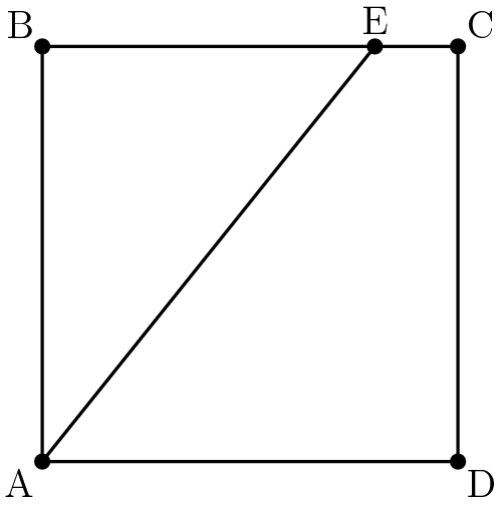
2013 AMC12A

Problem 1

Square $ABCD$ has side length 10. Point E is on $\overline{BC}$, and the area of $\triangle ABE$ is 40. What is BE?

正方形 *ABCD* 的边长为 10，点 *E* 在 *BC* 上，$\triangle ABE$的面积为 40，问 *BE* 的长是多少?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8



Problem 2

A softball team played ten games, scoring $1, 2, 3, 4, 5, 6, 7, 8, 9,$ and 10 runs. They lost by one run in exactly five games. In each of the other games, they scored twice as many runs as their opponent. How many total runs did their opponents score?

一支垒球队打了 10 场比赛，得分为 1，2，3，4，5，6，7，8，9 和 10 分，其中恰好 5 场比赛比对手少 1 分，在其他 5 场比赛中，得分是对手分数的 2 倍。那么他们的对手总共得分是多少?

(A) 35 **(B)** 40 **(C)** 45 **(D)** 50 **(E)** 55

Problem 3

A flower bouquet contains pink roses, red roses, pink carnations, and red carnations. One third of the pink flowers are roses, three fourths of the red flowers are carnations, and six tenths of the flowers are pink. What percent of the flowers are carnations?

一束花包含粉色玫瑰，红色玫瑰，粉色康乃馨和红色康乃馨，粉色花的三分之一是玫瑰，红色花的四分之三是康乃馨，所有花的十分之六是粉色的，问所有花的百分之多少是康乃馨？

(A) 15 **(B)** 30 **(C)** 40 **(D)** 60 **(E)** 70

Problem 4

What is the value of

$$\frac{2^{2014}+2^{2012}}{2^{2014}-2^{2012}}?$$

下列算式的值是

$$\frac{2^{2014}+2^{2012}}{2^{2014}-2^{2012}}?$$

(A) -1 **(B)** 1 **(C)** $\frac{5}{3}$ **(D)** 2013 **(E)** 2^{4024}

Problem 5

Tom, Dorothy, and Sammy went on a vacation and agreed to split the costs evenly. During their trip Tom paid $\$105$, Dorothy paid $\$125$, and Sammy paid $\$175$. In order to share the costs equally, Tom gave Sammy t dollars, and Dorothy gave Sammy d dollars. What is $t-d$?

Tom，Dorothy 和 Sammy 一起去度假，并且同意均摊费用，旅途中，Tom 支付了 105 美元，Dorothy 支付了 125 美元，Sammy 支付了 175 美元，为了平摊费用，Tom 给 Sammy 支付了 t 美元，Dorothy 給 Sammy 支付了 d 美元，问$t-d$是多少？

(A) 15 **(B)** 20 **(C)** 25 **(D)** 30 **(E)** 35

Problem 6

In a recent basketball game, Shenille attempted only three-point shots and two-point shots. She was successful on 20% of her three-point shots and 30% of her two-point shots. Shenille attempted 30 shots. How many points did she score?

