2018 AMC 12A

Problem 1

A large urn contains 100 balls, of which 36% are red and the rest are blue. How many of the blue balls must be removed so that the percentage of red balls in the urn will be 72%? (No red balls are to be removed.)

一个大瓮包含 100 个球，其中 36%的球是红色的，其余的球是蓝色的。必须要移除多少个蓝球，才能使得大瓮中的红球的比例变为 72%？ （红球不会被移除。）

$\textbf{(A)}\ 28 \qquad \textbf{(B)}\ 32 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 50 \qquad \textbf{(E)}\ 64$

Problem 2

While exploring a cave, Carl comes across a collection of 5-pound rocks worth $\$14$ each, 4-pound rocks worth $\$11$ each, and 1-pound rocks worth $\$2$ each. There are at least 20 of each size. He can carry at most 18 pounds. What is the maximum value, in dollars, of the rocks he can carry out of the cave?

Carl 在一个山洞里探险，他发现了一些 5 磅重的石头每块价值 14 美元，4 磅重的石头每块价值 11 美元，以及 1 磅重的石头每块价值 2 美元。每种大小的石头各至少有 20 块。他最多能 18 磅。他可以从山洞中带走的岩石的最大价值是多少美元？

$\textbf{(A)}\ 48 \qquad \textbf{(B)}\ 49 \qquad \textbf{(C)}\ 50 \qquad \textbf{(D)}\ 51 \qquad \textbf{(E)}\ 52$

Problem 3

How many ways can a student schedule 3 mathematics courses -- algebra, geometry, and number theory -- in a 6-period day if no two mathematics courses can be taken in consecutive periods? (What courses the student takes during the other 3 periods is of no concern here.)

一个学生想要在一天的 6 堂课中安排 3 门数学课-代数，几何和数论，要求任何两堂数学课不能连续安排（其他三堂课如何安排无需考虑），请问有多少种安排方法？

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 24$

Problem 4

Alice, Bob, and Charlie were on a hike and were wondering how far away the nearest town was. When Alice said, "We are at least 6 miles away," Bob replied, "We are at most 5 miles away." Charlie then remarked, "Actually the nearest town is at most 4 miles away." It turned out that none of the three statements were true. Let d be the distance in miles to the nearest town. Which of the following intervals is the set of all possible values of d?

Alice、Bob、和 Charlie 在徒步旅行，他们想知道离最近的城镇还有多远。Alice 说：“我们至少有 6 英里远。”Bob 回答说：“我们最多只有 5 英里远。”Charlie 评论道：“我们最多只有 4 英里远。”实际上这三个陈述都不正确。令 d 是到最近城镇的距离的英里数。以下哪个答案是 d 的所有可能值的集合？

$\textbf{(A)}\ (0,4) \qquad \textbf{(B)}\ (4,5) \qquad \textbf{(C)}\ (4,6) \qquad \textbf{(D)}\ (5,6) \qquad \textbf{(E)}\ (5,\infty)$

Problem 5

What is the sum of all possible values of k for which the polynomials $x^2 - 3x + 2$ and $x^2 - 5x + k$ have a root in common?

使得多项式$x^2 - 3x + 2$和$x^2 - 5x + k$有一个共同的根的所有可能的 k 的值的总和是多少？

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 10$

Problem 6

For positive integers m and n such that $m+10<n+1$, both the mean and the median of the set $\{m, m+4, m+10, n+1, n+2, 2n\}$ are equal to n. What is $m+n$?

对于满足$m+10<n+1$的正整数 m 和 n，集合$\{m, m+4, m+10, n+1, n+2, 2n\}$的平均值和中位数都等于 n， $m + n$ 是多少？

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 21 \qquad \textbf{(C)}\ 22 \qquad \textbf{(D)}\ 23 \qquad \textbf{(E)}\ 24$

Problem 7

For how many (not necessarily positive) integer values of n is the value of $4000\cdot \left(\tfrac{2}{5}\right)^n$ an integer?

