2013 AMC12B

Problem 1

On a particular January day, the high temperature in Lincoln, Nebraska, was 16 degrees higher than the low temperature, and the average of the high and low temperatures was 3. In degrees, what was the low temperature in Lincoln that day?

内布拉斯加州林肯市在一月份的某天，高温比低温高 16 度，且高温和低温的平均值是 3 度，问林肯市那天的低温是多少度?

$\textbf{(A)}\ -13 \qquad \textbf{(B)}\ -8 \qquad \textbf{(C)}\ -5 \qquad \textbf{(D)}\ -3 \qquad \textbf{(E)}\ 11$

Problem 2

Mr. Green measures his rectangular garden by walking two of the sides and finds that it is 15 steps by 20 steps. Each of Mr. Green's steps is 2 feet long. Mr. Green expects a half a pound of potatoes per square foot from his garden. How many pounds of potatoes does Mr. Green expect from his garden?

格林先生通过用自己的步长来测量他的矩形花园，发现大小是 15 步 x 20 步，格林先生的每一步是 2 英尺长，他期望他的花园每平方英尺的土地可以产出半磅的土豆，问格林先生期望他的花园总共可以产出多少磅的土豆?

$\textbf{(A)}\ 600 \qquad \textbf{(B)}\ 800 \qquad \textbf{(C)}\ 1000 \qquad \textbf{(D)}\ 1200 \qquad \textbf{(E)}\ 1400$

Problem 3

When counting from 3 to 201, 53 is the 51^{st} number counted. When counting backwards from 201 to 3, 53 is the n^{th} number counted. What is n?

当从 3 数到 201，数到的第 51 个数是 53，当从 201 倒数到 3，数到的第 *n* 个数是 53，问 *n* 是多少?

$\textbf{(A)}\ 146 \qquad \textbf{(B)}\ 147 \qquad \textbf{(C)}\ 148 \qquad \textbf{(D)}\ 149 \qquad \textbf{(E)}\ 150$

Problem 4

Ray's car averages 40 miles per gallon of gasoline, and Tom's car averages 10 miles per gallon of gasoline. Ray and Tom each drive the same number of miles. What is the cars' combined rate of miles per gallon of gasoline?

Ray 的汽车每加仑汽油可以行驶 40 英里，Tom 的汽车每加仑汽油可以行驶 10 英里。Ray 和 Tom 各自行驶了相同的英里数，问这两辆车平均每加仑汽油总共行驶了多少英里？

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 25 \qquad \textbf{(D)}\ 30 \qquad \textbf{(E)}\ 40$

Problem 5

The average age of 33 fifth-graders is 11. The average age of 55 of their parents is 33. What is the average age of all of these parents and fifth-graders?

33 个五年级学生的平均年龄是 11 岁，他们的 55 个父母的平均年龄是 33 岁，问所有这些父母和五年级学生的平均年龄是多少岁?

$\textbf{(A)}\ 22 \qquad \textbf{(B)}\ 23.25 \qquad \textbf{(C)}\ 24.75 \qquad \textbf{(D)}\ 26.25 \qquad \textbf{(E)}\ 28$

Problem 6

Real numbers x and y satisfy the equation $x^2 + y^2 = 10x - 6y - 34$. What is $x + y$?

实数 x 和 y 满足方程$x^2 + y^2 = 10x - 6y - 34$, 则 $x+y$ 是多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 8$

Problem 7

Jo and Blair take turns counting from 1 to one more than the last number said by the other person. Jo starts by saying "1", so Blair follows by saying "$1, 2$". Jo then says "$1, 2, 3$", and so on. What is the 53^{rd} number said?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 8$

Problem 8

Line l_1 has equation $3x - 2y = 1$ and goes through $A = (-1, -2)$. Line l_2 has equation $y = 1$ and meets line l_1 at point B. Line l_3 has positive slope, goes through point A, and meets l_2 at point C. The area of $\triangle ABC$ is 3. What is the slope of l_3?