在最近的篮球赛中，Shenille 只尝试三分球和二分球，三分球的成功率是 20%，二分球的成功率是 30%，已知 Shenille 总共尝试了 30 次投篮，问她总共得了多少 分？

(A) 12 **(B)** 18 **(C)** 24 **(D)** 30 **(E)** 36

Problem 7

The sequence $S_1, S_2, S_3, \cdots, S_{10}$ has the property that every term beginning with the third is the sum of the previous two. That is,

$$S_n = S_{n-2} + S_{n-1} \text{ for } n \geq 3.$$

Suppose that $S_9 = 110$ and $S_7 = 42$. What is S_4?

数列$S_1, S_2, S_3, \cdots, S_{10}$具有这样的性质：从第三项开始，每一项都是前两项之和，即对于$n \geq 3$，有$S_n = S_{n-2} + S_{n-1}$. 假设$S_9 = 110$，$S_7 = 42$，则S_4是多少？

(A) 4 **(B)** 6 **(C)** 10 **(D)** 12 **(E)** 16

Problem 8

Given that x and y are distinct nonzero real numbers such that $x + \frac{2}{x} = y + \frac{2}{y}$, what is xy?

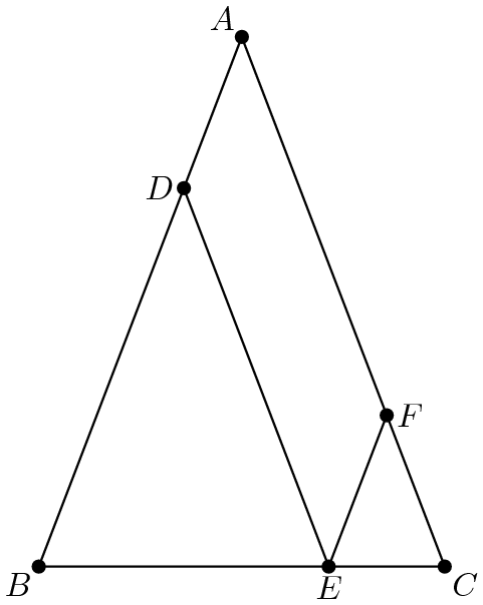
已知x，y是不等的非零实数，满足$x + \frac{2}{x} = y + \frac{2}{y}$，问$xy$是多少？

(A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** 4

Problem 9

In $\triangle ABC$, $AB = AC = 28$ and $BC = 20$. Points $D, E,$ and F are on sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively, such that $\overline{DE}$ and $\overline{EF}$ are parallel to $\overline{AC}$ and $\overline{AB}$, respectively. What is the perimeter of parallelogram $ADEF$?

在$\triangle ABC$中， *AB*=*AC*=28， *BC*=20，点 *D*，*E*，*F* 分别在边$\overline{AB}$，$\overline{BC}$和$\overline{AC}$上，*DE* 和 *EF* 分别平行于 *AC* 和 *AB*，问平行四边形 *ADEF* 的周长是多少？



(A) 48 **(B)** 52 **(C)** 56 **(D)** 60 **(E)** 72

Problem 10

Let S be the set of positive integers n for which $\frac{1}{n}$ has the repeating decimal representation $0.\overline{ab} = 0.ababab\cdots$, with a and b different digits. What is the sum of the elements of S?

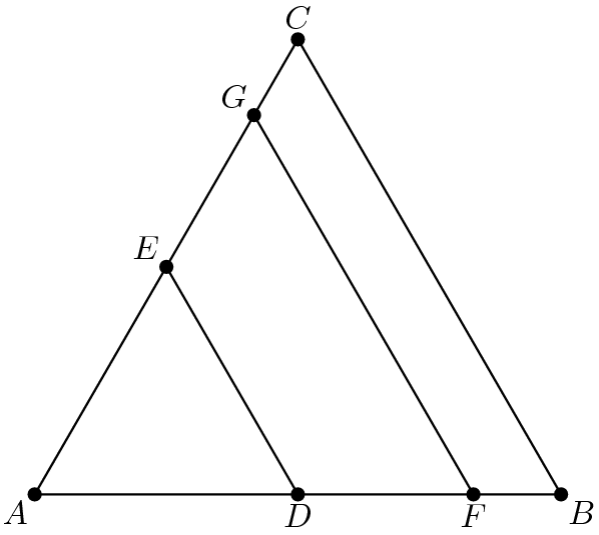
S 是所有满足以下条件的正整数 n 的集合：$\frac{1}{n}$的小数表示是个循环小数$0.\overline{ab} = 0.ababab\cdots$,其中 *a* 和 *b* 是不同的数字，问 S 中所有元素之和是多少？

