

2024 MAA AMC 10B

Problem 1

In a long line of people arranged left to right, the 1013th person from the left is also the 1010th person from the right. How many people are in the line?

一群人从左到右排成一个很长的队伍，从左边数起的第 1013 个人，也是从右边数起的第 1010 个人。问这个队伍中共有多少人？

- (A) 2021 (B) 2022 (C) 2023 (D) 2024 (E) 2025

Problem 2

What is $10! - 7! \cdot 6!$?

问 $10! - 7! \cdot 6!$ 是多少？

- (A) -120 (B) 0 (C) 120 (D) 600 (E) 720

Problem 3

For how many integer values of x is $|2x| \leq 7\pi$?

满足 $|2x| \leq 7\pi$ 的整数值 x 有多少个？

- (A) 16 (B) 17 (C) 19 (D) 20 (E) 21

Problem 4

Balls numbered $1, 2, 3, \dots$ are deposited in 5 bins, labeled A, B, C, D , and E using the following procedure. Ball 1 is deposited in bin A , and balls 2 and 3 are deposited in bin B . The next 3 balls are deposited in bin C , the next 4 in bin D , and so on, cycling back to bin A after balls are deposited in bin E . (For example, balls numbered $22, 23, \dots, 28$ are deposited in bin B at step 7 of this process.) In which bin is ball 2024 deposited?

编号为 $1, 2, 3, \dots$ 的球按照以下操作步骤存放在 5 个分别标记为 A, B, C, D, E 的箱子中。1 号球存放在箱子 A 中，2 号球和 3 号球存放在箱子 B 中。接下来

的 3 个球存放在箱子 C 中, 接下来的 4 个球存放在箱子 D 中, 依此类推, 球存放到箱子 E 中后, 再循环回到箱子 A. (例如, 编号为 22, 23, ..., 28 的球在操作的第 7 步存放在箱子 B 中.) 问 2024 号球存放在哪个箱子中?

- (A) A (B) B (C) C (D) D (E) E

Problem 5

In the following expression, Melanie changed some of the plus signs to minus signs:

$$1 + 3 + 5 + 7 + \cdots + 97 + 99$$

When the new expression was evaluated, it was negative. What is the least number of plus signs that Melanie could have changed to minus signs?

在下面的表达式中, Melanie 将一些加号改为减号:

$$1 + 3 + 5 + 7 + \cdots + 97 + 99$$

当对新的表达式求值时, 结果为负数. 问 Melanie 最少可能将多少个加号改为减号?

- (A) 14 (B) 15 (C) 16 (D) 17 (E) 18

Problem 6

A rectangle has integer side lengths and an area of 2024. What is the least possible perimeter of the rectangle?

一个矩形的边长为整数, 面积是 2024. 问该矩形的周长最小可能是多少?

- (A) 160 (B) 180 (C) 222 (D) 228 (E) 390

Problem 7

What is the remainder when $7^{2024} + 7^{2025} + 7^{2026}$ is divided by 19?

问 $7^{2024} + 7^{2025} + 7^{2026}$ 除以 19 的余数是多少?

- (A) 0 (B) 1 (C) 7 (D) 11 (E) 18

Problem 8

Let N be the product of all the positive integer divisors of 42. What is the units digit of N ?

设 N 为42的所有正整数因数的乘积. 问 N 的个位数字是多少?

- (A) 0 (B) 2 (C) 4 (D) 6 (E) 8

Problem 9

Real numbers a, b , and c have arithmetic mean 0. The arithmetic mean of a^2, b^2 , and c^2 is 10. What is the arithmetic mean of ab, ac , and bc ?

实数 a, b, c 的算术平均值为0. a^2, b^2, c^2 的算术平均值为10. 问 ab, ac, bc 的算术平均值是多少?

- (A) -5 (B) $-\frac{10}{3}$ (C) $-\frac{10}{9}$ (D) 0 (E) $\frac{10}{9}$

Problem 10

Quadrilateral $ABCD$ is a parallelogram, and E is the midpoint of the side \overline{AD} . Let F be the intersection of lines EB and AC . What is the ratio of the area of quadrilateral $CDEF$ to the area of $\triangle CFB$?

