Translating Research into Action

Conjuring Reality in the Ensemble vs. Restoring Temporal Skepticism: A Practitioner's List of Puzzles, Paradoxes, & Anomalies In Financial Economics

Speaker's Notes Prepared for the Ergodicity Economics 2021 Conference

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Introduction

This presentation started with a request from the London Mathematical Laboratory (LML) to submit proposals for presentations at the Ergodicity Economics 2021 Online Conference (EE2021) scheduled for January 18-20, 2021.

Here is the link to the conference webpage: http://lml.org.uk/ee2021/

See below the submission, including references, and the link to the RESEARCHERS.ONE paper that frames up the context for the submission.

Submission to the first Ergodicity Economics Conference

(EE2021 – January 18-20, 2021)

Title:

Conjuring Reality in the Ensemble vs. Restoring Temporal Skepticism: A Practitioner's List of Puzzles, Paradoxes & Anomalies

Abstract:

The narrated conceptual chronology (NCC 2021) that was recently published on RESEARCHERS.ONE[1], proved to be a helpful exercise to connect the dots between financial economics and ergodicity economics (EE).

It also proved to be a productive platform for the development of additional content, including a prioritized list of the empirical puzzles, paradoxes, and anomalies of financial economics that matter the most for a group of practitioners.

This presentation uses the format of the three-braided conceptual chronology to prioritize these issues, including questions such as the following:

Are EE-based decisions limited to growth optimal product selection, as contrasted with portfolio optimization or client-centric planning?

[1] See: https://researchers.one/articles/Expected-Value-Optimization-Client-Centric-Planning-and-Growth-Optimal-Selection-A-Centuries-long-Three-Braided-Conceptual-Chronology/e2198c63dc750310a3d6ffcb

Title Slide

Conjuring Reality in the Ensemble vs. Restoring Temporal Skepticism: A Practitioner's List of Puzzles, Paradoxes & Anomalies

Ergodicity Economics Conference

January 18 – 21, 2021

François Gadenne

CTRI

Slide 1 Speaker's Notes – Purpose and Structure of the Presentation

This presentation compares financial economics with ergodicity economics (EE) in the context of a framework that combines:

- Research findings of a previous venture, the Retirement Income Industry Association (RIIA), that are documented in the published paper "The Shapes of Retirement: Are you a Curve, a Triangle, or a Rectangle?" (Collins/Gadenne 2017).
    - https://img1.wsimg.com/blobby/go/9df5ee44-13e5-4d5d-b09e-3b10908c1568/downloads/1d1etmh1l_143128.pdf?ver=1605031955504

- CTRI's reading of research papers, which improved on the previous paper, and is documented in the published paper "Expected Value Optimization, Client-Centric Planning, & Growth Optimal Selection: A Centuries-long, Three-braided Conceptual Chronology" (Gadenne 2020).
    - https://researchers.one/articles/Expected-Value-Optimization-Client-Centric-Planning-and-Growth-Optimal-Selection-A-Centuries-long-Three-Braided-Conceptual-Chronology/e2198c63dc750310a3d6ffcb

- CTRI's Reading Notes and summary of "Ergodicity Economic's Logic, Notation, & Calculations," an internal document.

The primary purpose of this presentation is to **articulate a mapping as a method that systematically reveals and prioritizes a practitioner's list of puzzles, paradoxes, and anomalies in financial economics**, some of which are already solved by EE and others that await future resolution.

This mapping as a method also achieves the following:

- Compare financial economics to ergodicity economics
- Show how CTRI's statistical reading template for research papers augments the basic framework
- Illustrate how EE's process, calculations, and findings add to the framework
- Demonstrate how all of this comes together to identify relevant business problems that can be solved by EE

Slide 1 - Purpose and Structure of the Presentation

Purpose and Structure

- **Slide 1:** Purpose and Structure
  - ***A mapping as a method to reveal and prioritize a practitioner's list of financial economics puzzles, paradoxes, and anomalies for resolution by ergodicity economics***
- **Slide 2:** The Basic Framework
- **Slide 3:** Adding CTRI's Template for Reading Research Papers
- **Slide 4:** Adding Ergodicity Economics' Calculation Process
- **Slide 5:** Adding Puzzles, Paradoxes & Anomalies
- **Slide 6:** A Practitioner's List of Puzzles, Paradoxes & Anomalies
- **Slides 7 through 11 :** Vignettes for Discussion
  - **For additional information, see: "Expected Value Optimization, Client-Centric Planning, & Growth Optimal Selection: A Centuries-long, Three-braided Conceptual Chronology" on RESEARCHERS.ONE, and the Speaker's Notes for this presentation.**

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Slide 2 Speaker's Notes - The Basic Framework

The presentation introduces the comparative framework, one step at a time, starting with the basic process of **input/model/output**, as shown on the first slide.

