2016 AMC12A

Problem 1

What is the value of $\dfrac{11!-10!}{9!}$?

表达式$\dfrac{11!-10!}{9!}$的值是多少？

$\textbf{(A)}\ 99 \qquad \textbf{(B)}\ 100 \qquad \textbf{(C)}\ 110 \qquad \textbf{(D)}\ 121 \qquad \textbf{(E)}\ 132$

Problem 2

For what value of x does $10^{x}\cdot 100^{2x}=1000^{5}$?

x的值为多少时，$10^{x}\cdot 100^{2x}=1000^{5}$?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 3

The remainder can be defined for all real numbers x and y with $y \neq 0$ by

$$\text{rem}(x,y)=x-y\left\lfloor \frac{x}{y} \right\rfloor$$

where $\left\lfloor \tfrac{x}{y} \right\rfloor$ denotes the greatest integer less than or equal to $\tfrac{x}{y}$. What is the value of $\text{rem}(\tfrac{3}{8},-\tfrac{2}{5})$?

所有实数x和y的余数定义为: $\text{rem}(x,y)=x-y\left\lfloor \dfrac{x}{y} \right\rfloor$，其中$y \neq 0$，且$\left\lfloor \tfrac{x}{y} \right\rfloor$表示取小于或等于$\tfrac{x}{y}$的最大整数，$\text{rem}(\tfrac{3}{8},-\tfrac{2}{5})$的值是多少？

$\textbf{(A)}\ -\dfrac{3}{8} \qquad \textbf{(B)}\ -\dfrac{1}{40} \qquad \textbf{(C)}\ 0 \qquad \textbf{(D)}\ \dfrac{3}{8} \qquad \textbf{(E)}\ \dfrac{31}{40}$

Problem 4

The mean, median, and mode of the 7 data values $60, 100, x, 40, 50, 200, 90$ are all equal to x. What is the value of x?

60，100，x，40，50，200，90 这 7 个数字的平均值、中位数和众数都等子 x，那么 x 的值是？

$\textbf{(A)}\ 50 \qquad \textbf{(B)}\ 60 \qquad \textbf{(C)}\ 75 \qquad \textbf{(D)}\ 90 \qquad \textbf{(E)}\ 100$

Problem 5

Goldbach's conjecture states that every even integer greater than 2 can be written as the sum of two prime numbers (for example, $2016 = 13 + 2003$). So far, no one has been able to prove that the conjecture is true, and no one has found a counterexample to show that the conjecture is false. What would a counterexample consist of?

哥德巴赫猜想的内容是：任何大于 2 的偶数都可以写成 2 个质数之和。（例如，2016=13+2003）。到目前为止，还没人能够证明这个猜想是正确的，也没人能够找到一个反例证明这个猜想是错的。那么反例是什么？

(A)An odd integer greater than 2 that can be written as the sum of two prime numbers. ｜ 存在一个大于 2 的奇数，可以写成 2 个质数之和。

(B) An odd integer greater than 2 that cannot be written as the sum of two prime numbers. ｜存在一个大于 2 的奇数，无法写成 2 个质数之和。

(C) An even integer greater than 2 that can be written as the sum of two numbers that are not prime｜存在一个大于 2 的偶数，可以写成两个不是质数的数之和。

(D) An even integer greater than 2 that can be written as the sum of two prime numbers. ｜ 存在一个大于 2 的偶数，可以写成 2 个质数之和。

(E)An even integer greater than 2 that cannot be written as the sum of two prime numbers. ｜存在一个大于 2 的偶数，不能写成 2 个质数之和。

Problem 6

A triangular array of 2016 coins has 1 coin in the first row, 2 coins in the second row, 3 coins in the third row, and so on up to N coins in the Nth row. What is the sum of the digits of N ?

