2024 MAA AMC 12B

Problem 1

In a long line of people arranged left to right, the 1013th person from the left is also the 1010th person from the right. How many people are in the line?

一群人从左到右排成一个很长的队伍，从左边数起的第 1013 个人，也是从右边数起的第 1010 个人．问这个队伍中共有多少人?

(A) 2021 **(B)** 2022 **(C)** 2023 **(D)** 2024 **(E)** 2025

Problem 2

What is $10! - 7! \cdot 6!$?

问 $10! - 7! \cdot 6!$是多少?

(A) -120 **(B)** 0 **(C)** 120 **(D)** 600 **(E)** 720

Problem 3

For how many integer values of x is $|2x| \leq 7\pi$?

满足$|2x| \leq 7\pi$的整数值x有多少个?

(A) 16 **(B)** 17 **(C)** 19 **(D)** 20 **(E)** 21

Problem 4

Balls numbered $1,2,3,\cdots$ are deposited in 5 bins, labeled $A, B, C, D,$ and E using the following procedure. Ball 1 is deposited in bin A, and balls 2 and 3 are deposited in bin B. The next 3 balls are deposited in bin C, the next 4 in bin D, and so on, cycling back to bin A after balls are deposited in bin E. (For example, balls numbered $22,23,\cdots,28$ are deposited in bin B at step 7 of this process.) In which bin is ball 2024 deposited?

编号为 1，2，3，... 的球按照以下操作步骤存放在 5 个分别标记为 A，B，C，D，E 的箱子中．1 号球存放在箱子 A 中，2 号球和 3 号球存放在箱子 B 中．接下来

的 3 个球存放在箱子 C 中，接下来的 4 个球存放在箱子 D 中，依此类推，球存放到箱子 E 中后，再循环回到箱子 A.（例如，编号为 22， 23，...，28 的球在操作的第 7 步存放在箱子 B 中.）问 2024 号球存放在哪个箱子中?

(A) A **(B)** B **(C)** C **(D)** D **(E)** E

Problem 5

In the following expression, Melanie changed some of the plus signs to minus signs:

$$1+3+5+7+\cdots+97+99$$

When the new expression was evaluated, it was negative. What is the least number of plus signs that Melanie could have changed to minus signs?

在下面的表达式中，Melanie 将一些加号改为减号:

$$1+3+5+7+\cdots+97+99$$

当对新的表达式求值时，结果为负数. 问 Melanie 最少可能将多少个加号改为减号?

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

Problem 6

The national debt of the United States is on track to reach $5\cdot 10^{13}$ dollars by 2033. How many digits does this number of dollars have when written as a numeral in base 5? (The approximation of $\log_{10} 5$ as 0.7 is sufficient for this problem.)

美国的国债预计到 2033 年将达到5×10^{13}美元. 当这个数用 5 进制表示时，有多少位数? （在本题中，只需使用近似值$\log_{10} 5\approx 0.7$.）

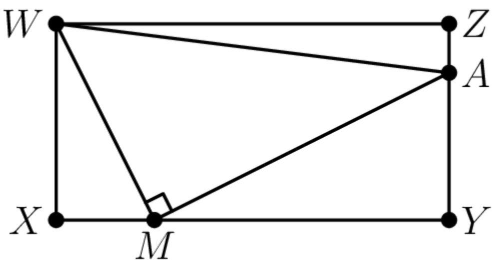
(A) 18 **(B)** 20 **(C)** 22 **(D)** 24 **(E)** 126

Problem 7

In the figure below $WXYZ$ is a rectangle with $WX=4$ and $WZ=8$. Point M lies on $\overline{XY}$, point A lies on $\overline{YZ}$, and $\angle WMA$ is a right angle. The areas of $\triangle$

WXM and $\triangle WAZ$ are equal. What is the area of $\triangle WMA$?

在矩形$WXYZ$中，$WX=4$且$WZ=8$．点M在$\overline{XY}$上，点A在$\overline{YZ}$上，且$\angle WMA$为直角．三角形$\triangle WXM$与$\triangle WAZ$的面积相等．问$\triangle WMA$的面积是多少?



(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

Problem 8

What value of x satisfies $\frac{\log_2 x \cdot \log_3 x}{\log_2 x+\log_3 x}=2$?

