding 64 + 16 + 4 = 84

Result: **84** in denary is equivalent to **1010100** in binary.

Bits and binary:

Computers use **binary** - the digits 0 and 1 - to store data. A binary digit, or **bit**, is the smallest unit of data in computing. It is represented by a 0 or a 1.

The circuits in a computer's processor are made up of billions of **transistors**. A transistor is a tiny switch that is activated by the electronic signals it receives. The digits **1** or **0** used in binary reflect the on or off states of a transistor.



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All **software**, music, documents, and any other information that is processed by a computer, is also stored using binary.

Bits and bytes

Bits can be grouped together to make them easier to work with. A group of 8 bits is called a byte.

Other aroupings include:

- Nibble 4 bits (half a byte)
- Byte 8 bits
- **Kilobyte** (KB) 1024 bytes (or 1024 x 8 bits)
- = 2¹⁰ Megabyte (MB) - 1024 kilobytes (or 1048576 bytes) = 2²⁰ $= 2^{30}$
- **Gigabyte** (GB) 1024 megabytes
- **Terabyte** (TB) 1024 gigabytes
- Petabyte (PB) -1024 Terabytes
- Exabyte (EB) -1024 Petabytes
- Zettabyte (ZB)- 1024 Exabytes
- Yottabyte (YB) 1024 Zettabytes

The IEC convention for computer internal memories (including RAM) becomes:

- 1 kilobyte = 1000 byte
- 1 megabyte = 1000000 bytes
- 1 gigabyte = 1000000000 bytes
- 1 terabyte = 100000000000 bytes and so on.

VS

- 1 kibibyte (1 KiB) = 1024 bytes
- **C** mebibyte (1 MiB) = 1048576 bytes
- ፪ 1 gibibyte (1 GiB) = 1073741824 bytes
- ᅊ 1 tebibyte (1 TiB) = 1099511627776 bytes and so on.

However, the IEC terms are not universally used and we still use the more conventional terms shown above. This also ties up with the Cambridge International Examinations computer science syllabus which uses the same terminology as in example above.



Outputs Inputs Camera Monito

= 2⁴⁰

= **2**⁵⁰

= **2**⁷⁰

= **2**⁸⁰

= 2⁶⁰

Binary addition (unsigned binary number)

Adding **binary** numbers is similar to adding **denary** numbers.

Example: Adding the binary numbers 011 and 100.

Write the numbers out using the column method. Start from the right, and simply add the binary number **3** to binary number **4**.

 $+\frac{100}{111}$

111 = 7 if converted back to denary.

Example: Adding two 1s in the same column.

Sometimes a binary addition will require you to carry over values into the next highest place-value column, eg when finding the sum of the binary numbers 0010 and 0111:

or ww

When adding two ones in the same column. In binary, 1+1 = 10 (2 in denary) - it has to become 0 in sum and with 1 carried over.

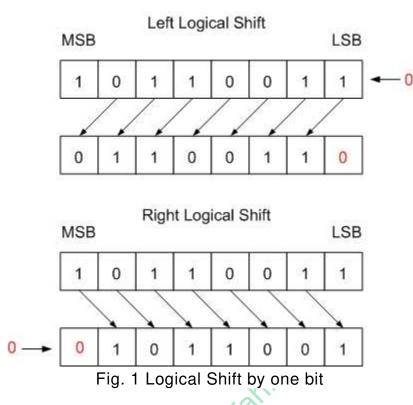
$$+ \frac{0}{1} \frac{0}{1} \frac{1}{1} \frac{$$

1001 = 9 if converted back to denary. **2 + 7 = 9** in denary.

Logical Shift

- A Left Logical Shift of one position moves each bit to the left by one. The vacant least significant bit (LSB) is filled with zero and the most significant bit (MSB) is discarded.
- A *Right Logical Shift* of one position moves each bit to the right by one. The least significant bit is discarded and the vacant MSB is filled with zero.





Effect of Logical Left Shift:

- A Left Logical Shift of one position moves each bit to the left by one. The vacant least significant bit (LSB) is filled with zero and the most significant bit (MSB) is discarded.
- If no 1's are lost (number is doubled) multiplied by 2

Effect of Logical Left Shift:

- A *Right Logical Shift* of one position moves each bit to the right by one. The least significant bit is discarded and the vacant MSB is filled with zero.
- If no 1's are lost (number is halved) divided by 2.