含有总共 2016 个硬币的三角形阵列的第一行有 1 个硬币，第二行有 2 个硬币，第三行有 3 个硬币，以此类推直到第 N 行有 N 个硬币。问 N 的各个位上数字之和为多少？

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 9 \qquad \textbf{(E)}\ 10$

Problem 7

Which of these describes the graph of $x^2(x+y+1)=y^2(x+y+1)$？

下面哪句话描述了方程$x^2(x+y+1)=y^2(x+y+1)$的图像？

（A）Two parallel lines | 两条平行线

（B）Two intersecting lines | 两条相交的线

（C）Three lines that all pass through a common point | 三条交于一点的直线

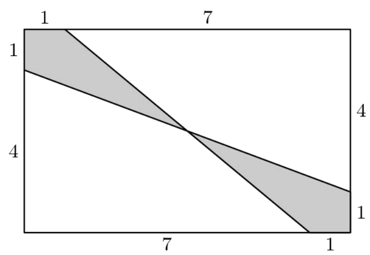
（D）Three lines that do not áll pass through a common point | 三条不交于一点的直线

（E）A line and a parabola | 一条直线和一个抛物线

Problem 8

What is the area of the shaded region of the given 8×5 rectangle?

下面给定的8×5的长方形内阴影部分的面积是多少？

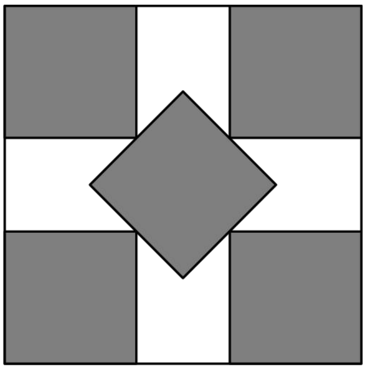


$\textbf{(A)}\ 4.75 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ 5.25 \qquad \textbf{(D)}\ 6.5 \qquad \textbf{(E)}\ 8$

Problem 9

The five small shaded squares inside this unit square are congruent and have disjoint interiors. The midpoint of each side of the middle square coincides with one of the vertices of the other four small squares as shown. The common side length is $\frac{a-\sqrt{2}}{b}$, where a and b are positive integers. What is $a+b$?

如图所示，单位正方形里有 5 个全等的小正方形，且内部不重合，中间正方形每条边的中点和其他 4 个小正方形的一个顶点重合。这些小正方形共同的边长是$\frac{a-\sqrt{2}}{b}$，其中 a 和 b 都是正整数。问$a+b$是多少？



$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 11$

Problem 10

Five friends sat in a movie theater in a row containing 5 seats, numbered 1 to 5 from left to right. (The directions "left" and "right" are from the point of view of the people as they sit in the seats.) During the movie Ada went to the lobby to get some popcorn. When she returned, she found that Bea had moved two seats to the right, Ceci had moved one seat to the left, and Dee and Edie had switched seats, leaving an end seat for Ada. In which seat had Ada been sitting before she got up?

5 个朋友在电影院里坐在一排从左向右编号为 1 到 5 的 5 个座位上（这里的"左"和"右"是指当这 5 人坐在座位上，从他们的角度看到的左和右）。在看电影的过程中，Ada 去休息室拿了一些爆米花，当她回来时，她发现 Bea 已经向右挪动了 2 个位置，Ceci 向左挪动了一个位置，Dee 和 Edie 交换了位置，空了一张最边上的位置给 Ada，那么 Ada 在她起身离开前，原先是坐在哪个座位的？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 11

Each of the 100 students in a certain summer camp can either sing, dance, or act. Some students have more than one talent, but no student has all three talents. There are 42 students who cannot sing, 65 students who cannot dance, and 29 students who cannot act. How many students have two of these talents?

某个夏令营的 100 个学生有的会唱歌，有的会跳舞，还有的会表演。有些学生具有不止一种技能，但是没有学生三种技能都会。已知有 42 名学生不会唱歌，65 个学生不会跳舞，29 个学生不会表演。问有多少个学生具备两种技能？

