

## Discussion Forum

### Magic equations for designing short RCC columns of different shapes with axial, uni-axial, bi-axial loads

Columns are the most common vertical load-bearing elements in reinforced concrete structures. The primary role of a column in a typical building is to support floor structures such as slabs, beams and girders and transmit the load to the lower levels and then to the foundations. Columns are mainly subjected to axial compression loads and are often called compression member. In reality however, few reinforced concrete columns are subjected to purely axial compression loads. More often, bending moments are also present due to eccentricity of applied loads, applied end moments, and lateral loading on the column. The moment in column may be due to gravity loads, wind loads or earthquake loads. Even the internal columns of a symmetrically framed building carry some amount of moments due to gravity loads when different types of live loads arrangement are applied. For wind and earthquake loads, all columns may carry some moments. So in effect, columns are subjected to combined axial load and flexure.

Columns have cross-sectional dimensions considerably less than their height. According to IS 456: 2000, column is defined as a compression member whose effective length exceeds three times the least lateral dimension. In terms of their load-carrying capacity relative to material usage, columns are among the most structurally efficient members. In many buildings, columns are the principal means of transmitting vertical loads to the foundation and failure of a single column can potentially lead to progressive collapse of the entire structure. Given the potential for catastrophic failure and the relatively low ratio of cost to additional load bearing capacity, it is recommended to design columns with some reserve capacity whenever possible.

The main objective of any column design exercise is to determine the required dimensions and reinforcement such that the column is able to carry the given design loads.

According to IS 456:2000 a column may be considered as short when both the slenderness ratios  $L_{ex}/D$  and  $L_{ey}/B$  are less than 12.

Where  $L_{ex}$  = effective length in respect of the minor axis

$D$  = depth in respect of the major axis

$L_{ey}$  = effective length in respect to the minor axis and

$B$  = width of the member.

It shall otherwise be considered as a slender compression member.

In the case of a short column subjected to combined factored axial load and bending, design must ensure that these loads are less than or equal to factored axial load resistance and factored moment resistance.

The behavior of reinforced columns subjected to axial load depends on their slenderness and the magnitude of the load eccentricity. In an axially loaded column where the load eccentricity is large, an increase in axial load leads to an increase in flexure (moment) resistance.

For design purposes, a column interaction diagram expresses the axial and flexural resistance of reinforced concrete column sections. Interaction curves for RC columns under axial load with uniaxial and biaxial

bending based on IS 456:2000 are given in SP-16 for different concrete strengths and steel bar arrangements. Each point on the interaction diagram corresponds to the column capacity at a specific load eccentricity. The points also represent the combination of axial forces and bending moment corresponding to the resistance of a column cross-section. Interaction diagrams are widely used in design of reinforced concrete columns.

A design proposal for RCC Column contributed by a Former Superintending Engineer of Central Design Organization, Delhi Development Authority, New Delhi is published here. The author claims that his proposal simplifies the arduous column design procedure.

He says "designing a column conventionally is a challenge to many structural engineers because it is cumbersome, complicated and time consuming in nature. The design method has also changed from time to time. The working stress method used in early 1970's was improved with the introduction of ultimate design, which remained in vogue from early 1970's until 1980's. Now the limit state approach is being used. In these methods, curves and charts are given in different RCC Hand Books. Now, based on fundamental principles, some equations which simplify the arduous column design procedure, have been developed. The procedure adopted is applicable to all countries' codes."

Our reviewer for this paper - a design consultant- has made valuable comments regarding the simplified equations. In his point-by-point reply, the author has responded to the reviewer's comments and suggestion. Despite being an unconventional method, we publish this paper in difference to the wishes of both the author and the reviewer to elicit response from the larger community of structural design professionals for appropriateness of using the proposed simplified method for designing columns. Figure 1 represents the concrete section.

The author has provided three different equations for rectangular, circular and L shaped columns. Table 1 presents the equations with common notations used in the equation.

