2011 AMC12B

Problem 1

What is

$$\frac{2+4+6}{1+3+5}-\frac{1+3+5}{2+4+6}?$$

表达式的值是多少

$$\frac{2+4+6}{1+3+5}-\frac{1+3+5}{2+4+6}?$$

$\textbf{(A)}\ -1 \qquad \textbf{(B)}\ \frac{5}{36} \qquad \textbf{(C)}\ \frac{7}{12} \qquad \textbf{(D)}\ \frac{147}{60} \qquad \textbf{(E)}\ \frac{43}{3}$

Problem 2

Josanna's test scores to date are $90, 80, 70, 60,$ and 85. Her goal is to raise her test average at least 3 points with her next test. What is the minimum test score she would need to accomplish this goal?

Josanna 目前的考试分数是 90，80，70，60 和 85，她的目标是靠下次考试将她的考试平均分提高至少 3 分，那么她下次至少需要考多少分才能完成这个目标？

$\textbf{(A)}\ 80 \qquad \textbf{(B)}\ 82 \qquad \textbf{(C)}\ 85 \qquad \textbf{(D)}\ 90 \qquad \textbf{(E)}\ 95$

Problem 3

LeRoy and Bernardo went on a week-long trip together and agreed to share the costs equally. Over the week, each of them paid for various joint expenses such as gasoline and car rental. At the end of the trip it turned out that LeRoy had paid A dollars and Bernardo had paid B dollars, where $A < B$. How many dollars must LeRoy give to Bernardo so that they share the costs equally?

LeRoy 和 Bernardo 一起出去旅游了一周，并且商定两人均摊费用，在这一周中，他们两人都支付了各种共同支出，例如汽油和租车费用。旅游结束后，发现 Leroy 花费 A 美元，Bernardo 花费 B 美元，这里 A<B，那么 LeRoy 需要支付 Bernardo 多少美元才能保证他俩均摊了费用？

$\textbf{(A)}\ \frac{A+B}{2} \qquad \textbf{(B)}\ \frac{A-B}{2} \qquad \textbf{(C)}\ \frac{B-A}{2} \qquad \textbf{(D)}\ B-A \qquad \textbf{(E)}\ A+B$

Problem 4

In multiplying two positive integers a and b, Ron reversed the digits of the two-digit number a. His erroneous product was 161. What is the correct value of the product of a and b?

Ron 在把 2 个正整数a和b相乘时，将两位数a的个位和十位数字弄反了，导致最后错误的乘积为 161，那么a和b相乘所得正确的乘积应该是多少?

$\textbf{(A)}\ 116 \qquad \textbf{(B)}\ 161 \qquad \textbf{(C)}\ 204 \qquad \textbf{(D)}\ 214 \qquad \textbf{(E)}\ 224$

Problem 5

Let N be the second smallest positive integer that is divisible by every positive integer less than 7. What is the sum of the digits of N?

N是能够同时被小于 7 的所有正整数整除的第二小的正整数.那么N的各个位上数字之和是多少?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 9$

Problem 6

Two tangents to a circle are drawn from a point A. The points of contact B and C divide the circle into arcs with lengths in the ratio $2:3$. What is the degree measure of $\angle BAC$?

从 A 点引出圆的两条切线，切点 B 和 C 将圆分成弧长比为 2:3 的两段弧，那么$\angle BAC$的度数是多少？

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 48 \qquad \textbf{(E)}\ 60$

Problem 7

Let x and y be two-digit positive integers with mean 60. What is the maximum value of the ratio $\frac{x}{y}$?

x和y为均为 2 位正整数，平均值是 60，则$\frac{x}{y}$的比值最大是多少?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ \frac{33}{7} \qquad \textbf{(C)}\ \frac{39}{7} \qquad \textbf{(D)}\ 9 \qquad \textbf{(E)}\ \frac{99}{10}$

Problem 8

Keiko walks once around a track at exactly the same constant speed every day. The sides of the track are straight, and the ends are semicircles. The track has width 6 meters, and it takes her 36 seconds longer to walk around the outside edge of the track than around the inside edge. What is Keiko's speed in meters per second?