(A) 11 **(B)** 44 **(C)** 110 **(D)** 143 **(E)** 155

Problem 11

Triangle ABC is equilateral with $AB = 1$. Points E and G are on $\overline{AC}$ and points D and F are on $\overline{AB}$ such that both $\overline{DE}$ and $\overline{FG}$ are parallel to $\overline{BC}$. Furthermore, triangle ADE and trapezoids $DFGE$ and $FBCG$ all have the same perimeter. What is $DE + FG$?

三角形 *ABC* 是等边三角形，*AB*＝1，点 *E* 和 *G* 在线段 *AC* 上，点 *D* 和 *F* 在线段 *AB* 上，*DE* 和 *FG* 都与 *BC* 平行，此外，三角形 *ADE* 和梯形 *DFGE*、*FBCG* 三者的周长都相等。问 *DE*＋*FG* 等于多少？



(A) 1 **(B)** $\frac{3}{2}$ **(C)** $\frac{21}{13}$ **(D)** $\frac{13}{8}$ **(E)** $\frac{5}{3}$

Problem 12

The angles in a particular triangle are in arithmetic progression, and the side lengths are $4, 5, x$. The sum of the possible values of x equals $a + \sqrt{b} + \sqrt{c}$ where a, b, and c are positive integers. What is $a + b + c$?

某个三角形的三个内角成等差数列，且边长分别是$4, 5, x$，已知 *x* 的所有可能值之和等于 $a + \sqrt{b} + \sqrt{c}$，这里 *a*，*b*，*c* 都是正整数，则 *a*+*b*+*c* 是多少？

(A) 36 **(B)** 38 **(C)** 40 **(D)** 42 **(E)** 44

Problem 13

Let points $A=(0,0),\ B=(1,2),\ C=(3,3)$, and $D=(4,0)$. Quadrilateral $ABCD$ is cut into equal area pieces by a line passing through A. This line intersects $\overline{CD}$ at point $\left(\frac{p}{q},\frac{r}{s}\right)$, where these fractions are in lowest terms. What is $p+q+r+s$?

已知四个点 *A*=（0，0），*B*=（1，2），*C*=（3，3），*D*=（4，0）,四边形 *ABCD* 被一条通过 *A* 点的直线分成面积相等的两块区域，这条直线和$\overline{CD}$交于点$\left(\frac{p}{q},\frac{r}{s}\right)$, 这里的两个分数已经是最简分数，问$p+q+r+s$是多少？

(A) 54 **(B)** 58 **(C)** 62 **(D)** 70 **(E)** 75

Problem 14

The sequence $\log_{12}162,\log_{12}x,\log_{12}y,\log_{12}z,\log_{12}1250$ is an arithmetic progression. What is x?

$\log_{12}162,\ \log_{12}x,\ \log_{12}y,\ \log_{12}z,\ \log_{12}1250$ 是个等差数列，问x是多少？

(A) $125\sqrt{3}$ **(B)** 270 **(C)** $162\sqrt{5}$ **(D)** 434 **(E)** $225\sqrt{6}$

Problem 15

Rabbits Peter and Pauline have three offspring—Flopsie, Mopsie, and Cotton-tail. These five rabbits are to be distributed to four different pet stores so that no store gets both a parent and a child. It is not required that every store gets a rabbit. In how many different ways can this be done?