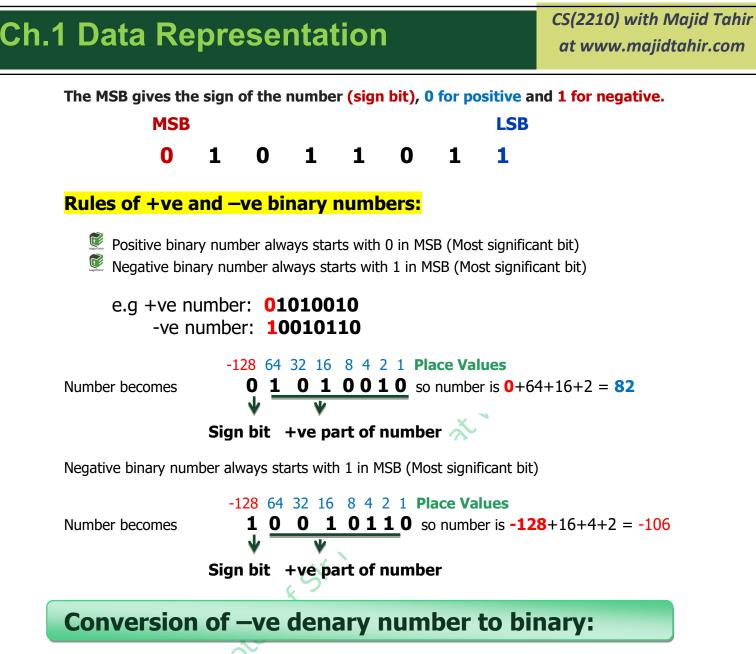
+ve and -ve binary numbers (signed numbers)

When computer stores binary numbers, we have to differentiate +ve binary numbers from -ve binary numbers. Unfortunately (-) or (+) sign cannot be displayed rather only 0/1 can be used in binary.

(MSB) Most significant bit and LSB least significant bit:

In a binary number, **the bit furthest to the left** is called the **most significant bit** (MSB) and the bit **furthest to the right** is called the **least significant bit** (LSB).





What is – 65 10 in binary?

Two's complement allows us to represent signed negative values in binary,

Binary addition

Up until now we have assumed all binary numbers have positive values. There are a number of methods to represent both positive and negative numbers. We will consider:

One's complement Two's complement.

In **one's complement**, each digit in the binary number is inverted (in other words, 0 becomes 1 and 1 becomes 0). For example, **Step 1**

65 = **01000001** in binary



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Step 2: invert 1 to 0 and 0 to 1 we get: 01000001 to its one's complement as below:

0100001 =10111110

In **two's complement**, binary digit 1 is added to one's complement

Step 3: Convert 10111110 Binary to its two's complement by

adding 1 to the one's complement.

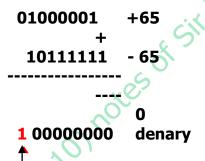
10111110 1 10111111 = Two's complement

ildtahir.com Two's compliment of a positive number will make it a negative number

> -128 64 32 16 8 4 2 1 Place values 1 0 1 1 1 1 1 1 = Two's complement

= -128 + 32 + 16 + 8 + 4 + 2 + 1 = -65

10111111 is - 65 in binary. We know this it true because if we add 01000001 (+65) to 10111111b (-65) and ignore the carry bit, the sum = 0.which is what we obtain if we add +65 + (-65) = 0.



Ignore the carry bit. What matters is that original number of bits (D7-D0) are all 0.

Two's complement sums:

Using two's complement, the **CPU** can perform arithmetic using **binary** addition. For -7 + 7 in two's complement binary would be calculated as: example:

> 1001 + ,0,1,1,1 10000



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In two's complement, if the final result **overflows** the remaining carry number is simply discarded. For example:

-3 + 4 in two's complement binary would be calculated as:

1 1 0 1 + 10,100 10001

Methods for converting a negative number expressed in two's complement form to the corresponding denary number

Consider the two's complement binary number **10110001**.

Method 1:

- Convert to the 1's compliment gives 01001110 and keep the minus sign with it
- Converting to **two's complement** gives us **01001111**.
- You ignore the leading zero in MSB as it is not a positive number and apply one of the methods to convert the remaining binary to denary which gives 79.
- Solution \mathbb{S} You add the minus sign to give -79.