求满足方程$\frac{\log_2 x \cdot \log_3 x}{\log_2 x+\log_3 x}=2$的$x$的取值?

(A) 25 **(B)** 32 **(C)** 36 **(D)** 42 **(E)** 48

Problem 9

A dartboard is the region B in the coordinate plane consisting of points (x, y) such that $|x|+|y| \leq 8$. A target T is the region where $(x^2+y^2-25)^2 \leq 49$. A dart is thrown and lands at a random point in B. The probability that the dart lands in T can be expressed as $\frac{m}{n} \cdot \pi$,where m and n are relatively prime positive integers. What is $m+n$?

在坐标平面上，由满足$|x|+|y| \leq 8$的点(x, y)组成靶盘区域B．由$(x^2+y^2-25)^2 \leq 49$ 界定目标区域T．飞镖被投掷后随机的落在B内的某个点上．飞镖落在T内的概率可以表示为$\frac{m}{n} \cdot \pi$，其中m和n是互质的正整数．问$m+n$是多少?

(A) 39 **(B)** 71 **(C)** 73 **(D)** 75 **(E)** 135

Problem 10

A list of 9 real numbers consists of 1, 2.2, 3.2, 5.2, 6.2, and 7, as well as x, y , and z with $x \le y \le z$. The range of the list is 7, and the mean and the median are both positive integers. How many ordered triples (x, y, z) are possible?

由 9 个实数组成的数据列表包含 1，2.2，3.2，5.2，6.2，7，以及满足$x \le y \le z$ 的数x，y，z．此数据列表的全距为 7，且平均数和中位数都是正整数．问有序三元组(x, y, z)有多少种可能？

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** infinitely many ｜无穷多个

Problem 11

Let $x_n = sin^2(n°)$. What is the mean of $x_1, x_2, x_3, \cdots, x_{90}$?

设$x_n = sin^2(n°)$，问$x_1, x_2, x_3, \cdots, x_{90}$的均值是多少？

(A) $\frac{11}{45}$ **(B)** $\frac{22}{45}$ **(C)** $\frac{89}{180}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{91}{180}$

Problem 12

Suppose z is a complex number with positive imaginary part, with real part greater than 1, and with $|z| = 2$. In the complex plane, the four points 0, z, z^2, and z^3 are the vertices of a quadrilateral with area 15. What is the imaginary part of z?

假设复数z的虚部为正，实部大于 1，且$|z| = 2$．在复平面上，以四个值 0，z，z^2, z^3为顶点构成的四边形的面积为 15．问 z的虚部是多少？

(A) $\frac{3}{4}$ **(B)** 1 **(C)** $\frac{4}{3}$ **(D)** $\frac{3}{2}$ **(E)** $\frac{5}{3}$

Problem 13

There are real numbers x, y, h and k that satisfy the system of equations

$$x^2 + y^2 - 6x - 8y = h$$

$$x^2 + y^2 - 10x + 4y = k$$

What is the minimum possible value of $h + k$?

已知存在实数x，y，h，k满足下面的方程组．问 $h + k$的最小可能值是多少？

$$x^2 + y^2 - 6x - 8y = h$$

$$x^2 + y^2 - 10x + 4y = k$$

(A) -54 **(B)** -46 **(C)** -34 **(D)** -16 **(E)** 16

Problem 14

How many different remainders can result when the 100th power of an integer is divided by 125?

当一个整数的 100 次方除以 125 时，可以得到多少种不同的余数?

(A) 1 **(B)** 2 **(C)** 5 **(D)** 25 **(E)** 125

Problem 15

A triangle in the coordinate plane has vertices $A(\log_2 1, \log_2 2)$, $B(\log_2 3, \log_2 4)$, and $C(\log_2 7, \log_2 8)$ .What is the area of $\triangle ABC$?

在坐标平面上有一个三角形，其顶点为$A(\log_2 1, \log_2 2)$，$A(\log_2 1, \log_2 2)$，$C(\log_2 7, \log_2 8)$. 问$\triangle ABC$的面积是多少?