This first slide is a 3x4 matrix where:

- Columns headers are "Financial Economics" and "Ergodicity Economics," and
- Row labels are "input," "model," and "output."

In addition, and to improve clarity when the matrix becomes more complicated:

- Input-related cells are yellow,
- Model-related cells are white, and
- Output-related cells are green.

Slide 2 - The Basic Framework

The Basic Framework

	Financial Economics	Ergodicity Economics
Input		
Model		
Output		

Slide 3 Speaker's Notes - Adding CTRI's Template for Reading Research Papers to the Basic Framework

CTRI uses a statistical template to document the reading of research papers.

Adding key components of this template to the basic framework increases the number of rows as follows and shown on Slide 2:

- Input: **Data Generation Process**
- Input: **Microstructure of Data**

- Model: **Design of Study**
- Model: **Descriptive Macrostructure Statistics**

- Output: **Inferential Macrostructure Statistics**
- Output: **Interpretation of Results**

Finally, we introduce the addition of EE content to the matrix with two column sub-headings, as follows:

- Under Financial Economics: "Conjuring Reality in the Ensemble"
- Under Ergodicity Economics: "Restoring Temporal Skepticism"

Financial economics focuses on expected values and thus maps reality with ensemble metrics.
However, these ensemble metrics are used in investment management and financial planning to conjure reality at the level of the individual client.

Ergodicity economics focuses on time average and thus restores temporal skepticism for individual client counseling. EE articulates the meaning of the means, showing that expected values summarize the growth of the ensemble and that time averages summarize the growth of the individual.

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Slide 3 - Adding CTRI's Template for Reading Research Papers to the Basic Framework

Adding CTRI's Template for Reading Research Papers

	Financial Economics **Conjuring Reality In the Ensemble**	Ergodicity Economics **Restoring Temporal Skepticism**
Data Generation Process		
Microstructure of Data		
Design of Study		
Descriptive Macrostructure Statistics		
Inferential Macrostructure Statistics		
Interpretation of Results		

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Slide 4 Speaker's Notes - Adding EE's Calculation Process to the Comparative Framework

EE has a calculation process that includes many logical and mathematical steps in order to develop values and insights for the meaning of the means.

Adding key components of this calculation process to the comparative framework increases the number of rows as follows, and shown in bold font below, and on Slide 4:

- Input: Data Generation Process: **Model of randomness**
- Input: Data Generation Process: **Growth dynamics**

- Input: Microstructure of Data: **Trajectory histories**
- Input: Microstructure of Data: **Increments**
- Input: Microstructure of Data: **Increments: Returns, multipliers, growth rates**
- Input: Microstructure of Data: **Probability distribution of the increments**

- Model: Design of Study: Relevance and coherence

- Model: Descriptive Macrostructure Statistics: **Measures of central tendency**
- Model: Descriptive Macrostructure Statistics: **Measures of variability**
- Model: Descriptive Macrostructure Statistics: **Time averages**
- Model: Descriptive Macrostructure Statistics: **Ergodic transformations**

- Output: Inferential Macrostructure Statistics: **The meaning of means**
- Output: Inferential Macrostructure Statistics: **Measures of relationships**
- Output: Inferential Macrostructure Statistics: **Precision and accuracy**

- Output: Interpretation of Results: Explaining and reframing for understanding

Slide 4 - Adding EE's Calculation Process to the Comparative Framework

	Financial Economics Conjuring Reality in the Ensemble	Ergodicity Economics Restoring Temporal Skepticism
Data Generation Process: **Model of randomness** **Growth dynamics, dx**		
Microstructure of Data: **Stochastic Process: Trajectory histories, x(t)** **Random Variable: Increments** **Increments: Returns, multipliers, growth rates** **Probability Distribution of the increments**		
Design of Study: Relevance and coherence		
Descriptive Macrostructure Statistics: **Measures of central tendency, μ** **Measures of variability, σ** **Time averages, $\bar{g}$** **Ergodic transformations, v(x)**		
Inferential Macrostructure Statistics: **The meaning of the means $\langle g \rangle = \bar{g} = \Upsilon$** Measures of relationships, r Measures of precision and accuracy		
Interpretation of Results: Explaining and reframing for understanding		

Slide 5 Speaker's Notes - Adding Puzzles, Paradoxes & Anomalies and EE Solutions to the Comparative Matrix

The comparative matrix is now ready to accept the headline descriptions of the puzzles, paradoxes, and anomalies that afflict financial economics. Each headline is font-and-color coded as used in the narrated conceptual chronology shown in the paper titled: "Expected Value Optimization, Client-Centric Planning, & Growth Optimal Selection: A Centuries-long, Three-braided Conceptual Chronology" and published on October 6, 2020, on RESEARCHERS.ONE, as follows and shown on slide 5:

- Purple font for items related to traditional expected-value portfolio optimization,
- Green font for items related to client-centric financial planning,
- Black font for resolved items related to growth optimal decision-making that have a reference to a published paper.