Keiko 每天都以同样的速度绕跑道走一圈，跑道的两边是直线，两端是半圆。跑道的宽为 6 米，她沿着跑道的外沿走一圈比沿着内沿走一圈多花 36 秒，问 Keiko 的速度是多少米每秒？

$\textbf{(A)}\ \frac{\pi}{3} \qquad \textbf{(B)}\ \frac{2\pi}{3} \qquad \textbf{(C)}\ \pi \qquad \textbf{(D)}\ \frac{4\pi}{3} \qquad \textbf{(E)}\ \frac{5\pi}{3}$

Problem 9

Two real numbers are selected independently and at random from the interval $[-20, 10]$. What is the probability that the product of those numbers is greater than zero?

从区间$[-20, 10]$内独立且随机地选择两个实数，那么这两个数的乘积大于 0 的概率是多少？

$\textbf{(A)}\ \frac{1}{9} \qquad \textbf{(B)}\ \frac{1}{3} \qquad \textbf{(C)}\ \frac{4}{9} \qquad \textbf{(D)}\ \frac{5}{9} \qquad \textbf{(E)}\ \frac{2}{3}$

Problem 10

Rectangle $ABCD$ has $AB = 6$ and $BC = 3$. Point M is chosen on side AB so that $\angle AMD = \angle CMD$. What is the degree measure of $\angle AMD$?

在矩形$ABCD$中，*AB*=6 ，*BC*=3，点 *M* 在边 *AB* 上，且$\angle AMD = \angle CMD$, 那么$\angle AMD$的度数是多少？

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 45 \qquad \textbf{(D)}\ 60 \qquad \textbf{(E)}\ 75$

Problem 11

A frog located at (x, y), with both x and y integers, makes successive jumps of length 5 and always lands on points with integer coordinates. Suppose that the frog starts at $(0, 0)$ and ends at $(1, 0)$. What is the smallest possible number of jumps the frog makes?

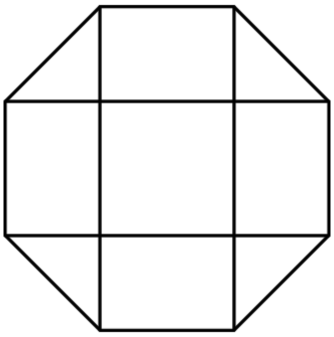
已知x和y都是整数。一只位于点(x, y)处的青蛙连续不断地跳，每次跳跃的长度都是 5，且每次都落在坐标为整数的点上。假设青蛙从（0，0）开始，最后落在点（1，0），那么这只青蛙最少可能跳多少次？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Problem 12

A dart board is a regular octagon divided into regions as shown below. Suppose that a dart thrown at the board is equally likely to land anywhere on the board. What is the probability that the dart lands within the center square?

有一种飞镖盘，它是一个被分割成如下围所示几个区域的正八边形。假设掷向飞镖盘的飞镖落在盘上任何点都是等可能的，那么飞镖落在位于中心位置的正方形的概率是多少？



$\textbf{(A)}\ \frac{\sqrt{2}-1}{2} \qquad \textbf{(B)}\ \frac{1}{4} \qquad \textbf{(C)}\ \frac{2-\sqrt{2}}{2} \qquad \textbf{(D)}\ \frac{\sqrt{2}}{4} \qquad \textbf{(E)}\ 2-\sqrt{2}$

Problem 13

Brian writes down four integers $w > x > y > z$ whose sum is 44. The pairwise positive differences of these numbers are $1, 3, 4, 5, 6$ and 9. What is the sum of the possible values of w?

Brian 写下了 4 个整数$w > x > y > z$，它们的和是 44，这些数两两之间的差的绝对值是 1，3，4，5，6 和 9，那么w的所有可能值的和是多少?

$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 31 \qquad \textbf{(C)}\ 48 \qquad \textbf{(D)}\ 62 \qquad \textbf{(E)}\ 93$

Problem 14

A segment through the focus F of a parabola with vertex V is perpendicular to $\overline{FV}$ and intersects the parabola in points A and B. What is $\cos(\angle AVB)$?

通过顶点为V的抛物线的焦点F的一条线段和线段$\overline{FV}$垂直，且和此抛物线交于点A和B，问$\cos(\angle AVB)$是多少？

$\textbf{(A)}\ -\frac{3\sqrt{5}}{7} \qquad \textbf{(B)}\ -\frac{2\sqrt{5}}{5} \qquad \textbf{(C)}\ -\frac{4}{5} \qquad \textbf{(D)}\ -\frac{3}{5} \qquad \textbf{(E)}\ -\frac{1}{2}$

Problem 15

How many positive two-digit integers are factors of $2^{24} - 1$?

