2024 MAA AMC 12A

Problem 1

What is the value of $9901 \cdot 101 - 99 \cdot 10101$?

9901 · 101 － 99 · 10101 的值是多少？

(A) 2 **(B)** 20 **(C)** 200 **(D)** 202 **(E)** 2020

Problem 2

A model used to estimate the time it will take to hike to the top of the mountain on a trail is of the form $T = aL + bG$,where a and b are constants, T is the time in minutes, L is the length of the trail in miles, and G is the altitude gain in feet. The model estimates that it will take 69 minutes to hike to the top if a trail is 1.5 miles long and ascends 800 feet, as well as if a trail is 1.2 miles long and ascends 1100 feet. How many minutes does the model estimates it will take to hike to the top if the trail is 4.2 miles long and ascends 4000 feet?

用于估算沿着山间小径登顶所需时间的模型表示为$T = aL + bG$，其中 a 和 b 是常数，T 是以分钟为单位计的时间，L 是以英里为单位计的小径长度，G 是以英尺为单位计的海拔增加量。该模型估算，沿着一条长 1.5 英里且海拔提升 800 英尺的小径需要 69 分钟登顶，沿着一条长 1.2 英里且海拔提升 1100 英尺的小径也同样需要 69 分钟登顶。根据该模型，沿着一条长 4.2 英里且海拔提升 4000 英尺的小径需要多少分钟才能登顶？

(A) 240 **(B)** 246 **(C)** 252 **(D)** 258 **(E)** 264

Problem 3

The number 2024 is written as the sum of not necessarily distinct two-digit numbers. What is the least number of two-digit numbers needed to write this sum?

将 2024 写成若干个两位数的和，这些两位数不一定互不相同。问写出这样的和式最少需要多少个两位数？

(A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

Problem 4

What is the least value of n such that $n!$ is a multiple of 2024?

最小的n值是多少，使得$n!$ 是 2024 的倍数?

(A) 11 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 253

Problem 5

A data set containing 20 numbers, some of which are 6, has mean 45. When all the 6s are removed, the data set has mean 66. How many 6s were in the original data set?

一个包含 20 个数的数据集中有一些数是 6，均值是 45．当移除所有的 6 后，数据集的均值变为 66．问数据集中原来有多少个 6?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

Problem 6

The product of three integers is 60. What is the least possible positive sum of the three integers?

三个整数的乘积是 60。如果它们的和是正数，那么这个正数最小可能是多少?

(A) 2 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 13

Problem 7

In $\triangle ABC, \angle ABC = 90°$ and $BA = BC = \sqrt{2}$. Points $P_1, P_2, \cdots, P_{2024}$ lie on hypotenuse $\overline{AC}$ so that $AP_1 = P_1P_2 = P_2P_3 = \cdots = P_{2023}P_{2024} = P_{2024}C$. What is the length of the vector sum $\overrightarrow{BP_1} + \overrightarrow{BP_2} + \overrightarrow{BP_3} + \cdots + \overrightarrow{BP_{2024}}$?

在$\triangle ABC$中，$\angle ABC = 90°$ 且$BA = BC = \sqrt{2}$．在斜边$\overline{AC}$上有点 $P_1, P_2, \cdots, P_{2024}$，使得$AP_1 = P_1P_2 = P_2P_3 = \cdots = P_{2023}P_{2024} = P_{2024}C$．问向量和$\overrightarrow{BP_1} + \overrightarrow{BP_2} + \overrightarrow{BP_3} + \cdots + \overrightarrow{BP_{2024}}$ 的长度是多少?

(A) 1011 **(B)** 1012 **(C)** 2023 **(D)** 2024 **(E)** 2025

Problem 8

How many angles θ with $0 \leq \theta \leq 2\pi$ satisfy $\log(\sin(3\theta)) + \log(\cos(2\theta)) = 0$?

