2019 AMC 12A

Problem 1

The area of a pizza with radius 4 inches is N percent larger than the area of a pizza with radius 3 inches. What is the integer closest to N?

半径为 4 英寸的比萨饼面积比半径为 3 英寸的比萨饼面积大的百分数是 N。问最接近 N的整数是什么？

$\textbf{(A) } 25 \qquad \textbf{(B) } 33 \qquad \textbf{(C) } 44 \qquad \textbf{(D) } 66 \qquad \textbf{(E) } 78$

Problem 2

Suppose a is 150% of b. What percent of a is $3b$?

假设 a 是 b 的 150%。 那么 a 的百分之多少是 $3b$?

$\textbf{(A) } 50 \qquad \textbf{(B) } 66+\frac{2}{3} \qquad \textbf{(C) } 150 \qquad \textbf{(D) } 200 \qquad \textbf{(E) } 450$

Problem 3

A box contains 28 red balls, 20 green balls, 19 yellow balls, 13 blue balls, 11 white balls, and 9 black balls. What is the minimum number of balls that must be drawn from the box without replacement to guarantee that at least 15 balls of a single color will be drawn?

一个盒子中有 28 个红球，20 个绿球，19 个黄球，13 个蓝球，11 个白球和 9 个黑球。为了保证至少取出 15 个单一颜色的球, 在不允许放回重取的情况下，必须从盒子里取出的球的数量最少是多少个？

$\textbf{(A) } 75 \qquad \textbf{(B) } 76 \qquad \textbf{(C) } 79 \qquad \textbf{(D) } 84 \qquad \textbf{(E) } 91$

Problem 4

What is the greatest number of consecutive integers whose sum is 45?

最多可以有多少个连续整数，它们的总和是 45?

$\textbf{(A) } 9 \qquad \textbf{(B) } 25 \qquad \textbf{(C) } 45 \qquad \textbf{(D) } 90 \qquad \textbf{(E) } 120$

Problem 5

Two lines with slopes $\dfrac{1}{2}$ and 2 intersect at $(2,2)$. What is the area of the triangle enclosed by these two lines and the line $x+y=10$?

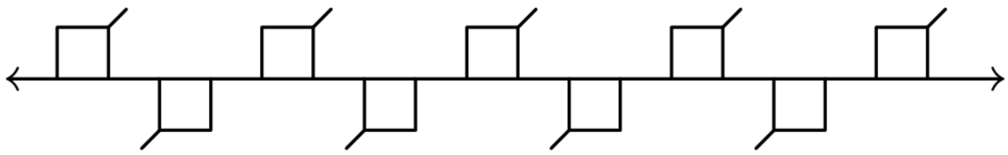
两条斜率分别为$\frac{1}{2}$和 2 的直线相交于(2, 2)。问由这两条直线和直线 $x + y = 10$ 所框出的三角形的面积是多少?

(A) 4 **(B)** $4\sqrt{2}$ **(C)** 6 **(D)** 8 **(E)** $6\sqrt{2}$

Problem 6

The figure below shows line ℓ with a regular, infinite, recurring pattern of squares and line segments.

下图显示了直线 l，该直线由规则的、无限的、重复出现的正方形和线段所组成的图形。



How many of the following four kinds of rigid motion transformations of the plane in which this figure is drawn, other than the identity transformation, will transform this figure into itself?

在下面的四种关于该图形所在平面的刚性变换中，有几种是除了恒等变换外，能够把上述图形变到自身的?

some rotation around a point of line ℓ

围绕直线 A 上的某个点的旋转

some translation in the direction parallel to line ℓ

沿着平行于直线 A 的方向的某个平移

the reflection across line ℓ

关于直线 A 的反射

some reflection across a line perpendicular to line ℓ

关于某条垂直于直线 A 的直线的反射

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

Problem 7

Melanie computes the mean μ, the median M, and the modes of the 365 values that are the dates in the months of 2019. Thus her data consist of 12 1s, 12 2s, . . . , 12 28s, 11 29s, 11 30s, and 7 31s. Let d be the median of the modes. Which of the following statements is true?