$\textbf{(A)}\ \frac{2}{3} \qquad \textbf{(B)}\ \frac{3}{4} \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ \frac{4}{3} \qquad \textbf{(E)}\ \frac{3}{2}$

Problem 9

What is the sum of the exponents of the prime factors of the square root of the largest perfect square that divides $12!$?

能够整除 12！的最大的完全平方数的平方根的质因数分解中，求这些质因数的指数之和是多少？

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 12$

Problem 10

Alex has 75 red tokens and 75 blue tokens. There is a booth where Alex can give two red tokens and receive in return a silver token and a blue token, and another booth where Alex can give three blue tokens and receive in return a silver token and a red token. Alex continues to exchange tokens until no more exchanges are possible. How many silver tokens will Alex have at the end?

Alex 有 75 张红色礼券和 75 张蓝色礼券，Alex 可以在某个报刊亭用 2 张红色礼券兑换一张银色礼券和一张蓝色礼券，而在另一个报刊亭他可以用 3 张蓝色礼券兑换一张银色礼券和一张红色礼券，Alex 一直持续地兑换礼券直到无法再继续兑换。问 Alex 最后将有多少张银色礼券？

$\textbf{(A)}\ 62 \qquad \textbf{(B)}\ 82 \qquad \textbf{(C)}\ 83 \qquad \textbf{(D)}\ 102 \qquad \textbf{(E)}\ 103$

Problem 11

Two bees start at the same spot and fly at the same rate in the following directions.
Bee A travels 1 foot north, then 1 foot east, then 1 foot upwards, and then continues to repeat this pattern. Bee B travels 1 foot south, then 1 foot west, and then continues to repeat this pattern. In what directions are the bees traveling when they are exactly 10 feet away from each other?

两只蜜蜂从同一地点出发，朝着下面的方向以相同的速率飞行，蜜蜂 A 先向北飞行 1 英尺，再向东 1 英尺，再向上 1 英尺，然后继续重复这种飞行模式, 蜜蜂 B 先向南飞行 1 英尺，再向西 1 英尺，之后继续重复这种飞行模式。当这两只蜜蜂相距恰好 10 英尺的时候，他们各自向什么方向飞行？

(A) A east, B west | 蜜蜂 A 向东，蜜蜂 B 向西

(B) A north, B south | 蜜蜂 A 向北，蜜蜂 B 向南

(C) A north, B west | 蜜蜂 A 向北，蜜蜂 B 向西

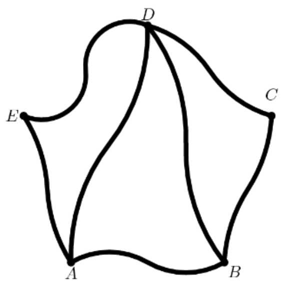
(D) A up, B south | 蜜蜂 A 向上，蜜蜂 B 向南

(E) A up, B west | 蜜蜂 A 向上，蜜蜂 B 向西

Problem 12

Cities A, B, C, D, and E are connected by roads $\widetilde{AB}$, $\widetilde{AD}$, $\widetilde{AE}$, $\widetilde{BC}$, $\widetilde{BD}$, $\widetilde{CD}$, and $\widetilde{DE}$. How many different routes are there from A to B that use each road exactly once? (Such a route will necessarily visit some cities more than once.)

A, B, C, D, 和E 这 5 座城市由道路$\widetilde{AB}$,$\widetilde{AD}$,$\widetilde{AE}$,$\widetilde{BC}$,$\widetilde{BD}$,$\widetilde{CD}$和$\widetilde{DE}$相连。从城市 A 到城市 B，若要求每条道路都走过且恰好走过一次，那么一共有多少条不同的路径？（这样一条路径可能会需要多次访问同一座城市。）



(A) 7 **(B)** 9 **(C)** 12 **(D)** 16 **(E)** 18

Problem 13

The internal angles of quadrilateral $ABCD$ form an arithmetic progression. Triangles ABD and DCB are similar with $\angle DBA = \angle DCB$ and $\angle ADB = \angle CBD$. Moreover, the angles in each of these two triangles also form an arithmetic progression. In degrees, what is the largest possible sum of the two largest angles of $ABCD$?