满足$0 \leq \theta \leq 2\pi$，以及$\log(\sin(3\theta)) + \log(\cos(2\theta)) = 0$ 的角度θ有多少个?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

Problem 9

Let M be the greatest integer such that both $M + 1213$ and $M + 3773$ are perfect squares. What is the units digit of M?

设M是使得$M + 1213$和$M + 3773$都是完全平方数的最大整数。问M的个位数字是几?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 8

Problem 10

Let α be the radian measure of the smallest angle in a $3-4-5$ right triangle. Let β be the radian measure of the smallest angle in a $7-24-25$ right triangle. In terms of α , what is β?

设α是三边长为 3，4，5 的直角三角形中最小角的弧度值，β是三边长为 7， 24，25 的直角三角形中最小角的弧度值．问β怎样用α表示?

(A) $\frac{\alpha}{3}$ **(B)** $\alpha - \frac{\pi}{8}$ **(C)** $\frac{\pi}{2} - 2\alpha$ **(D)** $\frac{\alpha}{2}$ **(E)** $\pi - 4\alpha$

Problem 11

There are exactly K positive integers b with $5 \leq b \leq 2024$ such that the base-b integer 2024_b is divisible by 16(where 16 is in base ten). What is the sum of the digits of K?

(A) 16 **(B)** 17 **(C)** 18 **(D)** 20 **(E)** 21

Problem 12

The first three terms of a geometric sequence are the integers a, 720 and b, where $a < 720 < b$. What is the sum of the digits of the least possible value of b?

一个等比数列的前三项是整数a，720，b，并且$a < 720 < b$。问b的最小可能值的各位数字之和是多少？

(A) 9　　**(B)** 12　　**(C)** 16　　**(D)** 18　　**(E)** 21

Problem 13

The graph of $y = e^{x+1} + e^{-x} - 2$ has an axis of symmetry. What is the reflection of the point $\left(-1, \frac{1}{2}\right)$ over this axis?

函数$y = e^{x+1} + e^{-x} - 2$的图像有一条对称轴，点$\left(-1, \frac{1}{2}\right)$关于这条对称轴的反射点是什么？

(A) $\left(-1, -\frac{3}{2}\right)$　　**(B)** $(-1, 0)$　　**(C)** $\left(-1, \frac{1}{2}\right)$　　**(D)** $\left(0, \frac{1}{2}\right)$　　**(E)** $\left(3, \frac{1}{2}\right)$

Problem 14

The numbers, in order, of each row and the numbers, in order, of each column of a 5×5 array of integers form an arithmetic progression of length 5. The numbers in positions (5,5),(2,4),(4,3) and (3,1) are 0, 48, 16, and 12, respectively. What number is in position (1,2)?

在5×5的整数数表中，每行的数按其现有顺序组成一个五项的等差数列，每列的数也按其现有顺序组成一个五项的等差数列。已知在位置(5,5)，(2,4)，(4,3)，(3,1)处的数分别是 0，48，16，12。问位置(1,2)处的数是多少？

$$\begin{bmatrix} . & ? & . & . & . \\ . & . & . & 48 & . \\ 12 & . & . & . & . \\ . & . & 16 & . & . \\ . & . & . & . & 0 \end{bmatrix}$$

(A) 19　　**(B)** 24　　**(C)** 29　　**(D)** 34　　**(E)** 39

Problem 15

The roots of x^3+2x^2-x+3 are p, q, and r. What is the value of $(p^2+4)(q^2+4)(r^2+4)$?

设x^3+2x^2-x+3的根为p，q，r．问$(p^2+4)(q^2+4)(r^2+4)$的值是多少?

(A) 64 **(B)** 75 **(C)** 100 **(D)** 125 **(E)** 144

Problem 16

A set of 12 tokens -- 3 red, 2 white, 1 blue, and 6 black - is to be distributed at random to 3 game players, 4 tokens per player. The probability that some player gets all the red tokens, another gets all the white tokens, and the remaining player gets the blue token can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

现有 3 个红色筹码，2 个白色筹码，1 个蓝色筹码，6 个黑色筹码共计 12 个筹码要随机分给 3 名玩家，每人 4 个筹码．某个玩家获得全部红色筹码，另一个玩家获得全部白色筹码，剩下的那个玩家获得蓝色筹码的概率可以表示为$\frac{m}{n}$，其中m和n是互质的正整数．问$m+n$是多少?