Melanie 计算 2019 年的各个月的日期这 365 个数值的平均数μ，中位数 M 以及众数。因此，她的数据包括 12 个 1，12 个 2，・・・，12 个 28，11 个 29，11 个 30 以及 7 个 31。设 d 为众数的中位数。下列哪个论断是正确的？

(A) $\mu < d < M$ **(B)** $M < d < \mu$ **(C)** $d = M = \mu$ **(D)** $d < M < \mu$ **(E)** $d < \mu < M$

Problem 8

For a set of four distinct lines in a plane, there are exactly N distinct points that lie on two or more of the lines. What is the sum of all possible values of N?

考虑平面上四条不同的直线构成的集合，在其中两条或更多条直线上的不同的点恰好有 N个。问 N的所有可能值的总和是多少？

(A) 14 **(B)** 16 **(C)** 18 **(D)** 19 **(E)** 21

Problem 9

A sequence of numbers is defined recursively by $a_1 = 1$, $a_2 = \frac{3}{7}$, and $a_n = \frac{a_{n-2} \cdot a_{n-1}}{2a_{n-2} - a_{n-1}}$ for all $n \geq 3$. Then a_{2019} can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p + q$?

数列的递归定义：$a_1 = 1$，$a_2 = \frac{3}{7}$，并且对于所有 $n \geq 3$，

$$a_n = \frac{a_{n-2} \cdot a_{n-1}}{2a_{n-2} - a_{n-1}}$$

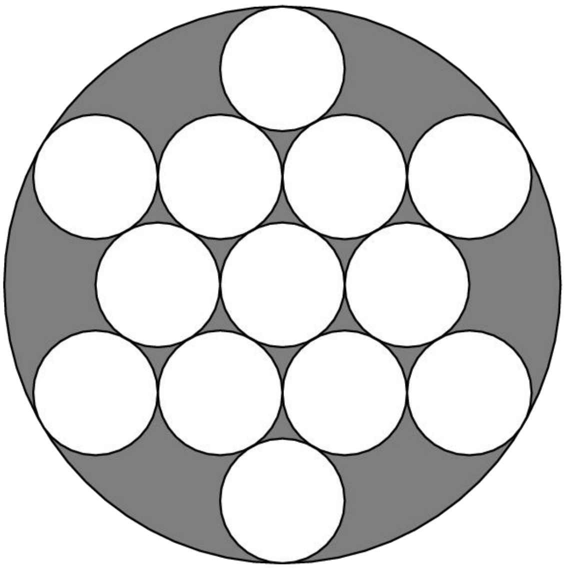
a_{2019} 可以写成 $\frac{p}{q}$ ，其中 p 和 q 是互质的正整数。问 $p+q$ 是多少？

(A) 2020 **(B)** 4039 **(C)** 6057 **(D)** 6061 **(E)** 8078

Problem 10

The figure below shows 13 circles of radius 1 within a larger circle. All the intersections occur at points of tangency. What is the area of the region, shaded in the figure, inside the larger circle but outside all the circles of radius 1?

下图显示了 13 个半径为 1 的圆在一个较大圆内。所有圆之间相交的点都是切点。在较大的圆内，但在所有半径为 1 的圆外的阴影部分的面积是多少？



(A) $4\pi\sqrt{3}$ **(B)** 7π **(C)** $\pi\left(3\sqrt{3}+2\right)$ **(D)** $10\pi\left(\sqrt{3}-1\right)$ **(E)** $\pi\left(\sqrt{3}+6\right)$

Problem 11

For some positive integer k, the repeating base-k representation of the (base-ten) fraction $\frac{7}{51}$ is $0.\overline{23}_k = 0.232323..._k$. What is k?