四边形 *ABCD* 的内角形成一个等差数列。三角形 *ABD* 和 *DCB* 是相似三角形，且满足 $\angle DBA = \angle DCB$，$\angle ADB = \angle CBD$，此外，这两个三角形各自的三个内角也都成等差数列。问 *ABCD* 的最大的两个内角度数之和最多可能是多少度？

$\textbf{(A)}\ 210 \qquad \textbf{(B)}\ 220 \qquad \textbf{(C)}\ 230 \qquad \textbf{(D)}\ 240 \qquad \textbf{(E)}\ 250$

Problem 14

Two non-decreasing sequences of nonnegative integers have different first terms. Each sequence has the property that each term beginning with the third is the sum of the previous two terms, and the seventh term of each sequence is N. What is the smallest possible value of N ?

两个非递减的非负整数数列的第一项不同，这两个数列都有这样两个性质：从第三项开始，每一项都是前面两项之和，且这两个数列的第七项都是 N，那么 *N* 的最小可能值是多少？

$\textbf{(A)}\ 55 \qquad \textbf{(B)}\ 89 \qquad \textbf{(C)}\ 104 \qquad \textbf{(D)}\ 144 \qquad \textbf{(E)}\ 273$

Problem 15

The number 2013 is expressed in the form

$$2013 = \frac{a_1!a_2!...a_m!}{b_1!b_2!...b_n!},$$

where $a_1 \ge a_2 \ge ... \ge a_m$ and $b_1 \ge b_2 \ge ... \ge b_n$ are positive integers and $a_1 + b_1$ is as small as possible. What is $|a_1 - b_1|$?

数字 2013 被写成如下形式：

$$2013 = \frac{a_1!a_2!...a_m!}{b_1!b_2!...b_n!}$$

这里$a_1 \ge a_2 \ge ... \ge a_m$和$b_1 \ge b_2 \ge ... \ge b_n$都是正整数，且$a_1 + b_1$取尽可能小的值，问$|a_1 - b_1|$是多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 16

Let $ABCDE$ be an equiangular convex pentagon of perimeter 1. The pairwise intersections of the lines that extend the sides of the pentagon determine a five-pointed star polygon. Let s be the perimeter of this star. What is the difference between the maximum and the minimum possible values of s?

ABCDE 是一个周长为 1 的等角凸五边形。五边形的五条边所在的直线两两相交，形成一个五角星。*s* 表示这个五角星的周长，那么 *s* 的最大可能值和最小可能值的差是多少？

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ \frac{1}{2} \qquad \textbf{(C)}\ \frac{\sqrt{5}-1}{2} \qquad \textbf{(D)}\ \frac{\sqrt{5}+1}{2} \qquad \textbf{(E)}\ \sqrt{5}$

Problem 17

Let a, b, and c be real numbers such that

$$a+b+c=2, \text{ and}$$
$$a^2+b^2+c^2=12$$

What is the difference between the maximum and minimum possible values of c?

实数 *a*，*b*，*c* 满足下列算式：

$$a+b+c=2 \text{ 且}$$

$$a^2+b^2+c^2=12$$

那么 *c* 的最大可能值和最小可能值的差是多少?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ \frac{10}{3} \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ \frac{16}{3} \qquad \textbf{(E)}\ \frac{20}{3}$

Problem 18

Barbara and Jenna play the following game, in which they take turns. A number of coins lie on a table. When it is Barbara's turn, she must remove 2 or 4 coins, unless only one coin remains, in which case she loses her turn. When it is Jenna's turn, she must remove 1 or 3 coins. A coin flip determines who goes first. Whoever removes the last coin wins the game. Assume both players use their best strategy. Who will win when the game starts with 2013 coins and when the game starts with 2014 coins?