Method 2:

Sum the individual place values but treat the most significant bit as a negative value You follow the approach illustrated in Table 1.02 to convert the original binary number 10110001 as follows:

		-							
	Place	-27	2 ⁶	2 ⁵	24	2 ³	2 ²	2 ¹	2 ⁰
		=	=	=	=	=	=	=	=
	value	-128	64	32	16	8	4	2	1
	Digit	1	0	1	1	0	0	0	1
)	Product	-128	0	32	16	0	0	0	1

.0

You now add the values in the bottom row to get -79.

Overflow :-

A **CPU** with a capacity of 8 **bits** has a capacity of up to 11111111 in **binary**. **If one more bit** was added there would be an **overflow error**.



Example: 8-bit overflow:

An example of an 8-bit overflow occurs in the binary sum 11111111 + 1 (denary: 255 + 1).

The total is a number bigger than 8 digits, and when this happens the **CPU drops the overflow digit** because the computer cannot store it anywhere, and **the computer thinks 255 + 1 = 0**.



Overflow errors happen when the largest number that a **register** can hold is exceeded. The number of bits that it can handle is called the **word size**.

Most CPUs use a much bigger word size than 8 bits. Many **PCs** have a 64-bit CPU. A 64-bit CPU can handle numbers larger than 18 quintillion (18,446,744,073,709,551,615 to be precise).



Hexadecimal Number System:

We often have to deal with large positive binary numbers. For instance, consider that computers connect to the Internet using a Network Interface Card (NIC). Every NIC in the world is assigned a unique 48-bit identifier as an Ethernet address. The intent is that no two NICs in the world will have the same address. iir.com

A sample Ethernet address might be:

- ើ Fortunately, large binary numbers can be made much more compactand hence easier to work with.
- You may wonder: Binary numbers would also be more compact if represented in base-10—why not just convert them to decimal?
- The answer, as you will soon see, is that converting between binary and hexadecimal is exceedingly easy.

The Hexadecimal Number System

(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)₁₆

The base 16 hexadecimal has 16 digits. Note that the single hexadecimal symbol A =10, B=11, C=12, D=13, E=14 and F=15.

Just as with decimal notation or binary notation, we again write a number as a string of symbols, but now each symbol is one of the 16 possible hexadecimal digits (0 through F). To interpret a hexadecimal number, we multiply each digit by the power of **16** associated with that digit's position.

Hexadecimal Place value:

1048576	65 536	4096	256	16	1
(165)	(164)	(163)	(162)	(161)	(160)

-H denotes hex prefix.

For example, consider the hexadecimal number 1 A 9 B.

Examples:

$$1 A 9 B_{16} = 1 A 9 B_{H}$$

= 1 × 16³ + 10 × 16² + 9 × 16¹ + 11 × 16⁰



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 $= 4096 + 2560 + 144 + 11 = 6811_{10}$

- (i) 2 F16 = 2 F_H = 2 × 16 +15 × 1= 47
- (ii) BC1216 = BC12_H = $11 \times 16^{3} + 12 \times 16^{2} + 1 \times 16^{1} + 2 \times 16^{0} = 48146$

Hexadecimal Numbers in Computing

There are two ways in which hex makes life easier.

- The first is that it can be used to write down very large integers in a compact form.
- For example, (A D 4 5)₁₆ is shorter than its decimal equivalent (44357)₁₀ and as values increase the difference in length becomes even more pronounced.

Converting Binary Numbers to Hexadecimal Numbers.

Let's assume we have a binary number of: 01010111

The binary number is 01010111

We will break number into 4 bits each as

0101

0111

Then we will start with the right side 4 bits

Starting from extreme right number

 for 0101
 for 0111

 0X2³+1X2²+0X2¹+1X2⁰
 0X2³+1X2²+1X2¹+1

 0X8+1X4+0X2+1X1
 0X8+1X4+1X2+1X

 0+4+0+1=5
 0+4+2+1=7

5

So Hexadecimal number is 57

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Converting Hexadecimal Numbers to Binary Numbers

To convert a hexadecimal number to a binary number, we reverse the above tahir.com procedure. We separate every digit of hexadecimal number and find its equivalent binary number and then we write it together.

Example 1:

To convert the hexadecimal number 9F216 to binary, each hex digit is converted into binary form.