对于某个正整数 k，十进制表示中的分数 $\frac{7}{51}$ 在 k 进制下的循环小数表示为 $0.\overline{23}_k = 0.232323..._k$。问 k 是多少？

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

Problem 12

Positive real numbers $x \neq 1$ and $y \neq 1$ satisfy $\log_2 x = \log_y 16$ and $xy = 64$. What is $(\log_2 \tfrac{x}{y})^2$?

正实数 $x \neq 1$ 和 $y \neq 1$ 满足 $\log_2 x = \log_y 16$ 和 $xy = 64$。问 $(\log_2 \frac{x}{y})^2$ 是多少？

$\textbf{(A)}\ \frac{25}{2} \qquad \textbf{(B)}\ 20 \qquad \textbf{(C)}\ \frac{45}{2} \qquad \textbf{(D)}\ 25 \qquad \textbf{(E)}\ 32$

Problem 13

How many ways are there to paint each of the integers $2, 3, \dots, 9$ either red, green, or blue so that each number has a different color from each of its proper divisors?

将整数 $2,3,\ldots,9$ 中的每个数染成红色，绿色或者蓝色，使得每个数和它的每个真约数都有不同的颜色，共有多少种方法？

$\textbf{(A)}\ 144 \qquad \textbf{(B)}\ 216 \qquad \textbf{(C)}\ 256 \qquad \textbf{(D)}\ 384 \qquad \textbf{(E)}\ 432$

Problem 14

For a certain complex number c, the polynomial $P(x) = (x^2 - 2x + 2)(x^2 - cx + 4)(x^2 - 4x + 8)$ has exactly 4 distinct roots. What is $|c|$?

对于某个复数 c，多项式 $P(x) = (x^2 - 2x + 2)(x^2 - cx + 4)(x^2 - 4x + 8)$ 恰好有 4 个不同的根。问 $|c|$ 是多少？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ \sqrt{6} \qquad \textbf{(C)}\ 2\sqrt{2} \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ \sqrt{10}$

Problem 15

Positive real numbers a and b have the property that$\sqrt{\log a} + \sqrt{\log b} + \log \sqrt{a} + \log \sqrt{b} = 100$

and all four terms on the left are positive integers, where $\log$ denotes the base-10 logarithm. What is ab?

正实数 a 和 b 具有性质 $\sqrt{\log a} + \sqrt{\log b} + \log \sqrt{a} + \log \sqrt{b} = 100$，

并且左边的四项都是正整数，其中 log 表示基数为 10 的对数。问 ab 是多少？

(A) 10^{52} **(B)** 10^{100} **(C)** 10^{144} **(D)** 10^{164} **(E)** 10^{200}

Problem 16

The numbers $1, 2, \dots, 9$ are randomly placed into the 9 squares of a 3×3 grid. Each square gets one number, and each of the numbers is used once. What is the probability that the sum of the numbers in each row and each column is odd?

数字$1,2,\dots,9$被随机放入 3 × 3 方格表的 9 个单位正方形中。每个单位正方形里有一个数字，

并且每个数字均使用一次。那么每个横行与每个竖列中的数字之和均是奇数的概率是多少？

(A) $\frac{1}{21}$ **(B)** $\frac{1}{14}$ **(C)** $\frac{5}{63}$ **(D)** $\frac{2}{21}$ **(E)** $\frac{1}{7}$

Problem 17

Let s_k denote the sum of the kth powers of the roots of the polynomial $x^3 - 5x^2 + 8x - 13$. In particular, $s_0 = 3$, $s_1 = 5$, and $s_2 = 9$. Let a, b, and c be real numbers such that $s_{k+1} = a\, s_k + b\, s_{k-1} + c\, s_{k-2}$ for $k = 2, 3, \dots$. What is $a + b + c$?