有多少个两位正整数是$2^{24} - 1$的因子？

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 14$

Problem 16

Rhombus $ABCD$ has side length 2 and $\angle B = 120^\circ$. Region R consists of all points inside of the rhombus that are closer to vertex B than any of the other three vertices. What is the area of R?

菱形 $ABCD$ 的边长为 2， $\angle B = 120°$，菱形内所有相对于顶点 A，C，D 来说，更靠近顶点 B 的点组成了区域 R，问区域 R 的面积是多少？

$\textbf{(A)}\ \frac{\sqrt{3}}{3} \qquad \textbf{(B)}\ \frac{\sqrt{3}}{2} \qquad \textbf{(C)}\ \frac{2\sqrt{3}}{3} \qquad \textbf{(D)}\ 1 + \frac{\sqrt{3}}{3} \qquad \textbf{(E)}\ 2$

Problem 17

Let $f(x)=10^{10x}, g(x)=\log_{10}\left(\frac{x}{10}\right), h_1(x)=g(f(x))$, and $h_n(x)=h_1(h_{n-1}(x))$ for integers $n\geq 2$. What is the sum of the digits of $h_{2011}(1)$?

令$f(x)=10^{10x}, g(x)=\log_{10}\left(\frac{x}{10}\right), h_1(x)=g(f(x))$，且$h_n(x)=h_1(h_{n-1}(x))$，对于整数$n\geq 2$，问$h_{2011}(1)$的各个位上数字之和是多少？

$\textbf{(A)}\ 16081 \qquad \textbf{(B)}\ 16089 \qquad \textbf{(C)}\ 18089 \qquad \textbf{(D)}\ 18098 \qquad \textbf{(E)}\ 18099$

Problem 18

A pyramid has a square base with side of length 1 and has lateral faces that are equilateral triangles. A cube is placed within the pyramid so that one face is on the base of the pyramid and its opposite face has all its edges on the lateral faces of the pyramid. What is the volume of this cube?

有这样一个四棱锥，它的底面是边长为 1 的正方形，侧面都是等边三角形，其内有一个正方体，正方体的一个面位于此棱锥的底面，而这个面的对面所有的棱都在棱锥的侧面上，则这个正方体的体积是多少？

$\textbf{(A)}\ 5\sqrt{2}-7 \qquad \textbf{(B)}\ 7-4\sqrt{3} \qquad \textbf{(C)}\ \frac{2\sqrt{2}}{27} \qquad \textbf{(D)}\ \frac{\sqrt{2}}{9} \qquad \textbf{(E)}\ \frac{\sqrt{3}}{9}$

Problem 19

A lattice point in an xy-coordinate system is any point (x,y) where both x and y are integers. The graph of $y=mx+2$ passes through no lattice point with $0<x\leq 100$ for all m such that $\frac{1}{2}<m<a$. What is the maximum possible value of a?

xy坐标系内的格点是指x和y均为整数的点(x,y)，对于$0<x\leq 100$，且满足$\frac{1}{2}<m<a$的所有m，都使得$y=mx+2$的图像不通过任何格点，那么a的最大可能值是多少？

$\textbf{(A)}\ \frac{51}{101} \qquad \textbf{(B)}\ \frac{50}{99} \qquad \textbf{(C)}\ \frac{51}{100} \qquad \textbf{(D)}\ \frac{52}{101} \qquad \textbf{(E)}\ \frac{13}{25}$

Problem 20

Triangle ABC has $AB = 13$, $BC = 14$, and $AC = 15$. The points D, E, and F are the midpoints of $\overline{AB}, \overline{BC}$, and $\overline{AC}$ respectively. Let $X \neq E$ be the intersection of the circumcircles of ΔBDE and ΔCEF. What is $XA + XB + XC$?

三角形 *ABC* 中， *AB*=13，*BC*=14，*AC*=15，点 *D*， *E* 和 *F* 分别是线段$\overline{AB}$，$\overline{BC}$和$\overline{AC}$的中点，已知ΔBDE的外接圆和ΔCEF的外接圆交于不同于 *E* 的另一点$X \neq E$，那么 *XA*+*XB*+*XC* 是多少？

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 14\sqrt{3} \qquad \textbf{(C)}\ \frac{195}{8} \qquad \textbf{(D)}\ \frac{129\sqrt{7}}{14} \qquad \textbf{(E)}\ \frac{69\sqrt{2}}{4}$

Problem 21

The arithmetic mean of two distinct positive integers x and y is a two-digit integer. The geometric mean of x and y is obtained by reversing the digits of the arithmetic mean. What is $|x - y|$?

