

## 2025 MAA AIME II

### Problem 1

Six points  $A, B, C, D, E$ , and  $F$  lie in a straight line in that order. Suppose that  $G$  is a point not on the line and that  $AC = 26$ ,  $BD = 22$ ,  $CE = 31$ ,  $DF = 33$ ,  $AF = 73$ ,  $CG = 40$ , and  $DG = 30$ . Find the area of  $\triangle BGE$ .

$A, B, C, D, E, F$ 是按此顺序排列在一条直线上的六个点. 假设 $G$ 是不在这条直线上的点, 并且 $AC = 26$ ,  $BD = 22$ ,  $CE = 31$ ,  $DF = 33$ ,  $AF = 73$ ,  $CG = 40$ ,  $DG = 30$ . 求三角形 $\triangle BGE$ 的面积.

### Problem 2

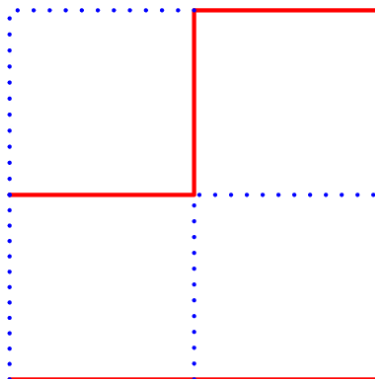
Find the sum of all positive integers  $n$  such that  $n + 2$  divides the product  $3(n + 3)(n^2 + 9)$ .

求所有这样正整数 $n$ 的和, 使得 $n + 2$ 能整除乘积 $3(n + 3)(n^2 + 9)$ .

### Problem 3

Four unit squares form a  $2 \times 2$  grid. Each of the 12 unit line segments forming the sides of the squares is colored either red or blue in such a way that each unit square has 2 red sides and 2 blue sides. One example is shown below (red is solid, blue is dashed). Find the number of such colorings.

四个单位正方形构成了 $2 \times 2$ 的网格. 构成这些正方形边的12条单位线段分别涂成红色或蓝色, 要求每个单位正方形必须恰好有2条红边和2条蓝边. 下面给出了一个示例(红色为实线, 蓝色为虚线). 问这样的涂色方式共有多少种?



### Problem 4

The product is equal to  $\frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers. Find  $m + n$ .

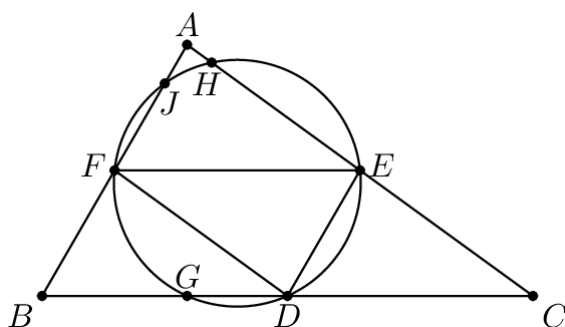
下面的乘积等于 $\frac{m}{n}$ ，其中 $m$ 和 $n$ 是互质的正整数。求 $m + n$ 的值。

$$\prod_{k=4}^{63} \frac{\log_k(5^{k^2-1})}{\log_{k+1}(5^{k^2-4})} = \frac{\log_4(5^{15})}{\log_5(5^{12})} \cdot \frac{\log_5(5^{24})}{\log_6(5^{21})} \cdot \frac{\log_6(5^{35})}{\log_7(5^{32})} \cdots \frac{\log_{63}(5^{3968})}{\log_{64}(5^{3965})}$$

### Problem 5

Suppose  $\triangle ABC$  has angles  $\angle BAC = 84^\circ$ ,  $\angle ABC = 60^\circ$ , and  $\angle ACB = 36^\circ$ . Let  $D$ ,  $E$ , and  $F$  be the midpoints of sides  $\overline{BC}$ ,  $\overline{AC}$ , and  $\overline{AB}$ , respectively. The circumcircle of  $\triangle DEF$  intersects  $\overline{BD}$ ,  $\overline{AE}$ , and  $\overline{AF}$  at points  $G$ ,  $H$ , and  $J$ , respectively. The points  $G, D, E, H, J$ , and  $F$  divide the circumcircle of  $\triangle DEF$  into six minor arcs, as shown. Find  $DE + 2 \cdot HJ + 3 \cdot FG$ , where the arcs are measured in degrees.