设 s_k 表示多项式 $x^3 - 5x^2 + 8x - 13$ 的根的 k 次幂之和。 特别的，$s_0 = 3$，$s_1 = 5$，$s_2 = 9$。 实数

a，b 和 c 使得 $s_{k+1} = as_k + bs_{k-1} + cs_{k-2}$ 对于 k = 2，3. . . . 成立。 求 $a+b+c$ 是多少？

(A) -6 **(B)** 0 **(C)** 6 **(D)** 10 **(E)** 26

Problem 18

A sphere with center O has radius 6. A triangle with sides of length $15, 15,$ and 24 is situated in space so that each of its sides is tangent to the sphere. What is the distance between O and the plane determined by the triangle?

中心为 O 的球体的半径是 6。在空间中有一个边长分别为 15、15、24 的三角形，它的每条边都与球体相切。问 O 和该三角形确定的平面之间的距离是多少？

$\textbf{(A)}\ 2\sqrt{3} \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 3\sqrt{2} \qquad \textbf{(D)}\ 2\sqrt{5} \qquad \textbf{(E)}\ 5$

Problem 19

In $\triangle ABC$ with integer side lengths, $\cos A = \frac{11}{16}, \qquad \cos B = \frac{7}{8}, \qquad \text{and} \qquad \cos C = -\frac{1}{4}.$ What is the least possible perimeter for $\triangle ABC$?

$\triangle ABC$的各边长为整数，$\cos A = \frac{11}{16}, \cos B = \frac{7}{8}$, and $\cos C = -\frac{1}{4}$. 问$\triangle ABC$的周长最小是多少？

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 23 \qquad \textbf{(D)}\ 27 \qquad \textbf{(E)}\ 44$

Problem 20

Real numbers between 0 and 1, inclusive, are chosen in the following manner. A fair coin is flipped. If it lands heads, then it is flipped again and the chosen number is 0 if the second flip is heads and 1 if the second flip is tails. On the other hand, if the first coin flip is tails, then the number is chosen uniformly at random from the closed interval $[0, 1]$. Two random numbers x and y are chosen independently in this manner. What is the probability that $|x - y| > \tfrac{1}{2}$?

0 与 1 之间（包括两端）的实数按下列方式选择。抛掷一枚均匀的硬币。如果是正面向上，那么它会再次抛掷，如果第二次是正面向上，则选择数 0，如果第二次是背面向上，则选择数 1。另一方面，如果第一次硬币是背面向上，则从闭区间[0, 1]中按随机均匀分布选择 1 个数。以这种方式独立选择两个数 x 和 y。那么 $|x-y| > \frac{1}{2}$ 的概率是多少？

$\textbf{(A)}\ \frac{1}{3} \qquad \textbf{(B)}\ \frac{7}{16} \qquad \textbf{(C)}\ \frac{1}{2} \qquad \textbf{(D)}\ \frac{9}{16} \qquad \textbf{(E)}\ \frac{2}{3}$

Problem 21

Let $z = \frac{1+i}{\sqrt{2}}$. What is $\left(z^{1^2}+z^{2^2}+z^{3^2}+\cdots+z^{12^2}\right)\cdot\left(\frac{1}{z^{1^2}}+\frac{1}{z^{2^2}}+\frac{1}{z^{3^2}}+\cdots+\frac{1}{z^{12^2}}\right)$?

设 $z=\frac{1+i}{\sqrt{2}}$，问下式的值是多少？

$$\left(z^{1^2}+z^{2^2}+z^{3^2}+\ldots+z^{12^2}\right)\cdot\left(\frac{1}{z^{1^2}}+\frac{1}{z^{2^2}}+\frac{1}{z^{3^2}}+\ldots+\frac{1}{z^{12^2}}\right)$$

(A) 18　**(B)** $72-36\sqrt{2}$　**(C)** 36　**(D)** 72　**(E)** $72+36\sqrt{2}$

Problem 22

Circles ω and γ, both centered at O, have radii 20 and 17, respectively. Equilateral triangle ABC, whose interior lies in the interior of ω but in the exterior of γ, has vertex A on ω, and the line containing side $\overline{BC}$ is tangent to γ. Segments $\overline{AO}$ and $\overline{BC}$ intersect at P, and $\frac{BP}{CP}=3$. Then AB can be written in the form $\frac{m}{\sqrt{n}}-\frac{p}{\sqrt{q}}$ for positive integers m, n, p, q with $\gcd(m,n)=\gcd(p,q)=1$. What is $m+n+p+q$?