(A) $\log_2 \frac{\sqrt{3}}{7}$ **(B)** $\log_2 \frac{3}{\sqrt{7}}$ **(C)** $\log_2 \frac{7}{\sqrt{3}}$ **(D)** $\log_2 \frac{11}{\sqrt{7}}$ **(E)** $\log_2 \frac{11}{\sqrt{3}}$

Problem 16

A group of 16 people will be partitioned into 4 indistinguishable 4-person committees. Each committee will have one chairperson and one secretary. The number of different ways to make these assignments can be written as $3^r M$, where r and M are positive integers and M is not divisible by 3. What is r?

现有一组共 16 人将被分成 4 个不加区别的委员会，每个委员会有 4 人. 每个委员会将有一位主席和一位秘书. 所有不同的分配方法的总数可以写成 $3^r M$，其中r和M是正整数，且M不能被 3 整除. 问r是多少?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

Problem 17

Integers a and b are randomly chosen without replacement from the set of integers with absolute value not exceeding 10. What is the probability that the polynomial x^3+ax^2+bx+6 has 3 distinct integer roots?

从绝对值不超过 10 的整数集合中随机选取两个不同的整数a和b. 问多项式x^3+ax^2+bx+6恰好有 3 个不同的整数根的概率是多少?

(A) $\frac{1}{240}$ **(B)** $\frac{1}{221}$ **(C)** $\frac{1}{105}$ **(D)** $\frac{1}{84}$ **(E)** $\frac{1}{63}$

Problem 18

The Fibonacci numbers are defined by $F_1=1, F_2=1,$ and $F_n=F_{n-1}+F_{n-2}$ for $n\geq 3$. What is $\frac{F_2}{F_1}+\frac{F_4}{F_2}+\frac{F_6}{F_3}+\cdots+\frac{F_{20}}{F_{10}}$?

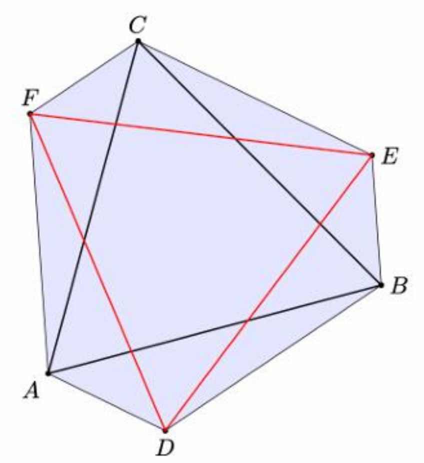
Fibonacci 数列定义为$F_1=1$，$F_2=1$，且对于$n\geq 3$，$F_n=F_{n-1}+F_{n-2}$. 问$\frac{F_2}{F_1}+\frac{F_4}{F_2}+\frac{F_6}{F_3}+\cdots+\frac{F_{20}}{F_{10}}$ 是多少?

(A) 318 **(B)** 319 **(C)** 320 **(D)** 321 **(E)** 322

Problem 19

Equilateral $\triangle ABC$ with side length 14 is rotated about its center by angle θ, where $0<\theta<60^\circ$, to form $\triangle DEF$. See the figure. The area of hexagon $ADBECF$ is $91\sqrt{3}$. What is $\tan\theta$?

如图所示，边长为 14 的等边三角形$\triangle ABC$绕中心旋转角度θ形成$\triangle DEF$，其中$0<\theta<60^\circ$. 若六边形 ADBECF 的面积为$91\sqrt{3}$，问$\tan\theta$的值是多少?



(A) $\frac{3}{4}$ **(B)** $\frac{5\sqrt{3}}{11}$ **(C)** $\frac{4}{5}$ **(D)** $\frac{11}{13}$ **(E)** $\frac{7\sqrt{3}}{13}$

Problem 20

Suppose A, B, and C are points in the plane with $AB = 40$ and $AC = 42$, and let x be the length of the line segment from A to the midpoint of $\overline{BC}$. Define a function f by letting $f(x)$ be the area of $\triangle ABC$. Then the domain of f is an open interval (p, q), and the maximum value r of $f(x)$ occurs at $x = s$. What is $p + q + r + s$?

假设平面上的点A，B，C满足$AB = 40$ 且 $AC = 42$，令x为连接A和$\overline{BC}$中点的线段的长度. 定义函数f，使得$f(x)$表示$\triangle ABC$的面积. 若f的定义域是开区间(p, q)，且$f(x)$的最大值r在$x = s$处取得. 问$p + q + r + s$的值是多少?