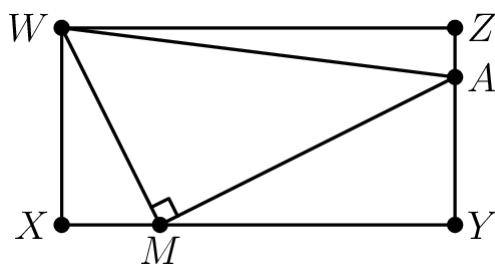
四边形 $ABCD$ 是平行四边形, E 是边 \overline{AD} 的中点. 设 F 是直线 EB 和 AC 的交点. 问四边形 $CDEF$ 的面积与 $\triangle CFB$ 的面积之比是多少?

- (A) 5:4 (B) 4:3 (C) 3:2 (D) 5:3 (E) 2:1

Problem 11

In the figure below $WXYZ$ is a rectangle with $WX = 4$ and $WZ = 8$. Point M lies on \overline{XY} , point A lies on \overline{YZ} , and $\angle WMA$ is a right angle. The areas of $\triangle WXM$ and $\triangle WAZ$ are equal. What is the area of $\triangle WMA$?

在矩形 $WXYZ$ 中, $WX = 4$ 且 $WZ = 8$. 点 M 在 \overline{XY} 上, 点 A 在 \overline{YZ} 上, 且 $\angle WMA$ 为直角. 三角形 $\triangle WXM$ 与 $\triangle WAZ$ 的面积相等. 问 $\triangle WMA$ 的面积是多少?



- (A) 13 (B) 14 (C) 15 (D) 16 (E) 17

Problem 12

A group of 100 students from different countries meet at a mathematics competition. Each student speaks the same number of languages, and, for every pair of students A and B, student A speaks some language that student B does not speak, and student B speaks some language that student A does not speak. What is the least possible total number of languages spoken by all the students?

在一场数学竞赛中，来自不同国家的 100 名学生相聚。每名学生会说相同数量的语言，且对于任意两名学生 A 和 B，A 会说某种 B 不会说的语言，B 也会说某种 A 不会说的语言。问有学生会说的各种语言的总数最少可能是多少？

- (A) 9 (B) 10 (C) 12 (D) 51 (E) 100

Problem 13

Positive integers x and y satisfy the equation $\sqrt{x} + \sqrt{y} = \sqrt{1183}$. What is the minimum possible value of $x + y$?

正整数 x 和 y 满足方程 $\sqrt{x} + \sqrt{y} = \sqrt{1183}$ ，求 $x + y$ 的最小可能取值？

- (A) 585 (B) 595 (C) 623 (D) 700 (E) 791

Problem 14

A dartboard is the region B in the coordinate plane consisting of points (x, y) such that $|x| + |y| \leq 8$. A target T is the region where $(x^2 + y^2 - 25)^2 \leq 49$. A dart is thrown and lands at a random point in B . The probability that the dart lands in T can be expressed as $\frac{m}{n} \cdot \pi$, where m and n are relatively prime positive integers. What is $m + n$?

在坐标平面上，由满足 $|x| + |y| \leq 8$ 的点 (x, y) 组成靶盘区域 B 。由 $(x^2 + y^2 - 25)^2 \leq 49$ 界定目标区域 T 。飞镖被投掷后随机的落在 B 内的某个点上。飞镖落在 T 内的概率可以表示为 $\frac{m}{n} \cdot \pi$ ，其中 m 和 n 是互质的正整数。问 $m + n$ 是多少？

- (A) 39 (B) 71 (C) 73 (D) 75 (E) 135

Problem 15

A list of 9 real numbers consists of 1, 2.2, 3.2, 5.2, 6.2, and 7, as well as x , y , and z with $x \leq y \leq z$. The range of the list is 7, and the mean and the median are both positive integers. How many ordered triples (x, y, z) are possible?

由 9 个实数组成的数据列表包含 1, 2.2, 3.2, 5.2, 6.2, 7, 以及满足 $x \leq y \leq z$ 的数 x, y, z 。此数据列表的全距为 7, 且平均数和中位数都是正整数。问有序三元组 (x, y, z) 有多少种可能？

- (A) 1 (B) 2 (C) 3 (D) 4 (E) infinitely many | 无穷多个

Problem 16

Jerry likes to play with numbers. One day, he wrote all the integers from 1 to 2024 on the whiteboard. Then he repeatedly chose four numbers on the whiteboard, erased them, and replaced them by either their sum or their product. (For example, Jerry's first step might have been to erase 1, 2, 3, and 5, and then write either 11, their sum, or 30, their product, on the whiteboard.) After repeatedly performing this operation, Jerry noticed that all the remaining numbers on the whiteboard were odd. What is the maximum possible number of integers on the whiteboard at that time?