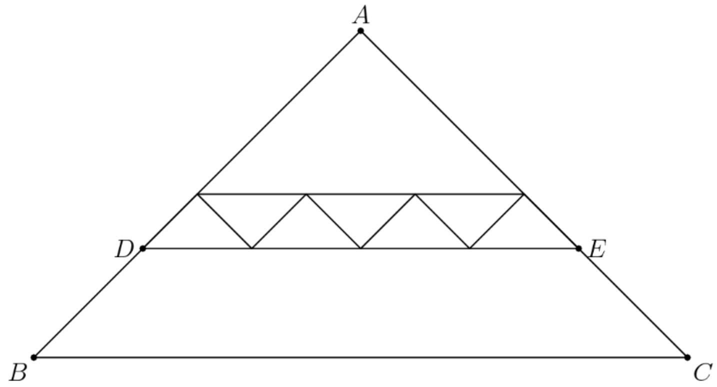
对于多少整数n，使得 $4000\cdot \left(\frac{2}{5}\right)^n$是一个整数？

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

Problem 8

All of the triangles in the diagram below are similar to isosceles triangle ABC, in which $AB = AC$. Each of the 7 smallest triangles has area 1, and $\triangle ABC$ has area 40. What is the area of trapezoid $DBCE$?

下图中的所有三角形与等腰三角形 ABC 类似，$AB = AC$。 7 个最小的三角形的面积都是 1，$\triangle ABC$的面积是 40。梯形 $DBCE$ 的面积是多少？



(A) 16 **(B)** 18 **(C)** 20 **(D)** 22 **(E)** 24

Problem 9

Which of the following describes the largest subset of values of y within the closed interval $[0, \pi]$ for which $\sin(x+y) \leq \sin(x) + \sin(y)$ for every x between 0 and π, inclusive?

以下哪一选项描述了在闭区间$[0, \pi]$内 y 的取值的最大子集，使得

$$\sin(x+y) \leq \sin(x) + \sin(y)$$

对于在 0 和 π 之间（包括 0 和 π）的每个 x 成立？

(A) $y = 0$ **(B)** $0 \leq y \leq \frac{\pi}{4}$ **(C)** $0 \leq y \leq \frac{\pi}{2}$ **(D)** $0 \leq y \leq \frac{3\pi}{4}$ **(E)** $0 \leq y \leq \pi$

Problem 10

How many ordered pairs of real numbers (x, y) satisfy the following system of equations?

$$x + 3y = 3$$
$$\big||x| - |y|\big| = 1$$

有多少有序数(x, y)对使得下面两个式子成立

$$x + 3y = 3$$

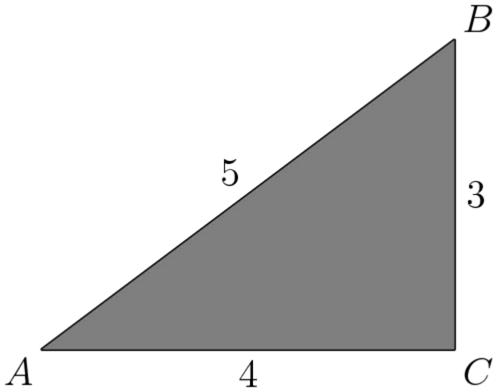
$$\big||x| - |y|\big| = 1$$

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 8$

Problem 11

A paper triangle with sides of lengths $3, 4,$ and 5 inches, as shown, is folded so that point A falls on point B. What is the length in inches of the crease?

如图所示，三边长度为 3，4 和 5 英寸的三角形纸被折叠，使得点 A 落在点 B 上。折痕的长度是多少英寸？



$\textbf{(A)}\ 1 + \frac{1}{2}\sqrt{2} \qquad \textbf{(B)}\ \sqrt{3} \qquad \textbf{(C)}\ \frac{7}{4} \qquad \textbf{(D)}\ \frac{15}{8} \qquad \textbf{(E)}\ 2$

Problem 12

Let S be a set of 6 integers taken from $\{1, 2, \ldots, 12\}$ with the property that if a and b are elements of S with $a < b$, then b is not a multiple of a. What is the least possible value of an element in S?