Barbara 和 Jenna 轮流玩游戏，规则如下：桌上放着一堆硬币，当轮到 Barbara 时，她只能移走 2 枚或 4 枚硬币，若桌上只有 1 枚硬币，那么她就丢失了这次移走硬币的机会；当轮到 Jemma 时，她只能移走 1 枚或者 3 枚硬币；游戏的最开始谁先走，由抛硬币决定，谁移走桌上最后一枚硬币谁就赢了。假设每个选手都使用他们各自的最佳策略。当游戏的一开始总共有 2013 枚硬币或 2014 枚硬币时，分别会是谁赢？

$\textbf{(A)}$ Barbara will win with 2013 coins and Jenna will win with 2014 coins. | 若一开始总共有 2013 枚硬币，那么 Barbara 会赢，若一开始总共有 2014 枚硬币，那么 Jenna 会赢。

$\textbf{(B)}$ Jenna will win with 2013 coins, and whoever goes first will win with 2014 coins. | 若一开始总共有 2013 枚硬币，那么 Jenna 会赢，若一开始总共有 2014 枚硬币，那么谁第一个走谁就会赢。

$\textbf{(C)}$ Barbara will win with 2013 coins, and whoever goes second will win with 2014 coins. | 若一开始总共有 2013 枚硬币，那么 Barbara 会赢，若一开始总共有 2014 枚硬币，那么谁第二个走谁就会赢。

$\textbf{(D)}$ Jenna will win with 2013 coins, and Barbara will win with 2014 coins. | 若一开始总共有 2013 枚硬币，那么 Jenna 会赢，若一开始总共有 2014 枚硬币，那么 Barbara 会赢。

$\textbf{(E)}$ Whoever goes first will win with 2013 coins, and whoever goes second will win with 2014 coins. | 若一开始总共有 2013 枚硬币，那么谁先走谁就会赢，若一开始总共有 2014 枚硬币，那么谁第二个走谁就会赢。

Problem 19

In triangle ABC, $AB = 13$, $BC = 14$, and $CA = 15$. Distinct points D, E, and F lie on segments $\overline{BC}$, $\overline{CA}$, and $\overline{DE}$, respectively, such that $\overline{AD} \perp \overline{BC}$, $\overline{DE} \perp \overline{AC}$, and $\overline{AF} \perp \overline{BF}$. The length of segment $\overline{DF}$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

三角形 ABC 中， AB=13，BC=14，CA=15,点 D，E，F 分别在线段$\overline{BC}$, $\overline{CA}$, $\overline{DE}$上，满足$\overline{AD} \perp \overline{BC}$,$\overline{DE} \perp \overline{AC}$,$\overline{AF} \perp \overline{BF}$,线段$\overline{DF}$的长度可以写成$\frac{m}{n}$，其中 m 和 n 是互质的正整数,问 $m+n$ 是多少？

(A) 18 **(B)** 21 **(C)** 24 **(D)** 27 **(E)** 30

Problem 20

For $135^\circ < x < 180^\circ$,points $P=(\cos x, \cos^2 x), Q=(\cot x, \cot^2 x), R=(\sin x, \sin^2 x)$ and $S=(\tan x, \tan^2 x)$ are the vertices of a trapezoid. What is $\sin(2x)$?

对于$135^\circ < x < 180^\circ$，点$P=(\cos x, \cos^2 x), Q=(\cot x, \cot^2 x), R=(\sin x, \sin^2 x)$，$S=(\tan x, \tan^2 x)$是一个梯形的 4 个顶点，问$\sin(2x)$是多少？

(A) $2-2\sqrt{2}$ **(B)**$3\sqrt{3}-6$ **(C)** $3\sqrt{2}-5$ **(D)** $-\frac{3}{4}$ **(E)** $1-\sqrt{3}$

Problem 21

Consider the set of 30 parabolas defined as follows: all parabolas have as focus the point $(0,0)$ and the directrix lines have the form $y=ax+b$ with a and b integers such that $a\in\{-2,-1,0,1,2\}$ and $b\in\{-3,-2,-1,1,2,3\}$. No three of these parabolas have a common point. How many points in the plane are on two of these parabolas?