Jerry 喜欢数的运算。有一天，他在白板上写下了从 1 到 2024 的所有整数。然后他反复执行以下操作：选择白板上的四个数，擦去它们，并用它们的和或积替换（例如，Jerry 第一步可能擦去 1, 2, 3, 5, 然后写下它们的和 11 或积 30）。在多次操作后，Jerry 发现白板上剩余的所有数都是奇数。问此时白板上最多可能有多少个整数？

- (A) 1010 (B) 1011 (C) 1012 (D) 1013 (E) 1014

Problem 17

In a race among 5 snails, there is at most one tie, but that tie can involve any number of snails. For example, the result of the race might be that Dazzler is first; Abby, Cyrus, and Elroy are tied for second; and Bruna is fifth. How many different results of the race are possible?

在一场 5 只蜗牛的比赛中，最多只会出现一次并列名次，但这个并列名次可以涉及任意数量的蜗牛。例如，比赛结果可能是 Dazzler 第一；Abby, Cyrus, Elroy 并列第二；Bruna 第五。问有多少种不同的可能比赛结果？

- (A) 180 (B) 361 (C) 420 (D) 431 (E) 720

Problem 18

How many different remainders can result when the 100th power of an integer is divided by 125?

当一个整数的 100 次方除以 125 时，可以得到多少种不同的余数？

- (A) 1 (B) 2 (C) 5 (D) 25 (E) 125

Problem 19

In the following table, each question mark is to be replaced by "Possible" or "Not Possible" to indicate whether a nonvertical line with the given slope can contain the given number of lattice points (points both of whose coordinates are integers). How many of the 12 entries will be "Possible"?

	zero	exactly one	exactly two	more than two
zero slope	?	?	?	?
nonzero rational slope	?	?	?	?
irrational slope	?	?	?	?

在下表中，每个问号需要被替换为“可能”或“不可能”，表示具有给定斜率的非竖直的直线是否可能包含给定数量的格点（即横纵坐标均为整数的点）。问这12个格中有多少个会填入“可能”？

	零个	恰好一个	恰好两个	多于两个
零斜率	?	?	?	?
非零有理数斜率	?	?	?	?
无理数斜率	?	?	?	?

- (A) 4 (B) 5 (C) 6 (D) 7 (E) 9

Problem 20

Three different pairs of shoes are placed in a row so that no left shoe is next to a right shoe from a different pair. In how many ways can these six shoes be lined up?

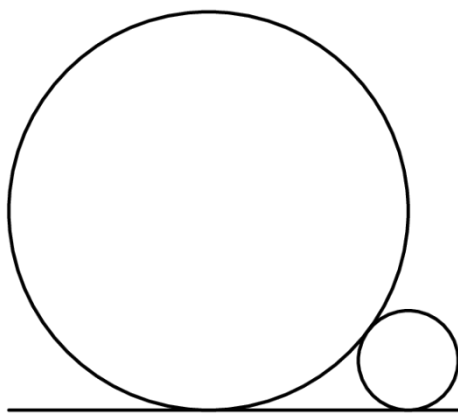
三双不同的鞋子排成一列，要求每只穿在左脚的鞋子不能和其他双鞋中穿在右脚的鞋子相邻。问这六只鞋共有多少种排列方式？

- (A) 60 (B) 72 (C) 90 (D) 108 (E) 120

Problem 21

Two straight pipes (circular cylinders), with radii 1 and $\frac{1}{4}$, lie parallel and in contact on a flat floor. The figure below shows a head-on view. What is the sum of the possible radii of a third parallel pipe lying on the same floor and in contact with both?

两根半径分别为 1 和 $\frac{1}{4}$ 的圆柱形直管平行地放置在平坦的地面上并相互接触。下图显示的是正视图。第三根直管平行放置在同一地面上，并且与这两根直管都接触，问它可能的半径之和是多少？



- (A) $\frac{1}{9}$ (B) 1 (C) $\frac{10}{9}$ (D) $\frac{11}{9}$ (E) $\frac{19}{9}$

Problem 22

A group of 16 people will be partitioned into 4 indistinguishable 4-person committees. Each committee will have one chairperson and one secretary. The number of different ways to make these assignments can be written as $3^r M$, where r and M are positive integers and M is not divisible by 3. What is r ?