(A) 387 **(B)** 388 **(C)** 389 **(D)** 390 **(E)** 391

Problem 17

Integers a, b, and c satisfy $ab+c=100$, $bc+a=87$, and $ca+b=60$. What is $ab+bc+ca$?

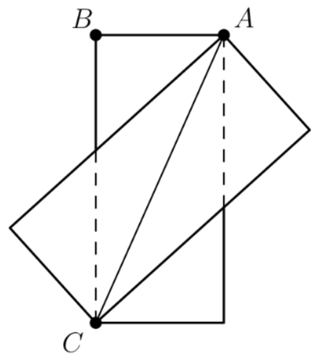
整数a，b，c满足$ab+c=100$，$bc+a=87$，$ca+b=60$．问$ab+bc+ca$是多少?

(A) 212 **(B)** 247 **(C)** 258 **(D)** 276 **(E)** 284

Problem 18

On top of a rectangular card with sides of length 1 and $2+\sqrt{3}$, an identical card is placed so that two of their diagonals line up, as shown ($\overline{AC}$, in this case).Continue the process, adding a third card to the second, and so on, lining up successive diagonals after rotating clockwise. In total, how many cards must be used until a vertex of a new card lands exactly on the vertex labeled B in the figure?

如图所示，在一个边长为 1 和$2+\sqrt{3}$的矩形卡片上方，放置一个相同的卡片，使它们的对角线重合（$\overline{AC}$）. 继续这样的操作，在第二张卡片上放置第三张卡片，顺时针旋转并使对角线重合，然后依此类推. 问总共需要使用多少张卡片，才能使新卡片的某个顶点恰好落在图中标记为B的顶点上?



(A) 6 (B) 8 (C) 10 (D) 12

(E) No new vertex will land on B｜不会有新的顶点落在 B 上

Problem 19

Cyclic quadrilateral $ABCD$ has lengths $BC=CD=3$ and $DA=5$ with $\angle CDA=120^\circ$. What is the length of the shorter diagonal of $ABCD$?

在圆内接四边形$ABCD$中，$BC=CD=3$，且$DA=5$，$\angle CDA=120^\circ$。问$ABCD$的较短对角线的长度是多少?

(A) $\frac{31}{7}$ **(B)** $\frac{33}{7}$ **(C)** 5 **(D)** $\frac{39}{7}$ **(E)** $\frac{41}{7}$

Problem 20

Points P and Q are chosen uniformly and independently at random on sides $\overline{AB}$ and $\overline{AC}$ respectively, of equilateral triangle $\triangle ABC$ Which of the following intervals contains the probability that the area of $\triangle APQ$ is less than half the area of $\triangle ABC$?

在等边三角形$\triangle ABC$的边$\overline{AB}$和$\overline{AC}$上分别均匀随机且独立地选取点P和Q.问下列

哪个区间包含三角形$\triangle APQ$的面积小于三角形$\triangle ABC$的面积一半的概率?

(A) $\left[\frac{3}{8},\frac{1}{2}\right]$ **(B)** $\left(\frac{1}{2},\frac{2}{3}\right]$ **(C)** $\left(\frac{2}{3},\frac{3}{4}\right]$ **(D)** $\left(\frac{3}{4},\frac{7}{8}\right]$ **(E)** $\left(\frac{7}{8},1\right]$

Problem 21

Suppose that $a_1 = 2$ and the sequence (a_n) satisfies the recurrence relation $\frac{a_n-1}{n-1} = \frac{a_{n-1}+1}{(n-1)+1}$ for all $n \geq 2$ .What is the greatest integer less than or equal to $\sum_{n=1}^{100} a_n^2$?