兔子 Peter 和 Pauline 有 3 只后代——Flopsie，Mopsie 和 Cotton-tail，这 5 只兔子要被分配到 4 家不同的宠物店，需满足父亲或母亲都不能和孩子分到同一家店，并且不要求每家店都分到兔子，问一共有多少种分配方法？

(A) 96 **(B)** 108 **(C)** 156 **(D)** 204 **(E)** 372

Problem 16

A, B, C are three piles of rocks. The mean weight of the rocks in A is 40 pounds, the mean weight of the rocks in B is 50 pounds, the mean weight of the rocks in the combined piles A and B is 43 pounds, and the mean weight of the rocks in the combined piles A and C is 44 pounds. What is the greatest possible integer value for the mean in pounds of the rocks in the combined piles B and C?

A，*B*，*C* 代表三堆石头，*A* 中石头的平均重量是 40 磅，*B* 中石头的平均重量是 50 磅，*A* 和 *B* 这两堆石头合起来后的平均重量是 43 磅，*A* 和 *C* 这两堆石头合起来后的平均重量是 44 磅，问 *B* 和 *C* 合起来后平均重量的最大可能的整数值是多少?

(A) 55 **(B)** 56 **(C)** 57 **(D)** 58 **(E)** 59

Problem 17

A group of 12 pirates agree to divide a treasure chest of gold coins among themselves as follows. The k^{th} pirate to take a share takes $\frac{k}{12}$ of the coins that remain in the chest. The number of coins initially in the chest is the smallest number for which this arrangement will allow each pirate to receive a positive whole number of coins. How many coins does the 12^{th} pirate receive?

12 个海盗同意按照以下方案分一箱金币：第 *k* 个海盗拿走箱子里剩下金币的$\frac{k}{12}$. 一开始箱子里金币的总数量是这种分配方案能够使得每个海盗都能分得整数枚金币的最小正整数，问第 12 个海盗得到多少枚金币?

(A) 720 **(B)** 1296 **(C)** 1728 **(D)** 1925 **(E)** 3850

Problem 18

Six spheres of radius 1 are positioned so that their centers are at the vertices of a regular hexagon of side length 2. The six spheres are internally tangent to a larger sphere whose center is the center of the hexagon. An eighth sphere is externally tangent to the six smaller spheres and internally tangent to the larger sphere. What is the radius of this eighth sphere?

6 个半径为 1 的球的球心位于一个边长为 2 的正六边形的 6 个顶点上，这 6 个球和一个球心在六边形中心的大球相内切，第 8 个球和这 6 个小球都外切，且和这个大球相内切，那么这第 8 个球的半径是多少?

(A) $\sqrt{2}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{5}{3}$ **(D)** $\sqrt{3}$ **(E)** 2

Problem 19

In $\triangle ABC$, $AB = 86$, and $AC = 97$. A circle with center A and radius AB intersects $\overline{BC}$ at points B and X. Moreover $\overline{BX}$ and $\overline{CX}$ have integer lengths. What is BC?

在$\triangle ABC$中，*AB*=86，AC=97，一个以 *A* 为圆心，*AB* 为半径的圆和$\overline{BC}$线段交于点 *B* 和 *X*，并且线段$\overline{BX}$和$\overline{CX}$的长度都是整数，问 *BC* 的长是多少？

(A) 11 **(B)** 28 **(C)** 33 **(D)** 61 **(E)** 72

Problem 20

Let S be the set $\{1, 2, 3, ..., 19\}$. For $a, b \in S$, define $a \succ b$ to mean that

either $0 < a - b \leq 9$ or $b - a > 9$. How many ordered triples (x, y, z) of elements of S have the property that $x \succ y$, $y \succ z$, and $z \succ x$?

S 代表集合｛1，2，3，...，19｝，对于 *S* 中的元素$a, b \in S$，定义$a \succ b$为$0 < a - b \leq 9$或者$b - a > 9$，问 *S* 中有多少有序三元组(x, y, z)有这样的性质：$x \succ y$，$y \succ z$且$z \succ x$?

(A) 810 **(B)** 855 **(C)** 900 **(D)** 950 **(E)** 988

Problem 21

Consider $A = \log(2013 + \log(2012 + \log(2011 + \log(\cdots + \log(3 + \log 2) \cdots))))$.