在 $\triangle ABC$ 中， $\angle BAC = 84^\circ$ ， $\angle ABC = 60^\circ$ ， $\angle ACB = 36^\circ$ 。  $D, E, F$  分别是边 $BC, AC, AB$ 的中点。 三角形 $DEF$ 的外接圆与 $BD, AE, AF$ 在点 $G, H, J$ 处相交。 如图所示，点 $G, D, E, H, J, F$ 将三角形 $DEF$ 的外接圆分成六段小圆弧。 求 $DE + 2 \cdot HJ + 3 \cdot FG$ 的度数。

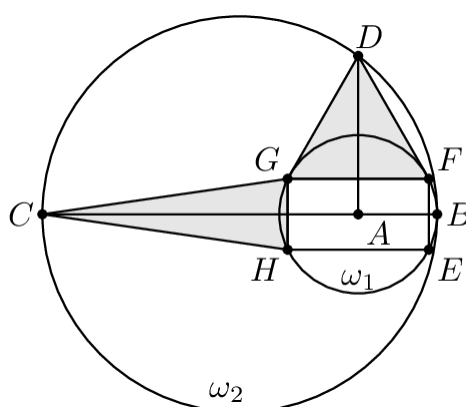


### Problem 6

Circle  $\omega_1$  with radius 6 centered at point  $A$  is internally tangent at point  $B$  to circle  $\omega_2$  with radius 15. Points  $C$  and  $D$  lie on  $\omega_2$  such that  $\overline{BC}$  is a diameter of  $\omega_2$  and  $\overline{BC} \perp \overline{AD}$ . The rectangle  $EFGH$  is inscribed in  $\omega_1$  such

that  $\overline{EF} \perp \overline{BC}$ ,  $C$  is closer to  $\overline{GH}$  than to  $\overline{EF}$ , and  $D$  is closer to  $\overline{FG}$  than to  $\overline{EH}$ , as shown. Triangles  $\triangle DGF$  and  $\triangle CHG$  have equal areas. The area of rectangle  $EFGH$  is  $\frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers. Find  $m + n$ .

圆 $\omega_1$ 的半径为6, 圆心为A, 与半径为15的圆 $\omega_2$ 在点B处内切. 点C和D位于圆 $\omega_2$ 上, 使得BC是圆 $\omega_2$ 的直径, 并且 $BC \perp AD$ . 如图所示, 矩形EFGH内接于圆 $\omega_1$ , 使得 $EF \perp BC$ , C更靠近GH, 而不是EF, 并且D更靠近FG, 而不是EH. 三角形 $\triangle DGF$ 和三角形 $\triangle CHG$ 的面积相等. 矩形EFGH的面积为 $\frac{m}{n}$ , 其中m和n是互质的正整数. 求 $m + n$ 的值.



### Problem 7

Let  $A$  be the set of positive integer divisors of 2025. Let  $B$  be a randomly selected subset of  $A$ . The probability that  $B$  is a nonempty set with the property that the least common multiple of its elements is 2025 is  $\frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers. Find  $m + n$ .

设 $A$ 为2025的正整数约数构成的集合. 设 $B$ 为随机选择的 $A$ 的子集.  $B$ 是一个非空集合且其元素的最小公倍数是2025的概率为 $\frac{m}{n}$ , 其中 $m$ 和 $n$ 是互质的正整数. 求 $m + n$ 的值.