假设$a_1 = 2$，且对所有$n \geq 2$，序列(a_n)满足递推关系$\frac{a_n-1}{n-1} = \frac{a_{n-1}+1}{(n-1)+1}$. 问不超过$\sum_{n=1}^{100} a_n^2$的最大整数是多少?

(A) 338,550 **(B)** 338,551 **(C)** 338,552 **(D)** 338,553 **(E)** 338,554

Problem 22

The figure below shows a dotted grid 8 cells wide and 3 cells tall consisting of $1'' \times 1''$ squares. Carl places 1-inch toothpicks along some of the sides of the squares to create a closed loop that does not intersect itself. The numbers in the cells indicate the number of sides of that square that are to be covered by toothpicks, and any number of toothpicks are allowed if no number is written. In how many ways can Carl place the toothpicks?

下图显示了由 1 英寸×1 英寸的方格组成的，宽是 8 个方格，高是 3 个方格的虚线网格. Carl 要沿着一些方格的边放置长为 1 英寸的牙签，以形成一个不自相交的闭合折线. 方格中的数表示该方格被牙签覆盖的边的个数; 对于没有数的方格，其被牙签覆盖的边的个数没有限制。问 Carl 有多少种不同的放置牙签的方法?

1	1	1	1	1	1	1	1

(A) 130 **(B)** 144 **(C)** 146 **(D)** 162 **(E)** 196

Problem 23

What is the value of $tan^2\frac{\pi}{16}\cdot tan^2\frac{3\pi}{16}+tan^2\frac{\pi}{16}\cdot tan^2\frac{5\pi}{16}+tan^2\frac{3\pi}{16}\cdot tan^2\frac{7\pi}{16}+tan^2\frac{5\pi}{16}\cdot tan^2\frac{7\pi}{16}$?

问 $tan^2\frac{\pi}{16}\cdot tan^2\frac{3\pi}{16}+tan^2\frac{\pi}{16}\cdot tan^2\frac{5\pi}{16}+tan^2\frac{3\pi}{16}\cdot tan^2\frac{7\pi}{16}+tan^2\frac{5\pi}{16}\cdot tan^2\frac{7\pi}{16}$ 的值是多少?

(A) 28 **(B)** 68 **(C)** 70 **(D)** 72 **(E)** 84

Problem 24

A disphenoid is a tetrahedron whose triangular faces are congruent to one another. What is the least total surface area of a disphenoid whose faces are scalene triangles with integer side lengths?

如果一个四面体的所有三角形面都互相全等，那么称该四面体为“等面四面体”. 问每个面的三边长都互不相同，并且是整数的等面四面体的最小表面积是多少?

(A) $\sqrt{3}$ **(B)** $3\sqrt{15}$ **(C)** 15 **(D)** $15\sqrt{7}$ **(E)** $24\sqrt{6}$

Problem 25

A graph is *symmetric* about a line if the graph remains unchanged after reflection in that line. For how many quadruples of integers (a, b, c, d), where $|a|, |b|, |c|, |d| \leq 5$ and c and d are not both 0, is the graph of $y = \frac{ax+b}{cx+d}$ symmetric about the line $y = x$?

如果一个图像经过一条直线反射后仍保持不变，那么称此图像关于该直线“对称”。使得函数$y = \frac{ax+b}{cx+d}$的图像关于直线$y = x$对称，并且满足 $|a|, |b|, |c|, |d| \leq 5$，$c$和$d$不同时为 0 的四元整数组$(a, b, c, d)$有多少个?

(A) 1282 **(B)** 1292 **(C)** 1310 **(D)** 1320 **(E)** 1330

2024 AMC 12A Answer Key													
题目	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
答案	A	B	B	D	D	B	D	A	E	C	D	E	D
题目	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
答案	C	D	C	D	A	D	D	B	C	B	D	B	

2024 AMC12A Solution

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