圆 ω 和 γ 都以 O 为中心，半径分别为 20 和 17。等边三角形 ABC 的内部位于圆 ω 的内部，但在 γ 的外部，顶点 A 在 ω 上，包含边 BC 的直线与 γ 相切。线段 AO 和 BC 相交于点 P，并且 $\frac{BP}{CP}=3$。AB 可以写成 $\frac{m}{\sqrt{n}}-\frac{p}{\sqrt{q}}$ 的形式， 其中 m，n，p，q 为正整数， 并且 $\gcd(m,n)=\gcd(p,q)=1$。问 $m+n+p+q$ 是多少？

(A) 42　**(B)** 86　**(C)** 92　**(D)** 114　**(E)** 130

Problem 23

Define binary operations $\diamondsuit$ and $\heartsuit$ by $a \diamondsuit b = a^{\log_7(b)}$ and $a \heartsuit b = a^{\frac{1}{\log_7(b)}}$ for all real numbers a and b for which these expressions are defined. The sequence (a_n) is defined recursively by $a_3 = 3 \heartsuit 2$ and $a_n = (n \heartsuit (n-1)) \diamondsuit a_{n-1}$ for all integers $n \geq 4$. To the nearest integer, what is $\log_7(a_{2019})$?

定义二元运算♦和♥ 如下：对所有使得表达式有意义的实数 a 和 b，$a♦b = a^{\log_7(b)}$，$a♥b = a^{\frac{1}{\log_7(b)}}$。序列 (a_n) 递归的定义如下：$a_3 = 3♥2$，并且对于所有整数 $n \geq 4$，$a_n = (n♥(n-1))♦a_{n-1}$ 问将 $\log_7(a_{2019})$ 舍入到最接近的整数，是多少？

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

Problem 24

For how many integers n between 1 and 50, inclusive, is $\frac{(n^2-1)!}{(n!)^n}$ an integer? (Recall that $0! = 1$.)

在从 1 到 50 的整数（包括首尾两数）n 中，有多少个数使得

$$\frac{(n^2-1)!}{(n!)^n}$$

是整数？（注意 0! = 1。）

(A) 31 **(B)** 32 **(C)** 33 **(D)** 34 **(E)** 35

Problem 25

Let $\triangle A_0B_0C_0$ be a triangle whose angle measures are exactly 59.999°, 60°, and 60.001°. For each positive integer n, define A_n to be the foot of the altitude from A_{n-1} to line $B_{n-1}C_{n-1}$. Likewise, define B_n to be the foot of the altitude from B_{n-1} to line $A_{n-1}C_{n-1}$, and C_n to be the foot of the altitude from C_{n-1} to line $A_{n-1}B_{n-1}$. What is the least positive integer n for which $\triangle A_nB_nC_n$ is obtuse?

设$\triangle A_0B_0C_0$是一个三角形，各个角的大小是59.999°，60°和60.001°。对于每个正整数 n， 定义 A_n 为从 A_{n-1} 到直线 $B_{n-1}C_{n-1}$ 的高的垂足。类似的，定义 B_n 为从 B_{n-1} 到直线 $A_{n-1}C_{n-1}$ 的高的垂足，C_n 为 C_{n-1} 到直线 $A_{n-1}B_{n-1}$ 的高的垂足。问使得$\triangle A_0B_0C_0$是钝角三角形的最小正整数 n 是多少？

(A) 10 **(B)** 11 **(C)** 13 **(D)** 14 **(E)** 15

2019 AMC 12A Answer Key

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
E	D	B	D	C	C	E	D	E	A	D	B	E
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
E	D	B	D	D	A	B	C	E	D	D	E	

2019 AMC 12A Solution

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