设 S 是从{1，2，…，12}中取出 6 个整数组成的集合，具有性质：如果 a 和 b 是 S 中的元素并满足 $a<b$，那么 b 不是 a 的倍数。 S 中元素的最小可能值是多少？

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 7

Problem 13

How many nonnegative integers can be written in the form $a_7 \cdot 3^7 + a_6 \cdot 3^6 + a_5 \cdot 3^5 + a_4 \cdot 3^4 + a_3 \cdot 3^3 + a_2 \cdot 3^2 + a_1 \cdot 3^1 + a_0 \cdot 3^0$,where

$a_i \in \{-1, 0, 1\}$ for $0 \leq i \leq 7$?

有多少个非负整数可以写成以下形式

$a_7 \cdot 3^7 + a_6 \cdot 3^6 + a_5 \cdot 3^5 + a_4 \cdot 3^4 + a_3 \cdot 3^3 + a_2 \cdot 3^2 + a_1 \cdot 3^1 + a_0 \cdot 3^0$,

其中对于$0 \leq i \leq 7$有$a_i \in \{-1, 0, 1\}$?

(A) 512 **(B)** 729 **(C)** 1094 **(D)** 3281 **(E)** 59, 048

Problem 14

The solution to the equation $\log_{3x} 4 = \log_{2x} 8$, where x is a positive real number other than $\frac{1}{3}$ or $\frac{1}{2}$, can be written as $\frac{p}{q}$ where p and q are relatively prime positive integers. What is $p + q$?

方程 $\log_{2x} 4 = \log_{3x} 8$ 的解 x 是不等于 $\frac{1}{3}$ 或者 $\frac{1}{2}$ 的正实数，它的解可以表示成$\frac{p}{q}$，其中 p 和 q 是互质的正整数。$p + q$ 是多少?

(A) 5 **(B)** 13 **(C)** 17 **(D)** 31 **(E)** 35

Problem 15

A scanning code consists of a 7×7 grid of squares, with some of its squares colored black and the rest colored white. There must be at least one square of each color in this grid of 49 squares. A scanning code is called $symmetric$ if its look does not change when the entire square is rotated by a multiple of 90° counterclockwise around its center, nor when it is reflected across a line joining opposite corners or a line joining midpoints of opposite sides. What is the total number of possible symmetric scanning codes?

一个扫描代码由一个 7 x 7 的正方形网格组成，其中一些正方形的颜色为黑色，其余为白色。这 49 格子里每种颜色的正方形至少有一个。如果整个扫描码以中心为轴逆时针旋转 90°的倍数，或是沿着正方形的对角线以及连接对边中点的联机翻转时，整个图形都不改变，那么这个扫面代码被称为对称的。所有可能的对称扫描代码的总数是多少？

$\textbf{(A)}\ 510 \qquad \textbf{(B)}\ 1022 \qquad \textbf{(C)}\ 8190 \qquad \textbf{(D)}\ 8192 \qquad \textbf{(E)}\ 65,534$

Problem 16

Which of the following describes the set of values of a for which the curves $x^2+y^2=a^2$ and $y=x^2-a$ in the real xy-plane intersect at exactly 3 points?

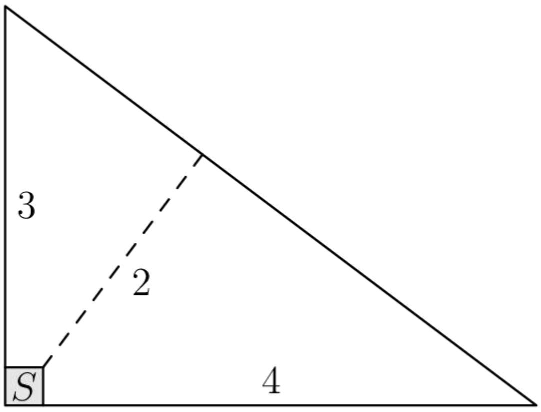
使得曲线 $x^2+y^2=a^2$ 和 $y=x^2-a$ 在 xy 实坐标系恰好有 3 个交点的 a 的取值范围是什么?

$\textbf{(A)}\ a=\frac{1}{4} \qquad \textbf{(B)}\ \frac{1}{4}<a<\frac{1}{2} \qquad \textbf{(C)}\ a>\frac{1}{4} \qquad \textbf{(D)}\ a=\frac{1}{2} \qquad \textbf{(E)}\ a>\frac{1}{2}$

Problem 17

Farmer Pythagoras has a field in the shape of a right triangle. The right triangle's legs have lengths 3 and 4 units. In the corner where those sides meet at a right angle, he leaves a small unplanted square S so that from the air it looks like the right angle symbol. The rest of the field is planted. The shortest distance from S to the hypotenuse is 2 units. What fraction of the field is planted?