### Problem 8

From an unlimited supply of 1-cent coins, 10-cent coins, and 25-cent coins, Silas wants to find a collection of coins that has a total value of  $N$  cents, where  $N$  is a positive integer. He uses the so-called *greedy algorithm*, successively choosing the coin of greatest value that does not cause the value of his collection

to exceed  $N$ . For example, to get 42 cents, Silas will choose a 25-cent coin, then a 10-cent coin, then 7 1-cent coins. However, this collection of 9 coins uses more coins than necessary to get a total of 42 cents; indeed, choosing 4 10-cent coins and 2 1-cent coins achieves the same total value with only 6 coins.

In general, the greedy algorithm succeeds for a given  $N$  if no other collection of 1-cent, 10-cent, and 25-cent coins gives a total value of  $N$  cents using strictly fewer coins than the collection given by the greedy algorithm. Find the number of values of  $N$  between 1 and 1000 inclusive for which the greedy algorithm succeeds.

从数量充足的 1 美分硬币、10 美分硬币和 25 美分硬币中, Silas 想找出一组硬币, 使其总值为  $N$  美分, 其中  $N$  是正整数. 他使用所谓的贪心算法, 在保证硬币总价值不超过  $N$  美分的前提下, 依次选择最大面值的硬币. 例如, 要得到 42 美分, Silas 会先选择一个 25 美分硬币, 然后是一个 10 美分硬币, 然后是 7 个 1 美分硬币. 然而, 形成这样的总价值并不需要有 9 个硬币那么多, 如果选择 4 个 10 美分和 2 个 1 美分硬币, 仅用 6 个硬币也能达到相同的总价值.

一般来说, 对于给定的  $N$ , 所谓贪心算法是成功的是指: 如果没有其他总价值为  $N$  美分的 1 美分, 10 美分, 25 美分的硬币组合使用严格少于贪心算法给出的组合的硬币数量. 当  $N$  从 1 到 1000 (包括 1 和 1000) 取值时, 求贪心算法成功的总次数.

### Problem 9

There are  $n$  values of  $x$  in the interval  $0 < x < 2\pi$  where  $f(x) = \sin(7\pi \cdot \sin(5x)) = 0$ . For  $t$  of these  $n$  values of  $x$ , the graph of  $y = f(x)$  is tangent to the  $x$ -axis. Find  $n + t$ .

在区间  $0 < x < 2\pi$  中, 有  $n$  个值  $x$  使得  $f(x) = \sin(7\pi \cdot \sin(5x)) = 0$ . 在这  $n$  个值中有  $t$  个值, 使得  $y = f(x)$  的图像与  $x$  轴相切. 求  $n + t$  的值.

### Problem 10

Sixteen chairs are arranged in a row. Eight people each select a chair in which to sit so that no person sits next to two other people. Let  $N$  be the number of subsets of 16 chairs that could be selected. Find the remainder when  $N$  is

divided by 1000.

十六把椅子排成一排. 八个人各自选择一把椅子坐下, 要求没有人同时与两个其他人相邻而坐, 有人就坐的椅子构成 16 把椅子的一个子集. 设所有这样的子集的数量为  $N$ . 求  $N$  除以 1000 的余数.

### Problem 11

Let  $S$  be the set of vertices of a regular 24-gon. Find the number of ways to draw 12 segments of equal lengths so that each vertex in  $S$  is an endpoint of exactly one of the 12 segments.

设  $S$  为正 24 边形顶点构成的集合. 现在要画出 12 条相等长度的对角线, 使得  $S$  中的每个顶点恰好是这 12 条对角线中某一条的一个端点. 求满足上述条件的画法的总数.

### Problem 12

Let  $A_1A_2 \dots A_{11}$  be an 11-sided non-convex simple polygon with the following properties:

- For every integer  $2 \leq i \leq 10$ , the area of  $A_iA_1A_{i+1}$  is 1,
- For every integer  $2 \leq i \leq 10$ ,  $\cos(\angle A_iA_1A_{i+1}) = \frac{12}{13}$ ,
- The perimeter of the 11-gon  $A_1A_2 \dots A_{11}$  is equal to 20.