两个不等的正整数x和y的算数平均值是个两位数， x和y的几何平均值是由它们的算数平均值交换十位和个位数字得到，那么$|x - y|$是多少？

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 48 \qquad \textbf{(C)}\ 54 \qquad \textbf{(D)}\ 66 \qquad \textbf{(E)}\ 70$

Problem 22

Let T_1 be a triangle with side lengths $2011, 2012$, and 2013. For $n \geq 1$, if $T_n = \Delta ABC$ and D, E, and F are the points of tangency of the incircle of ΔABC to the sides AB, BC, and AC, respectively, then T_{n+1} is a triangle with side lengths AD, BE, and CF, if it exists. What is the perimeter of the last triangle in the sequence (T_n)?

T_1是个边长为 2011，2012 和 2013 的三角形。对于$n \geq 1$，若$T_n = \Delta ABC$，且点 *D*，*E*，*F* 是ΔABC的内切圆分别和边 *AB*，*BC* 和 *AC* 相切的切点，那么T_{n+1}是以线段 *AD*，*BE* 和 *CF* 为三边的三角形（如果+1 存在的话）。问数列(T_n)中最后一个三角形的周长是多少？

$\textbf{(A)}\ \frac{1509}{8} \qquad \textbf{(B)}\ \frac{1509}{32} \qquad \textbf{(C)}\ \frac{1509}{64} \qquad \textbf{(D)}\ \frac{1509}{128} \qquad \textbf{(E)}\ \frac{1509}{256}$

Problem 23

A bug travels in the coordinate plane, moving only along the lines that are parallel to the x-axis or y-axis. Let $A = (-3, 2)$ and $B = (3, -2)$. Consider all possible paths of the bug from A to B of length at most 20. How many points with integer coordinates lie on at least one of these paths?

一只虫子沿着坐标系里平行于 x 轴或 y 轴的直线移动。令 A=（-3，2），B=（3，-2），考虑虫子从 A 到 B 长度最多为 20 的所有可能路径，问有多少坐标为整数的点在至少其中一条路径上？

(A) 161 **(B)** 185 **(C)** 195 **(D)** 227 **(E)** 255

Problem 24

Let $P(z) = z^8 + \left(4\sqrt{3} + 6\right) z^4 - \left(4\sqrt{3} + 7\right)$. What is the minimum perimeter among all the 8-sided polygons in the complex plane whose vertices are precisely the zeros of $P(z)$?

令$P(z) = z^8 + \left(4\sqrt{3} + 6\right) z^4 - \left(4\sqrt{3} + 7\right)$，那么在复平面内，$P(z)$的零点所形成的所有八边形中，周长最短的是多少？

(A) $4\sqrt{3}+4$ **(B)** $8\sqrt{2}$ **(C)** $3\sqrt{2}+3\sqrt{6}$ **(D)** $4\sqrt{2}+4\sqrt{3}$ **(E)** $4\sqrt{3}+6$

Problem 25

For every m and k integers with k odd, denote by $\left[\frac{m}{k}\right]$ the integer closest to $\frac{m}{k}$. For every odd integer k, let $P(k)$ be the probability that

$$\left[\frac{n}{k}\right]+\left[\frac{100-n}{k}\right]=\left[\frac{100}{k}\right]$$

for an integer n randomly chosen from the interval $1 \leq n \leq 99!$. What is the minimum possible value of $P(k)$ over the odd integers k in the interval $1 \leq k \leq 99$?

m 和 k 为整数，且 k 为奇数，定义 $\left[\frac{m}{k}\right]$ 为最靠近 $\frac{m}{k}$ 的整数。对于任何奇数 k， $P(k)$表示，当 n 是从区间 1 ⩽ n ⩽ 99！中随机选择的一个整数时，n 满足下面方程的概率：

$$\left[\frac{n}{k}\right]+\left[\frac{100-n}{k}\right]=\left[\frac{100}{k}\right]$$

那么当奇数 k 从区间 1⩽k⩽99 中取值时，$P(k)$能取到的最小可能值是多少？

(A) $\frac{1}{2}$ **(B)** $\frac{50}{99}$ **(C)** $\frac{44}{87}$ **(D)** $\frac{34}{67}$ **(E)** $\frac{7}{13}$

2011 AMC 12B Answer Key

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
C	E	C	E	A	C	B	A	D	E	B	A	B
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
D	D	C	B	A	B	C	D	D	C	B	D	

2011 AMC 12B Solution

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