考虑如下定义的 30 个抛物线所组成的集合：所有的抛物线的焦点都是点（0，0），并且它们的准线都有这样的形式：*y=ax+b*，其中 *a* 和 *b* 都是整数，满足$a\in\{-2,-1,0,1,2\}$，$b\in\{-3,-2,-1,1,2,3\}$，不存在其中三个抛物线交于同一点的情况，问平面内有多少个同时在其中两条抛物线上的点？

(A) 720　**(B)** 760　**(C)** 810　**(D)** 840　**(E)** 870

Problem 22

Let $m>1$ and $n>1$ be integers. Suppose that the product of the solutions for x of the equation $8(\log_n x)(\log_m x)-7\log_n x-6\log_m x-2013=0$ is the smallest possible integer. What is $m+n$?

令$m>1$，$n>1$，且都是整数。假设方程

$$8(\log_n x)(\log_m x)-7\log_n x-6\log_m x-2013=0$$

的根的乘积是可能的最小的整数，问$m+n$是多少？

(A) 12　**(B)** 20　**(C)** 24　**(D)** 48　**(E)** 272

Problem 23

Bernardo chooses a three-digit positive integer N and writes both its base-5 and base-6 representations on a blackboard. Later LeRoy sees the two numbers Bernardo has written. Treating the two numbers as base-10 integers, he adds them to obtain an integer S. For example, if $N = 749$, Bernardo writes the numbers 10444 and 3245, and LeRoy obtains the sum $S = 13,689$. For how many choices of N are the two rightmost digits of S, in order, the same as those of $2N$?

Bernardo 选择了一个三位正整数，并把它的五进制和六进制表示写在黑板上，之后 LeRoy 看到了 Bernardo 写的这两个数，他把这两个数当成了十进制的数，把它们相加，得到和为 *S*，例如，若 *N*＝749，Bernardo 就在黑板上写下数字 10444 和 3245，LeRoy 得到两数之和 *S*＝13689，问有多少个这样的 *N*，使得 *S* 的最右边的两位数字与 *2N* 最右边的两位数字依照从左到右的顺序分别相等？

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 15 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 25$

Problem 24

Let ABC be a triangle where M is the midpoint of $\overline{AC}$, and $\overline{CN}$ is the angle bisector of $\angle ACB$ with N on $\overline{AB}$. Let X be the intersection of the median $\overline{BM}$ and the bisector $\overline{CN}$. In addition $\triangle BXN$ is equilateral with $AC = 2$. What is BX^2?

在三角形 *ABC* 中，*M* 是边$\overline{AC}$的中点，$\overline{CN}$是$\angle ACB$B 的角平分线，且点 *N* 在$\overline{AB}$上。*X* 是中线$\overline{BM}$和角平分线$\overline{CN}$的交点，且$\triangle BXN$是等边三角形，已知 *AC*=2，问BX^2是多少？

$\textbf{(A)}\ \frac{10-6\sqrt{2}}{7} \qquad \textbf{(B)}\ \frac{2}{9} \qquad \textbf{(C)}\ \frac{5\sqrt{2}-3\sqrt{3}}{8} \qquad \textbf{(D)}\ \frac{\sqrt{2}}{6} \qquad \textbf{(E)}\ \frac{3\sqrt{3}-4}{5}$

Problem 25

Let G be the set of polynomials of the form $P(z) = z^n + c_{n-1}z^{n-1} + \cdots + c_2z^2 + c_1z + 50$, where $c_1, c_2, \cdots, c_{n-1}$ are integers and $P(z)$ has distinct roots of the form $a + ib$ with a and b integers. How many polynomials are in G?

G 是一个由如下形式的多项式所组成的集合：

$$P(z) = z^n + c_{n-1}z^{n-1} + \cdots + c_2z^2 + c_1z + 50,$$

这里$c_1, c_2, \cdots, c_{n-1}$都是整数，且$P(z)$的根都是不同的，并具有$a + ib$的形式，其中 a 和 b 都是整数，问 G 中有多少个多项式？

$\textbf{(A)}\ 288 \qquad \textbf{(B)}\ 528 \qquad \textbf{(C)}\ 576 \qquad \textbf{(D)}\ 992 \qquad \textbf{(E)}\ 1056$

2013 AMC 12B Answer Key

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
C	A	D	B	C	B	E	B	C	E	A	D	D
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
C	B	A	D	B	B	A	C	A	E	A	B	

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