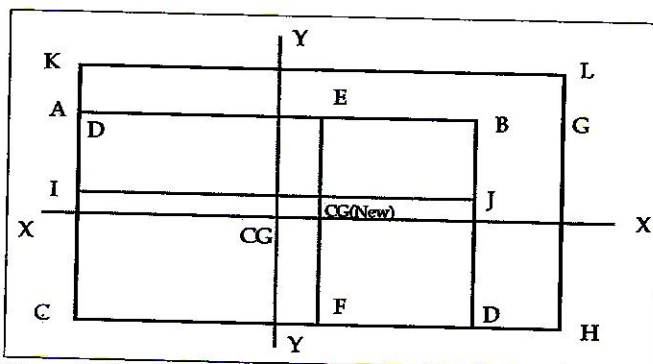


Figure 1. Concrete section

Table 1. Equations for designing short RCC columns for  $f_y=415 \text{ N/mm}^2$  and  $f_{ck}=20 \text{ N/mm}^2$

No	Equation (Variables are $f_{ck}$ and $f_y$ )	For RCC Short column
A	$As=(3.307) \times P + (-0.0295) \times B \times D + (57.317) \times D \times (M_x) / P + (57.317) \times B \times (M_y) / P + (114634.538) \times (M_x) \times (M_y) / P^2$	Rectangular columns
B	$As=(3.307)P + (-0.0231)D \times D + (90.033)D \times M / P + (90033.755)M \times M / P^2$	Circular columns
C	$As=(3.332)P + (-0.0371)AREA + (71.646)D.M_x / P + (71.646)B.M_y / P + (143293.172)M_x.M_y / P^2$	L- columns
	<p>where,</p> <p><math>As = As_1 + As_2</math> <math>As_1</math> is the area of steel required for purely axially loaded RCC column and <math>As_2</math> is the equivalent area of steel required for eccentricity in proportion to respective designed compressive stresses of concrete and steel to a strain of 0.002 .</p> <p><math>P</math> = axial load in kN</p> <p><math>B \times B</math> = size of column along X-direction</p> <p><math>D \times D</math> = size of column along Y-direction</p> <p><math>M_x</math> = moment parallel to x-direction in kNm</p> <p><math>M_y</math> = moment parallel to y-direction in kNm</p> <p><math>f_{ck}</math> = characteristic strength of concrete</p> <p><math>f_y</math> = characteristic strength of steel</p>	
<p>NOTE: This equation is valid only if the summation of first two terms is not less than zero or <math>P &gt; 0.446 f_{ck} B D</math></p> <p>The first two terms are <math>(3.307) \times P + (-0.0295) \times B \times D</math>, <math>(3.307) \times P + (-0.0231) \times D \times D</math> and <math>(3.332) \times P + (-0.0371) \times AREA</math> in equations A, B, and C respectively.</p>		



The author has prescribed certain conditions for validity of these equations and has made available the excel sheets explaining the calculations. For want of space, we are unable to publish the entire set of excel sheets. However, a typical calculation for rectangular short column is shown below.

#### Illustrative calculation

$f_{ck}$  = characteristic strength of concrete, 20.00 N/mm<sup>2</sup>

$f_y$  = characteristic strength of steel, 415.0 N/mm<sup>2</sup>

$P$  = axial load, 4000.00 kN

$M_x$  = moment parallel to x-direction, 264.00 kNm

$M_y$  = moment parallel to y-direction in, 264.00 kNm

$B_x$  OR  $B$  = size of column along x-direction, 600.00 mm

$D_y$  OR  $D$  = size of column along y-direction, 600.00 mm

$A_1$  and  $A_2$  = are equivalent areas of concrete

$A_s = A_{s1} + A_{s2} = 7647.95 \text{ mm}^2$

$e_x$  = eccentricity of load in x- direction, 66.00mm

$e_y$  = eccentricity of load in y- direction 66.00mm

$A_1$  = AREA ABCD =  $A_1$  refer Figure 1, 360000.00 mm<sup>2</sup>

$A_2$  = AREA ABDHLK =  $A_2$  refer Figure 1 535824.00 mm<sup>2</sup>

$A_{s1} = 2609.07 \text{ mm}^2$

$A_{s2} = 5038.88 \text{ mm}^2$

( $A_{s1}$  is the area of steel required for purely axially loaded RCC column and  $A_{s2}$  is the equivalent area of steel required for eccentricity in proportion to respective designed compressive stresses of concrete and steel to a strain of 0.002.)

As the moments  $M_x$  and  $M_y$  are not zero, hence eccentricity  $e_x$  and  $e_y$  will be governing. Due to this eccentricity of load in x-direction and y-direction, the load  $P$  will be at the junction of EF & IJ.

Where AB or CD is the width of section and AC or BD is the depth of section.