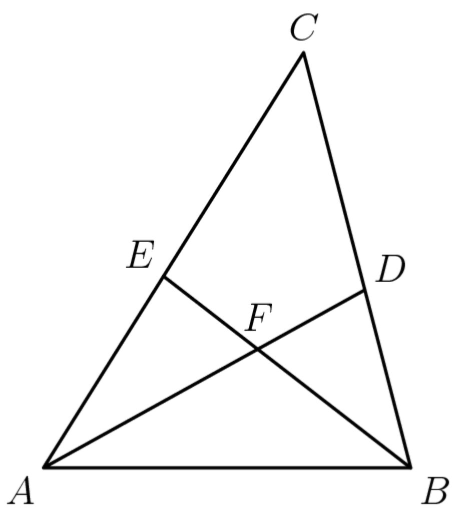
$\textbf{(A)}\ 16 \qquad \textbf{(B)}\ 25 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 49 \qquad \textbf{(E)}\ 64$

Problem 12

In $\triangle ABC$, $AB = 6$, $BC = 7$, and $CA = 8$. Point D lies on $\overline{BC}$, and $\overline{AD}$ bisects $\angle BAC$. Point E lies on $\overline{AC}$, and $\overline{BE}$ bisects $\angle ABC$. The bisectors intersect at F. What is the ratio $AF : FD$?

在$\triangle ABC$中， AB=6 ，BC=7 ，CA=8，点 D 在线段$\overline{BC}$上，$\overline{AD}$平分$\angle BAC$，.点

E 在$\overline{AC}$上，$\overline{BE}$平分$\angle ABC$，这两条角平分线交于点 F。问 $AF : FD$是多少？



$\textbf{(A)}\ 3:2 \qquad \textbf{(B)}\ 5:3 \qquad \textbf{(C)}\ 2:1 \qquad \textbf{(D)}\ 7:3 \qquad \textbf{(E)}\ 5:2$

Problem 13

Let N be a positive multiple of 5. One red ball and N green balls are arranged in a line in random order. Let $P(N)$ be the probability that at least $\frac{3}{5}$ of the green balls are on the same side of the red ball. Observe that $P(5)=1$ and that $P(N)$ approaches $\frac{4}{5}$ as N grows large. What is the sum of the digits of the least value of N such that $P(N)<\frac{321}{400}$?

N 是一个正整数且是 5 的倍数，一个红球和 N 个绿球被随机排成一排。令$P(N)$表示至少$\frac{3}{5}$的绿球在红球的同一边的概率，注意到，$P(5)=1$，且当 N 很大时，$P(N)$接近$\frac{4}{5}$，问满足$P(N)<\frac{321}{400}$的最小的 N 的各个位上数字之和是多少？

P（5）=1

$\textbf{(A) }12\qquad\textbf{(B) }14\qquad\textbf{(C) }16\qquad\textbf{(D) }18\qquad\textbf{(E) }20$

Problem 14

Each vertex of a cube is to be labeled with an integer from 1 through 8, with each integer being used once, in such a way that the sum of the four numbers on the vertices of a face is the same for each face. Arrangements that can be obtained from each other through rotations of the cube are considered to be the same. How many different arrangements are possible?

一个立方体的每个顶点被 1 到 8 这 8 个整数标记，且每个整数只能使用 1 次，满足每个面的 4 个顶点上的数字之和都相同，两种排列方法如果可以通过旋转立方体变成同一种，那么这两种排列方法被认为是同一种方法，问一共有多少种不同的排列方法？

$\textbf{(A) }1\qquad\textbf{(B) }3\qquad\textbf{(C) }6\qquad\textbf{(D) }12\qquad\textbf{(E) }24$

Problem 15

Circles with centers P, Q and R, having radii $1, 2$ and 3, respectively, lie on the same side of line l and are tangent to l at P', Q' and R', respectively, with Q' between P' and R'. The circle with center Q is externally tangent to each of the other two circles. What is the area of triangle PQR?

圆心为 P，Q 和 R 的圆半径分别为 1，2，3，它们在直线 l 的同一边，且和 l 相切，切点分别为P', Q'，R'，其中Q'位于P'和R'之间，圆心为 Q 的圆和另外两个圆外切，三角形 PQR 的面积是多少？

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ \sqrt{6}/3 \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ \sqrt{6}-\sqrt{2} \qquad \textbf{(E)}\ \sqrt{6}/2$

Problem 16

The graphs of $y = \log_3 x, y = \log_x 3, y = \log_{\frac{1}{3}} x,$ and $y = \log_x \dfrac{1}{3}$ are plotted on the same set of axis. How many points in the plane with positive x-coordinates lie on two or more of the graphs?