现有一组共 16 人将被分成 4 个不加区别的委员会，每个委员会有 4 人。每个委员会将有一位主席和一位秘书。所有不同的分配方法的总数可以写成 $3^r M$ ，其中 r 和 M 是正整数，且 M 不能被 3 整除。问 r 是多少？

- (A) 5 (B) 6 (C) 7 (D) 8 (E) 9

Problem 23

The Fibonacci numbers are defined by $F_1 = 1, F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. What is $\frac{F_2}{F_1} + \frac{F_4}{F_2} + \frac{F_6}{F_3} + \cdots + \frac{F_{20}}{F_{10}}$?

Fibonacci 数列定义为 $F_1 = 1, F_2 = 1$, 且对于 $n \geq 3$, $F_n = F_{n-1} + F_{n-2}$. 问 $\frac{F_2}{F_1} + \frac{F_4}{F_2} + \frac{F_6}{F_3} + \cdots + \frac{F_{20}}{F_{10}}$ 是多少?

- (A) 318 (B) 319 (C) 320 (D) 321 (E) 322

Problem 24

Let

$$P(m) = \frac{m}{2} + \frac{m^2}{4} + \frac{m^4}{8} + \frac{m^8}{8}$$

How many of the values $P(2022)$, $P(2023)$, $P(2024)$, and $P(2025)$ are integers?

设

$$P(m) = \frac{m}{2} + \frac{m^2}{4} + \frac{m^4}{8} + \frac{m^8}{8}.$$

问在 $P(2022)$, $P(2023)$, $P(2024)$, $P(2025)$ 这些值中, 有几个是整数?

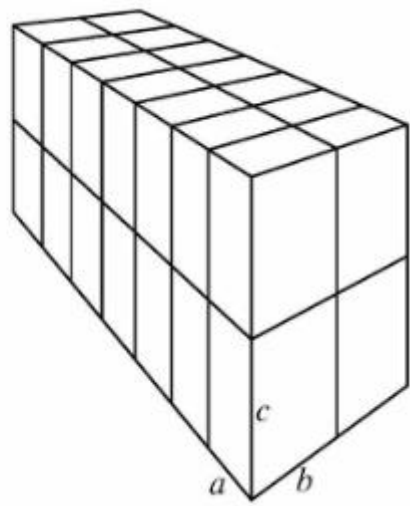
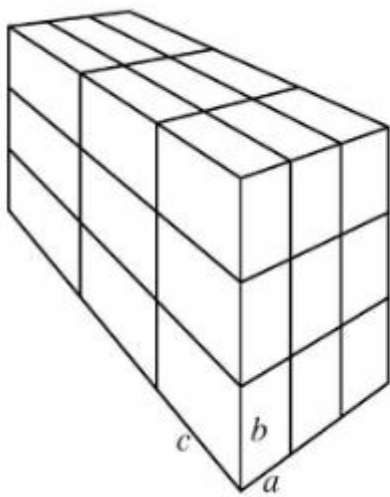
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Problem 25

Each of 27 bricks (right rectangular prisms) has dimensions $a \times b \times c$, where a , b , and c are pairwise relatively prime positive integers. These bricks are arranged to form a $3 \times 3 \times 3$ block, as shown on the left below. A 28th brick with the same dimensions is introduced, and these bricks are reconfigured into a $2 \times 2 \times 7$ block, shown on the right. The new block is 1 unit taller, 1 unit wider, and 1 unit deeper than the old one. What is $a + b + c$?

有 27 块长方体形状的砖块, 其尺寸均为 $a \times b \times c$, 其中 a, b, c 是两两互质的正整数. 如左下图所示, 这些砖块按照 $3 \times 3 \times 3$ 的样式堆砌成立体图形. 现在添加相同尺寸的第 28 块砖, 然后如右下图所示, 将所有砖块按照 $2 \times 2 \times 7$ 的形式重新堆砌成新的立体图形. 新的立体图形比原来的高 1 个单位, 宽 1 个单位, 深 1

个单位. 问 $a + b + c$ 是多少?



- (A) 88 (B) 89 (C) 90 (D) 91 (E) 92

2024 AMC 10B Answer Key													
题目	1	2	3	4	5	6	7	8	9	10	11	12	13
答案	B	B	E	D	B	B	A	D	A	A	C	A	B
题目	14	15	16	17	18	19	20	21	22	23	24	25	
答案	B	C	A	D	B	C	A	C	A	B	E	E	

2024 AMC10B Solution



扫码观看视频解析