9 F 2
$$_{16}$$
 = (1001 1111 0010)₂

F=1111 9 = 1001

2 = 0010

So Binary equivalent of Hexadecimal number is: 9F2= 100111110010

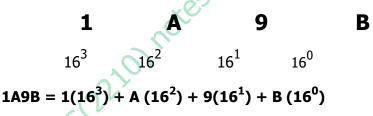
Work on the given question below :

Convert hexadecimal 2BF9 to its binary equivalent.

Convert binary **110011100001** to its hexadecimal equivalent.

Converting a Hexadecimal Number to a (Denary) Decimal Number

To convert a hexadecimal number to a decimal number, write the hexadecimal number as a sum of powers of 16. For example, considering the hexadecimal number 1A9B above, we convert this to decimal as:



$$= 4096 + 10(256) + 9(16) + 11(1) = 6811$$

So $1A9B_{16} = 6811_{10}$

Converting a (Denary) Decimal Number into Hexadecimal Number

The easiest way to convert from decimal to hexadecimal is to use the same division algorithm that you used to convert from decimal to binary, but repeatedly dividing by 16 instead of by 2. As before, we keep track of the remainders.

For example, to convert the decimal number **746** to hexadecimal, we proceed as follows:



<u>1 Data Representation</u>

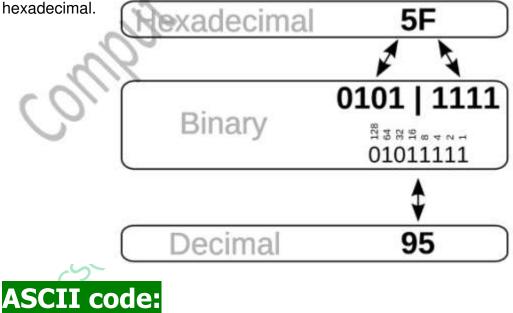
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Remainder 16 | 746 | 46 - 10 = A | 2 14 = E | 0 2

We read the number as last is first and first is last. The denary **746 = 2EA** in **hexadecimal**

Converting Between Bases

To convert from denary to hexadecimal, it is recommended to just convert the number to binary first, and then use the simple method above to convert from binary to



If text is to be stored in a computer it is necessary to have a coding scheme that provides a unique binary code for each distinct individual component item of the text.

Such a code is referred to as a **character code**.



- The scheme which has been used for the longest time is the ASCII (American Standard Code for Information Interchange) coding scheme.
- This is an internationally agreed standard. There are some variations on ASCII coding schemes but the major one is the 7-bit code. It is customary to present the codes in a table for which a number of different designs have been used.
- The full table shows the 27 (128) different codes available for a 7-bit code. You should not try to remember any of the individual codes but there are certain aspects of the coding scheme which you need to understand.
- Computers store text documents, both on disk and in memory, using ASCII codes. For example, if you use Notepad in Windows OS to create a text file containing the words, "Four score and seven years ago," Notepad would use 1 byte of memory per character (including 1 byte for each space character between the words
- It is worth emphasizing here that these codes for numbers are exclusively for use in the context of stored, displayed or printed text.
- All of the other coding schemes for numbers are for internal use in a computer system and would not be used in a text.
- There are some special features that make the coding scheme easy to use in certain circumstances.
 - The first is that the codes for numbers and for letters are in sequence in each case so that, for example,
 - \circ if 1 is added to the code for seven the code for eight is produced.
 - The second is that the codes for the upper-case letters differ from the codes for the corresponding lower-case letters only in the value of bit 6.
 - This makes conversion of upper case to lower case, or the reverse, a simple operation.



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Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char	
32	20	<space></space>	64	40	@	96	60	`	
33	21	!	65	41	A	97	61	a	
34	22	"	66	42	В	98	62	b	
35	23	#	67	43	C	99	63	с	
36	24	\$	68	44	D	100	64	d	
37	25	%	69	45	E	101	65	e	
38	26	&	70	46	F	102	66	f	
39	27	1	71	47	G	103	67	g	
40	28	(72	48	Н	104	68	h	
41	29)	73	49	I	105	69	i	
42	2A	*	74	4A	J	106	6A	j	
43	2B	+	75	4B	К	107	6B	k	
44	2C	,	76	4C	L	108	6C	l	
45	2D	-	77	4D	М	109	6D	m	
46	2E		78	4E	Ν	110	6E	n	
47	2F	/	79	4F	0	111	6F	0	

Notice the storage of characters with uppercase and lowercase. For example:

а	1	1	0	0	0	0	1	hex 61 (lower case)
А	1	0	0	0	0	0	1	hex 41 (upper case)
У	1	1	1	1	0	0	1	hex 79 (lower case)
Υ	1	0	1	1	0	0	1	hex 59 (uppercase)

Extended ACSII codes:

Extended ASCII codes use 8 bits per character allowing 256 codes including 128 codes of ASCII and some other symbols for other languages.