将函数$y = \log_3 x, y = \log_x 3, y = \log_{\frac{1}{3}} x,$ $y = \log_x \dfrac{1}{3}$的图像画在同一坐标系内，那么平面内有多少个横坐标为正的点位于这里所画的两条或者多条曲线上？

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Problem 17

Let $ABCD$ be a unit square. Let E, F, G and H be the centers, respectively, of equilateral triangles with bases $\overline{AB}, \overline{BC}, \overline{CD},$ and $\overline{DA},$ each exterior to the square. What is the area of square $EFGH$?

$ABCD$ 是一个单位正方形（边长为 1 的正方形）。分别以$\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA},$为底，向外作 4 个等边三角形，它们的中心分别是点 E，F，G 和 H。问正方形 $EFGH$ 的面积是多少？

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ \dfrac{2+\sqrt{3}}{3} \qquad \textbf{(C)}\ \sqrt{2} \qquad \textbf{(D)}\ \dfrac{\sqrt{2}+\sqrt{3}}{2} \qquad \textbf{(E)}\ \sqrt{3}$

Problem 18

For some positive integer n, the number $110n^3$ has 110 positive integer divisors, including 1 and the number $110n^3$. How many positive integer divisors does the number $81n^4$ have?

n 是个正整数，数字$110n^3$有 110 个正整数因子，包括 1 和$110n^3$，那么数字$81n^4$前有多少个正整数因子？

$\textbf{(A) }110 \qquad \textbf{(B) }191 \qquad \textbf{(C) }261 \qquad \textbf{(D) }325 \qquad \textbf{(E) }425$

Problem 19

Jerry starts at 0 on the real number line. He tosses a fair coin 8 times. When he gets heads, he moves 1 unit in the positive direction; when he gets tails, he moves 1 unit in the negative direction. The probability that he reaches 4 at some time during this process is $\frac{a}{b}$, where a and b are relatively prime positive integers. What is $a + b$? (For example, he succeeds if his sequence of tosses is $HTHHHHHH$.)

Jerry 从数轴的 0 点开始移动。他将一枚标准的硬币抛了 8 次。当硬币正面朝上时，他就沿着数轴正方向移动 1 个单位；当反面朝上时，他就朝负方向移动 1 个单位。他在移动过程中的某个时间点到达 4 的概率是$\frac{a}{b}$,其中 a 和 b 是互质的正整敛。那么 $a+b$ 等于多少？（例如，若用 H 表示正面朝上，T 表示反面朝上，那么如果某 8 次抛币结果为 *HTHHHHHH*，他就成功了）

$\textbf{(A) }69 \qquad \textbf{(B) }151 \qquad \textbf{(C) }257 \qquad \textbf{(D) }293 \qquad \textbf{(E) }313$

Problem 20

A binary operation $\diamondsuit$ has the properties that $a\,\diamondsuit\, (b\,\diamondsuit\, c) = (a\,\diamondsuit\, b)\cdot c$ and that $a\,\diamondsuit\, a = 1$ for all nonzero real numbers a, b and c. (Here the dot . represents the usual multiplication operation.) The solution to the equation $2016 \,\diamondsuit\, (6\,\diamondsuit\, x)=100$ can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p + q$?

二元运算符$\diamondsuit$有这样的性质：对于所有的非零实数 a，b，c，有$a\,\diamondsuit\, (b\,\diamondsuit\, c) = (a\,\diamondsuit\, b)\cdot c$，且 $a\,\diamondsuit\, a = 1$，（这里·表示乘法）。方程$2016 \,\diamondsuit\, (6\,\diamondsuit\, x)=100$的解可写成$\frac{p}{q}$，其中$p$和$q$是互质的正整数，则 $p+q$ 等于多少？

$\textbf{(A) }109 \qquad \textbf{(B) }201 \qquad \textbf{(C) }301 \qquad \textbf{(D) }3049 \qquad \textbf{(E) }33{,}601$

Problem 21

A quadrilateral is inscribed in a circle of radius $200\sqrt{2}$. Three of the sides of this quadrilateral have length 200. What is the length of its fourth side?