农场主 Pythagoras 有一块形状为直角三角形的土地，两条直角边的长度分别为 3 个单位　和 4 个单位。在两条直角边相交形成直角的角落处，他留下了一小块正方形土地没有进行种植，如图标记为 S 的区域，使得从空中看下去很像是直角的标识。其余的土地都进行了种植。从 S 到斜边的最短距离为 2 个单位。整块土地的种植比例是多少？



$\textbf{(A) } \frac{25}{27} \qquad \textbf{(B) } \frac{26}{27} \qquad \textbf{(C) } \frac{73}{75} \qquad \textbf{(D) } \frac{145}{147} \qquad \textbf{(E) } \frac{74}{75}$

Problem 18

Triangle ABC with $AB=50$ and $AC=10$ has area 120. Let D be the midpoint of $\overline{AB}$, and let E be the midpoint of $\overline{AC}$. The angle bisector of $\angle BAC$ intersects $\overline{DE}$ and $\overline{BC}$ at F and G, respectively. What is the area of quadrilateral $FDBG$?

在三角形 ABC 中，$AB = 50$，$AC = 10$，其面积为 120。令 D 为$\overline{AB}$的中点并且 E 为$\overline{AC}$的中点。$\angle BAC$ 的角平分线分别与$\overline{DE}$和$\overline{BC}$相交于 F 和 G。四边形 $FDBG$ 的面积是多少？

$\textbf{(A) } 60 \qquad \textbf{(B) } 65 \qquad \textbf{(C) } 70 \qquad \textbf{(D) } 75 \qquad \textbf{(E) } 80$

Problem 19

Let A be the set of positive integers that have no prime factors other than 2, 3, or 5. The infinite sum $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}+\frac{1}{12}+\frac{1}{15}+\frac{1}{16}+\frac{1}{18}+\frac{1}{20}+\cdots$ of the reciprocals of the elements of A can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

令 A 是质因子只有 2，3 或 5 的正整数组成的集合。集合 A 中的所有元素的倒数的无限的总和 $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}+\frac{1}{12}+\frac{1}{15}+\frac{1}{16}+\frac{1}{18}+\frac{1}{20}+\cdots$ 可表示为 $\frac{m}{n}$，其中 m 和 n 是互质的正整数。 $m+n$ 是多少?

$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 17 \qquad \textbf{(C)}\ 19 \qquad \textbf{(D)}\ 23 \qquad \textbf{(E)}\ 36$

Problem 20

Triangle ABC is an isosceles right triangle with $AB=AC=3$. Let M be the midpoint of hypotenuse $\overline{BC}$. Points I and E lie on sides $\overline{AC}$ and $\overline{AB}$, respectively, so that $AI>AE$ and $AIME$ is a cyclic quadrilateral. Given that triangle EMI has area 2, the length CI can be written as $\frac{a-\sqrt{b}}{c}$, where a, b, and c are positive integers and b is not divisible by the square of any prime. What is the value of $a+b+c$?

三角形 ABC 是一个等腰直角三角形，$AB = AC = 3$。令 M 是斜边 BC 的中点。 I 点和 E 点分别位于 AC 和 AB 上， 使得 $AI > AE$， 并且 $AIME$ 是一个圆内接四边形。已知三角形 EMI 的面积是 2，CI 的长度可以被写为 $\frac{a-\sqrt{b}}{c}$ ，其中 a, b, 和 c 是正整数并且 b 不能被任何质数的平方所整除。 $a + b + c$ 是多少?

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 11 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 13$

Problem 21

Which of the following polynomials has the greatest real root?

以下哪个多项式的实数根最大?