Despite still being widely used, the ASCII codes are far from adequate for many purposes.

Unicode is an international encoding standard for use with different languages and scripts.



It works by providing a unique number for every character, this creates a consistent encoding, representation, and handling of text.

Basically Unicode is like a Universal Alphabet that covers the majority of different languages across the world, it transforms characters into numbers.

It achieves this by using character encoding, which is to assign a

Unicode has its own special terminology. For example, a character code is a referred to as a 'code point'. In any documentation there is a constant example is 11.22 example is U+0041 which is the code point corresponding to the alphabetic character A.

The 0041 are hexadecimal characters representing two bytes. The interesting point is that in a text where the coding has been identified as Unicode it is only necessary to use a one-byte representation for the 128 codes corresponding to ASCII. To ensure such a code cannot be misinterpreted, the codes where more than one byte is needed have restrictions applied.

I	L	ĩ	Ĺ
FE8D	FE8E	FE81	FE82
ب FE8F	ج FE91	FE92	・ FE90
پ	ţ	÷	پ
FB56	Fb58	FB59	FB57



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At its core, Unicode is like ASCII: a list of characters that people want to type into a computer. Every character gets a numeric codepoint, whether it's capital A, lowercase or lambda.

A = 65 λ = 923

So Unicode says things like, —Allright, this character exists, we assigned it an official name and a codepoint, here are its lowercase or uppercase equivalents (if any), and here's a picture of what it could look like. Font designers, it's up to you to draw this in your font if you want to.

Just like ASCII, Unicode strings (imagine —codepoint 121, codepoint 111...||) have to be encoded to ones and zeros before you can store or transmit them. But unlike ASCII, Unicode has more than a million possible codepoints, so



they can't possibly all fit in one byte. And unlike ASCII, there's no One True Way to encode it.

What can we do? One idea would be to **always** use, say, 3 bytes per character. That would be nice for string traversal, because the 3rd codepoint in a string would always start at the 9th byte. But it would be inefficient UTF-8 gives you four templates to choose from: a one-byte template, a difficult two-byte template, a three-byte template, and a four-byte template **Oxxxxxx** when it comes to storage space and bandwidth.Instead, the most common

.ate

110XXXXX **10**XXXXXX

1110xxxx 10xxxxxx 10xxxxxx

11110xxx 10xxxxxx 10xxxxxx 10xxxxxx

Each of those templates has some headers which are always the same (**shown here in red**) and some slots where your code point data can go (shown here as black).

The four-byte template gives us 21 bits for our data, which would let us represent 2,097,151 different values. There are only about 128,000 codepoints right now, so UTF-8 can easily encode any Unicode codepoint for the foreseeable future. Unicode to represent any possible text in code form.

Unicode is a computing industry standard for the consistent encoding, representation, and handling of text expressed in most of the world's writing systems.

Developed in conjunction with the Universal Coded Character Set (UCS) standard and published as The Unicode Standard, the latest version of Unicode contains a repertoire of more than 128,000 characters covering 135 modern and historic scripts, as well as multiple symbol sets.

As of June 2016, the most recent version is *Unicode 9.0*. The standard is maintained by the Unicode Consortium.

Unicode's success at unifying character sets has led to its widespread and predominant use in the internationalization and localization of computer software. The standard has been implemented in many recent technologies,



including modern operating systems, XML, Java (and other programming languages), and the .NET Framework

References:

http://www.bfoit.org/itp/ComputerContinuum/RobotComputer.html https://en.wikibooks.org/wiki/GCSE_Computer_Science/Binary_representation Free cs 20, notes of sit mail to a more of site of s http://bssbmi.com/olevel/computer-science-2210/class-9/binary-systems/ http://www.math10.com/en/algebra/systems-of-counting/binary-system.html