一个四边形内接在半径为$200\sqrt{2}$.的圆内，四边形的三条边长度都是 200，那么第四条边长度是多少？

$\textbf{(A) } 200 \qquad \textbf{(B) } 200\sqrt{2} \qquad \textbf{(C) } 200\sqrt{3} \qquad \textbf{(D) } 300\sqrt{2} \qquad \textbf{(E) } 500$

Problem 22

How many ordered triples (x,y,z) of positive integers satisfy $\text{lcm}(x,y) = 72, \text{lcm}(x,z) = 600$ and $\text{lcm}(y,z) = 900$?

有多少个这样的有序正整数三元组(x,y,z)，满足$\text{lcm}(x,y) = 72, \text{lcm}(x,z) = 600$，且$\text{lcm}(y,z) = 900$？

$\textbf{(A) } 15 \qquad \textbf{(B) } 16 \qquad \textbf{(C) } 24 \qquad \textbf{(D) } 27 \qquad \textbf{(E) } 64$

Problem 23

Three numbers in the interval $[0,1]$ are chosen independently and at random. What is the probability that the chosen numbers are the side lengths of a triangle with positive area?

从区间［0，1］中独立随机地选择 3 个数。问选择的这 3 个数是一个面积为正的三角形的三条边的边长的概率是多少？

$\textbf{(A) } \frac{1}{6} \qquad \textbf{(B) } \frac{1}{3} \qquad \textbf{(C) } \frac{1}{2} \qquad \textbf{(D) } \frac{2}{3} \qquad \textbf{(E) } \frac{5}{6}$

Problem 24

There is a smallest positive real number a such that there exists a positive real number b such that all the roots of the polynomial $x^3 - ax^2 + bx - a$ are real. In fact, for this value of a the value of b is unique. What is the value of b?

存在一个最小的正实数 a，对应这个 a 值，存在一个正实数 b，使得多项式$x^3 - ax^2 + bx - a$ 的所有根都是实根。实际上，对于这个 a 值，正实数 b 是唯一的。那么 b 的值是多少？

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 10 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 12$

Problem 25

Let k be a positive integer. Bernardo and Silvia take turns writing and erasing numbers on a blackboard as follows: Bernardo starts by writing the smallest perfect square with $k+1$ digits. Every time Bernardo writes a number, Silvia erases the last k digits of it. Bernardo then writes the next perfect square, Silvia erases the last k digits of it, and this process continues until the last two numbers that remain on the board differ by at least 2. Let $f(k)$ be the smallest positive integer not written on the board. For example, if $k = 1$, then the numbers that Bernardo writes are $16, 25, 36, 49, 64$, and the numbers showing on the board after Silvia erases are $1, 2, 3, 4,$ and 6, and thus $f(1) = 5$. What is the sum of the digits of $f(2) + f(4) + f(6) + ... + f(2016)$?

k 是个正整数。Bernardo 和 Silvia 以如下方式轮流在黑板上写数字和擦数字：Bemardo 一开始写下的数字是最小的 k+1 位的平方数。每次 Bernardo 写下一个数字，Silvia 就把这个数字的最后 k 位擦除。然后 Bernardo 继续写出下一个平方数，Silvia 又把这个数字的最后 k 位擦除。这个过程一直持续下去，直到在黑板上留下的最后两个数字的差至少为 2，令$f(k)$表示在黑板上没有写的最小正整数。例如，若 k=1，那么 Bernardo 写下的数字分别是,16，25，36，49，64，当 Silvia 擦除后，黑板上留下的数字是 1,2,3,4 和 6, 因此$f(1) = 5$。问 $f(2) + f(4) + f(6) + ... + f(2016)$的结果的各个位上的数字之和是多少

$\textbf{(A)}\ 7986 \qquad \textbf{(B)}\ 8002 \qquad \textbf{(C)}\ 8030 \qquad \textbf{(D)}\ 8048 \qquad \textbf{(E)}\ 8064$

2016 AMC 12A Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
B	C	B	D	E	D	D	D	E	B	E	C	A
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
C	D	D	B	D	B	A	E	A	C	B	E	

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