Which of the following intervals contains A?

考虑$A = \log(2013 + \log(2012 + \log(2011 + \log(\cdots + \log(3 + \log 2) \cdots))))$，下面哪个区间包含$A$?

(A) $(\log 2016, \log 2017)$ **(B)** $(\log 2017, \log 2018)$ **(C)** $(\log 2018, \log 2019)$

(D) $(\log 2019, \log 2020)$ **(E)** $(\log 2020, \log 2021)$

Problem 22

A palindrome is a nonnegative integer number that reads the same forwards and backwards when written in base 10 with no leading zeros. A 6-digit palindrome n is chosen uniformly at random. What is the probability that $\frac{n}{11}$ is also a palindrome?

回环数是指从左往右和从右往左读，读数都相同的首位不为 0 的十进制非负整数。从所有 6 位回环数中随机选择一个，问$\frac{n}{11}$也是回环数的概率是多少？

$\textbf{(A)}\ \frac{8}{25}\qquad \textbf{(B)}\ \frac{33}{100}\qquad \textbf{(C)}\ \frac{7}{20}\qquad \textbf{(D)}\ \frac{9}{25}\qquad \textbf{(E)}\ \frac{11}{30}$

Problem 23

$ABCD$ is a square of side length $\sqrt{3}+1$. Point P is on $\overline{AC}$ such that $AP=\sqrt{2}$. The square region bounded by $ABCD$ is rotated 90° counterclockwise with center P, sweeping out a region whose area is $\frac{1}{c}(a\pi+b)$, where a, b, and c are positive integers and $\gcd(a,b,c)=1$. What is $a+b+c$?

正方形 *ABCD* 边长为$\sqrt{3}+1$，点P在线段$\overline{AC}$上， $AP=2$.由 *ABCD* 所包围的正方形区域绕着点 *P* 逆时针旋转 90°，扫出的区域的面积是$\frac{1}{c}(a\pi+b)$，这里 *a*，*b*，*c* 都是正整数，且$\gcd(a,b,c)=1$，问 *a* +*b* +*c* 是多少？

$\textbf{(A)}\ 15\qquad \textbf{(B)}\ 17\qquad \textbf{(C)}\ 19\qquad \textbf{(D)}\ 21\qquad \textbf{(E)}\ 23$

Problem 24

Three distinct segments are chosen at random among the segments whose end-points are the vertices of a regular 12-gon. What is the probability that the lengths of these three segments are the three side lengths of a triangle with positive area?

一个正十二边形的顶点两两相连所形成的所有线段中，任意选择 3 根不同的线段（这 3 条线段中可能存在等长的线段），问这三根线段能够组成一个面积为正的三角形的概率是多少？

$\textbf{(A)}\ \frac{553}{715}$ $\textbf{(B)}\ \frac{443}{572}$ $\textbf{(C)}\ \frac{111}{143}$ $\textbf{(D)}\ \frac{81}{104}$ $\textbf{(E)}\ \frac{223}{286}$

Problem 25

Let $f : \mathbb{C} \to \mathbb{C}$ be defined by $f(z) = z^2 + iz + 1$. How many complex numbers z are there such that $\text{Im}(z) > 0$ and both the real and the imaginary parts of $f(z)$ are integers with absolute value at most 10?

f是定义在复数域上的一个函数，$f : \mathbb{C} \to \mathbb{C}$，且$f(z) = z^2 + iz + 1$，有多少个这样的复数 z 存在，满足$\text{Im}(z) > 0$ (z 的虚部大于 0)，且$f(z)$的实部和虚部都是绝对值至多为 10 的整数？

$\textbf{(A)}\ 399$ $\textbf{(B)}\ 401$ $\textbf{(C)}\ 413$ $\textbf{(D)}\ 431$ $\textbf{(E)}\ 441$

2013 AMC 12A Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
E	C	E	C	B	B	C	D	C	D	C	A	B
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
B	D	E	D	B	D	B	A	E	C	E	A	

2013 AMC 12A Solution

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