Then  $A_1A_2 + A_1A_{11}$  can be expressed as  $\frac{m\sqrt{n}-p}{q}$  where  $m, n, p, q$  are positive integers,  $n$  is not divisible by any square, and no prime divides all of  $m$ ,  $p$ , and  $q$ . Find  $m + n + p + q$ .

设  $A_1A_2 \dots A_{11}$  是一个有 11 条边的非凸的简单多边形, 具有以下性质:

- 对于每个整数  $2 \leq i \leq 10$ , 三角形  $A_iA_1A_{i+1}$  的面积等于 1.
- 对于每个整数  $2 \leq i \leq 10$ ,  $\cos(\angle A_iA_1A_{i+1}) = \frac{12}{13}$ .
- 11 边形  $A_1A_2 \dots A_{11}$  的周长等于 20.

那么  $A_1A_2 + A_1A_{11}$  可以表示成  $\frac{m\sqrt{n}-p}{q}$  的形式, 其中  $m, n, p, q$  是正整数,  $n$  不可被任

何平方数整除，并且没有同一个质数同时整除 $m$ ,  $p$ 和 $q$ 。求 $m + n + p + q$ 的值。

### Problem 13

Let the sequence of rationals  $x_1, x_2, \dots$  be defined such that  $x_1 = \frac{25}{11}$  and

$$x_{k+1} = \frac{1}{3} \left( x_k + \frac{1}{x_k} - 1 \right)$$

for all  $k \geq 1$ . Then  $x_{2025}$  can be expressed as  $\frac{m}{n}$  for relatively prime positive integers  $m$  and  $n$ . Find the remainder when  $m + n$  is divided by 1000.

定义有理数数列 $x_1, x_2, \dots$ ，其中 $x_1 = \frac{25}{11}$ 且对所有 $k \geq 1$ ，有

$$x_{k+1} = \frac{1}{3} \left( x_k + \frac{1}{x_k} - 1 \right)$$

那么 $x_{2025}$ 可以表示为 $\frac{m}{n}$ ，其中 $m$ 和 $n$ 是互质的正整数。求 $m + n$ 除以1000的余数。

### Problem 14

Let  $\triangle ABC$  be a right triangle with  $\angle A = 90^\circ$  and  $BC = 38$ . There exist points  $K$  and  $L$  inside the triangle such

$$AK = AL = BK = CL = KL = 14.$$

The area of the quadrilateral  $BKLC$  can be expressed as  $n\sqrt{3}$  for some positive integer  $n$ . Find  $n$ .

设 $\triangle ABC$ 是一个直角三角形， $\angle A = 90^\circ$ 且 $BC = 38$ 。在三角形内部存在点 $K$ 和 $L$ ，使得

$$AK = AL = BK = CL = KL = 14.$$

四边形 $BKLC$ 的面积可以表示为 $n\sqrt{3}$ ，其中 $n$ 为正整数。求 $n$ 的值。

### Problem 15

There are exactly three positive real numbers  $k$  such that the function

$$f(x) = \frac{(x-18)(x-72)(x-98)(x-k)}{x}$$

defined over the positive real numbers achieves its minimum value at exactly two positive real numbers  $x$ . Find the sum of these three values of  $k$ .

恰好存在三个正实数 $k$ ，使得定义在正实数集上的函数

$$f(x) = \frac{(x - 18)(x - 72)(x - 98)(x - k)}{x}$$

的最小值恰好在两个正实数 $x$ 处取到。求这三个 $k$ 的值之和。

2025 AIME II Answer Key

题目	1	2	3	4	5	6	7	8
答案	468	049	082	106	336	293	237	610
题目	9	10	11	12	13	14	15	
答案	149	907	113	019	248	104	240	