Consider CG of the section is at the origin of x-axis and y-axis

Hence distance of CG from line AC =  $B_x/2$

Eccentricity  $e_x = M_x/P$  i.e. Position of load due to moment  $M_x$  from CG

Hence distance of CG from line CD =  $D_y/2$

Eccentricity  $e_y = M_y/P$  i.e. Position of load due to moment  $M_y$  from CG

To neutralize the effect of eccentricity load has to be placed at CG (new) i.e. by a distance  $e_x$  away from y- axis and  $e_y$  away from x-axis at junction of line EF and IJ

By keeping the compressive stresses in concrete core as constant

The revised width of the concrete core block shall be double of CF i.e. CH

Hence revised half width  $CF = B_x/2 + e_x$

Therefore revised full width  $CH = 2 \times (B_x/2 + e_x)$  i.e.  $(B_x + 2e_x)$

The revised depth of the concrete core block shall be double of CI i.e. CK

Hence revised half depth  $CI = D_y/2 + e_y$

Therefore revised full depth  $CK = 2 \times (D_y/2 + e_y)$  i.e.  $(D_y + 2e_y)$

Hence equivalent core section without the effect of eccentricity shall be area  $KLCH = A_K \times CH$  i.e.  $(D_y + 2e_y) \times (B_x + 2e_x)$

$= (D_y) \times (B_x) + 2e_x(D_y) + 2e_y(B_x)$

$= (D_y) \times (B_x) + 2e_x(D_y) \times (M_x/P) + 2e_y(B_x) \times (M_y/P)$

This equivalent area of concrete is divided in to two parts i.e.

AREA ABCD =  $A_1 = (D_y) \times (B_x)$  and

AREA ABDHLK =  $A_2 = 2e_x(D_y) \times (M_x/P) + 2e_y(B_x) \times (M_y/P) + 4e_x e_y (M_x/P) \times (M_y/P)$

As maximum stresses in concrete and steel at strain of 0.002 are  $0.446 \times (f_{ck})$  and  $0.75 \times (f_y)$  respectively.

$P$  is the ultimate load without eccentricity

$A_1 = (D_y) \times (B_x)$

$P = (0.446) \times (f_{ck}) \times (A_c) + (0.75) \times (f_y) \times (A_{sc})$

$= (0.446) \times (f_{ck}) \times (A_g - A_{sc}) + (0.75) \times (f_y) \times (A_{sc})$

$A_{s1} = (P - (0.446) \times (f_{ck}) \times (A_g)) / ((0.75) \times (f_y) - 0.446 \times (f_{ck}))$

$A_{s1} = 1 / ((0.75) \times (f_y) - 0.446 \times (f_{ck})) \times (P) - 0.446 \times (f_{ck}) / ((0.75) \times (f_y) - 0.446 \times (f_{ck})) \times (B_x) \times (D_y)$

As2= equivalent area of steel in proportion to concrete area in excess or original core area i.e.

$$A2 = 0.446 \times f_{ck} / (0.75 \times f_y) \times [A2]$$

$$= 0.446 \times (f_{ck}) / (0.75 \times f_y) \times [2 \times (D_y) \times (M_x / P) + 2 \times (B_x) \times (M_y / P) + 4 \times (M_x / P) \times (M_y / P)]$$

$$A_s = A_{s1} + A_{s2}$$

$$= 1 / (0.75 \times f_y - 0.446 \times f_{ck}) \times (P) - 0.446 \times (f_{ck}) / (0.75 \times f_y - 0.446 \times f_{ck}) \times (B_x) \times (D_y) + 0.446 \times (f_{ck}) / (0.75 \times f_y) \times [2 \times (D_y) \times (M_x / P)$$

$$+ 0.446 \times (f_{ck}) / (0.75 \times f_y) \times [2 \times (B_x) \times (M_y / P) + 0.446 \times (f_{ck}) / (0.75 \times f_y) \times 4 \times (M_x / P) \times (M_y / P)]$$

$$C1 = 1 / (0.75 \times f_y - 0.446 \times f_{ck})$$

$$C2 = 0.446 \times (f_{ck}) / (0.75 \times f_y - 0.446 \times f_{ck})$$

$$C3 = 0.446 \times (f_{ck}) / (0.75 \times f_y) \times [2]$$

$$C4 = 0.446 \times (f_{ck}) / (0.75 \times f_y) \times [2]$$

$$C5 = 0.446 \times (f_{ck}) / (0.75 \times f_y) \times [4]$$

$$A_s = (C1) \times (P) + (-C2) \times (B_x) \times (D_y) + (C3) \times (D_y) \times (M_x / P) + (C4) \times (B_x) \times (M_y / P) + (C5) \times (M_x) \times (M_y) / P^2$$

For further discussion and clarifications on these equations, readers are requested to contact the author.



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