$\textbf{(A)}\ x^{19}+2018x^{11}+1 \qquad \textbf{(B)}\ x^{17}+2018x^{11}+1 \qquad \textbf{(C)}\ x^{19}+2018x^{13}+1 \qquad \textbf{(D)}\ x^{17}+2018x^{13}+1 \qquad \textbf{(E)}\ 2019x+2018$

Problem 22

The solutions to the equations $z^2 = 4 + 4\sqrt{15}i$ and $z^2 = 2 + 2\sqrt{3}i$, where $i = \sqrt{-1}$, form the vertices of a parallelogram in the complex plane. The area of this parallelogram can be written in the form $p\sqrt{q} - r\sqrt{s}$, where p, q, r, and s are positive integers and neither q nor s is divisible by the square of any prime number. What is $p + q + r + s$?

方程$z^2 = 4 + 4\sqrt{15}i$和$z^2 = 2 + 2\sqrt{3}i$,的解构成复平面的一个平行四边形。该平行四边形的面积可以用$p\sqrt{q} - r\sqrt{s}$,来表示，其中的 p,q,r,s 是正整数，并且 q 和 s 不被任何质数的平方所整除。问 $p+q+r+s$ 的值是？

(A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

Problem 23

In $\triangle PAT$, $\angle P = 36^\circ$, $\angle A = 56^\circ$, and $PA = 10$. Points U and G lie on sides $\overline{TP}$ and $\overline{TA}$, respectively, so that $PU = AG = 1$. Let M and N be the midpoints of segments $\overline{PA}$ and $\overline{UG}$, respectively. What is the degree measure of the acute angle formed by lines MN and PA?

在$\triangle PAT$,中，$\angle P = 36^\circ$，$\angle A = 56^\circ$，并且 $PA = 10$。U 点和 G 点分别位于$\overline{TP}$和$\overline{TA}$,上，使得 $PU = AG = 1$。令 M 和 N 点分别是$\overline{PA}$和$\overline{UG}$,的中点。直线 MN 和 PA 形成的锐角的角度是多少？

(A) 76 **(B)** 77 **(C)** 78 **(D)** 79 **(E)** 80

Problem 24

Alice, Bob, and Carol play a game in which each of them chooses a real number between 0 and 1. The winner of the game is the one whose number is between the numbers chosen by the other two players. Alice announces that she will choose her number uniformly at random from all the numbers between 0 and 1, and Bob announces that he will choose his number uniformly at random from all the numbers between $\frac{1}{2}$ and $\frac{2}{3}$. Armed with this information, what number should Carol choose to maximize her chance of winning?

Alice,Bob,和 Carol 玩一个游戏，其中每个玩家选择一个介于 0 和 1 之间的实数。如果一个玩家选择的数介于其他两个玩家所选择的数之间，那么这个玩家就是胜利者。Alice 宣布她将从 0 到 1 之间的所有按照均匀分布随机地选择她的数，Bob 宣布他将从 $\frac{1}{2}$ 和 $\frac{2}{3}$ 之间的所有数中按照均匀分布随机选择他的数。有了这些信息，Carol 应该选择什么数来最大化她的获胜机会？

$\textbf{(A)}\ \frac{1}{2} \qquad \textbf{(B)}\ \frac{13}{24} \qquad \textbf{(C)}\ \frac{7}{12} \qquad \textbf{(D)}\ \frac{5}{8} \qquad \textbf{(E)}\ \frac{2}{3}$

Problem 25

For a positive integer n and nonzero digits a, b, and c, let A_n be the n-digit integer each of whose digits is equal to a; let B_n be the n-digit integer each of whose digits is equal to b, and let C_n be the $2n$-digit (not n-digit) integer each of whose digits is equal to c. What is the greatest possible value of $a + b + c$ for which there are at least two values of n such that $C_n - B_n = A_n^2$?

对于正整数 n 和非零数字 a，b 以及 c，令 A_n 为 n 位整数，其每一位数字都等于 a；令 B_n 为 n 位整数，其每一位数字都等于 b；令 C_n 为 $2n$ 位整数（非 n 位整数），其每一位数字都等于 c。使得等式 $C_n - B_n = A^2$ 对于至少两个 n 值成立的 $a + b + c$ 最大可能取值为多少？

$\textbf{(A)}\ 12 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 20$

2018 AMC 12A Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
D	C	E	D	E	B	E	E	E	C	D	C	D
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
D	B	E	D	D	C	D	B	A	E	B	D	

2018 AMC 12A Solution

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