(A) 909 **(B)** 910 **(C)** 911 **(D)** 912 **(E)** 913

Problem 21

The measures of the smallest angles of three different right triangles sum to 90°. All three triangles have side lengths that are primitive Pythagorean triples. Two of them are $3-4-5$ and $5-12-13$. What is the perimeter of the third triangle?

三个不同的直角三角形中最小角的角度之和为 90°. 这三个三角形的边长均为互质的勾股三元数组. 其中两个三角形的边长分别为$3-4-5$和$5-12-13$. 问第三个三角形的周长是多少?

(A) 40 **(B)** 126 **(C)** 154 **(D)** 176 **(E)** 208

Problem 22

Let $\triangle ABC$ be a triangle with integer side lengths and the property that $\angle B = 2\angle A$. What is the least possible perimeter of such a triangle?

设$\triangle ABC$是边长均为整数的三角形，且满足$\angle B = 2\angle A$. 求满足条件的三角形的最小可能周长?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

Problem 23

A right pyramid has regular octagon $ABCDEFGH$ with side length 1 as its base and apex V .Segments $\overline{AV}$ and $\overline{DV}$ are perpendicular. What is the square of the height of the pyramid?

正棱锥的底面是边长为 1 的正八边形$ABCDEFGH$，顶点是V. 线段$\overline{AV}$和$\overline{DV}$ 相互垂直. 问这个棱锥的高的平方是多少?

(A) 1 **(B)** $\frac{1+\sqrt{2}}{2}$ **(C)** $\sqrt{2}$ **(D)** $\frac{3}{2}$ **(E)** $\frac{2+\sqrt{2}}{3}$

Problem 24

What is the number of ordered triples (a, b, c) of positive integers, with $a \leq b \leq c \leq 9$, such that there exists a (non-degenerate) triangle $\triangle ABC$ with an integer inradius for which a, b, and c are the lengths of the altitudes from A to $\overline{BC}$, B to $\overline{AC}$, and C to $\overline{AB}$, respectively? (Recall that the inradius of a triangle is the radius of the largest possible circle that can be inscribed in the triangle.)

考虑有序三元组(a, b, c)：a，b，c均为正整数，且$a \leq b \leq c \leq 9$，存在一个（非退化）的三角形$\triangle ABC$，其内切圆半径为整数，且a，b，c分别为从A到$\overline{BC}$，从B到$\overline{AC}$，从C到$\overline{AB}$的高的长度. 问具有上述性质的有序三元组(a, b, c)有多少个?（注意：三角形的内切圆半径是指可以内接于该三角形的最大圆的半径.）

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

Problem 25

Pablo will decorate each of 6 identical white balls with either a striped or a dotted pattern, using either red or blue paint. He will decide on the color and pattern for each ball by flipping a fair coin for each of the 12 decisions he must make. After the paint dries, he will place the 6 balls in an urn. Frida will randomly select one ball from the urn and note its color and pattern. The events "the ball Frida selects is red" and "the ball Frida selects is striped" may or may not be independent, depending on the outcome of Pablo's coin flips. The probability that these two events are independent can be written as $\frac{m}{n}$,where m and n are relatively prime positive integers. What is m (Recall that two events A and B are independent if $P(A \; and \; B) = P(A) \cdot P(B)$.)

Pablo 将装饰 6 个相同的白球，每个球都会用红色或蓝色油漆涂上条纹或圆点图案．他将通过抛掷均匀硬币的方式来决定每个球的颜色和图案，总共需要做 12 次决定．油漆干燥后，他将这 6 个球放入罐中．Frida 将从罐中随机选取一个球，并记录其颜色和图案．事件「Frida 选中的球是红色的」和「Frida 选中的球是条纹的」可能相互独立，也可能不独立，这取决于 Pablo 抛硬币的结果．这两个事件相互独立的概率可以表示为 $\frac{m}{n}$，其中m 和n互质．问m的值是多少？（注意：两个事件 A和B相互独立，当且仅当P(A 和 B 同时发生) $= P(A) \cdot P(B)$.）

(A) 243 **(B)** 245 **(C)** 247 **(D)** 249 **(E)** 251

2024 AMC 12B Answer Key													
题目	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
答案	B	B	E	D	B	B	C	C	B	C	E	D	C
题目	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
答案	B	B	A	C	B	B	C	C	C	B	B	A	

2024 AMC12B Solution

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