Key Take-Aways:

Ergodicity economics is a precise mathematical concept that requires three levels of reframing for financial practitioners:

- New distinctions about the microstructure of the data (including models of randomness, growth dynamics, stochastic process, and the distribution of growth rates),
- Specific properties of the macrostructure of the data (including time averaging for the distribution of growth rates, functional transformations, and ergodic observables), and
- Reframing the meaning of the means (including ensemble versus individual, optimal leverage, and minimum holding periods).

Specifically, ergodicity is a mathematical property of macrostructure observables (i.e., averages) in models based on the distribution of growth rates for a system of trajectories.

Financial economics focuses on expected values and thus maps reality with macrostructure observables that represent the ensemble. However, these ensemble metrics are used in investment management and financial planning to conjure reality at the level of the individual client. This creates the basis for the many puzzles, paradoxes, and anomalies in financial economics.

Ergodicity economics focuses on time averages and thus restores temporal skepticism for client counseling with macrostructure observables that represent the individual. EE articulates the meaning of the means, showing that expected values summarize the growth of the ensemble, and that time averages summarize the growth of the individual

The middle column in Slide 5 has nineteen headlines that relate to puzzles, paradoxes, and anomalies that are of interest to practitioners. The right column has eleven topics that provide resolutions to such puzzles, paradoxes, and anomalies. It also includes five questions that are of interest to practitioners because EE may be able to provide innovative answers and resolutions.

Note to the Reader: To keep the focus on the key points, the matching presentation slide was edited down from the table on the following page.

Slide 5 – Adding Selected Puzzles, Paradoxes & Anomalies and EE Solutions to the Comparative Matrix

	Financial Economics Conjuring Reality in the Ensemble	Ergodicity Economics Restoring Temporal Skepticism
Data Generation Process: Model of Randomness Growth Dynamics, dx	**Instability of financial risk tolerance**	**Empirical evidence of context-switching growth dynamics**
Microstructure of Data: Stochastic Process: Trajectory histories, x(t) Random Variables: Increments Increments: Returns, multipliers, growth rates Probability distribution of the increments	**Puzzle of mean reversion** **Gambler's fallacy** **Limits of central limit theorems** **Puzzle of reversion to the tail**	
Design of Study: Relevance and coherence	**Puzzle of behavioral finance best practices**	**Growth optimal probability weighting**
Descriptive Macrostructure Statistics: Measures of central tendency, μ Measures of variability, σ Time averages, $\bar{g}$ Ergodic transformations, v(x)	**Instability of the 4% rule** **Puzzle of prevalence of certain utility functions** **Limits of the law of large numbers**	**Growth optimal decision criterion** **Growth optimal insurance** **Growth optimal preference reversals** **Growth optimal utility/transformation functions** **Growth optimal risk preferences in time lotteries**
Inferential Macrostructure Statistics: The meaning of the means $\langle g \rangle = \bar{g} = \Upsilon$ Measures of relationships, r Measures of precision and accuracy	**Modern Portfolio Theory** **Capital Asset Pricing Model** **Optimizers' over-sensitivity to input changes** **Puzzle of unstable correlations** **Limits of Monte Carlo simulations** **Puzzle regression to the mean**	**Growth optimal equity premium** **The value of cooperation** **Growth optimal leverage**
Interpretation of results Explaining and reframing for understanding	**Asset allocation** **Style investing anomalies** **Factor investing anomalies** **Paradox of over-diversification**	**Irreproducible research results due to non-ergodicity**

Slide 6 Speaker's Notes – A Practitioner's Focused List of Puzzles, Paradoxes & Anomalies

This next slide changes our direction from expanding to focusing by eliminating the financial economics as well as the ergodicity economics entries that are not related to the following questions:

- **Can EE's growth dynamics provide directly measurable, and more reliable metrics than psychological risk tolerance as an input for financial planning?**
- **Can EE stabilize the 4% rule with time average?**
- **Can EE's growth dynamics explain, and justify the most frequently used utility functions, their matching risk aversion properties, and how they relate to client risk tolerance?**
- **Can EE's time average provide new signal-from-noise explanations to the puzzle of regression to the mean?**
- **Can EE extend from growth optimal selection to growth optimal portfolio construction?**

The rest of this presentation organizes the five financial economics puzzles, paradoxes, and anomalies that are related to these questions into three groups (vignettes for discussion) that are based on the form of their potential answers provided by ergodicity economics, including: (i) growth dynamics, (ii) time average, and (iii) growth optimality.

These vignettes present each question, its matching puzzle, paradox, and anomaly from financial economics, and the potential directions for resolution by EE, keeping the color coding from both this presentation (blue for the selected puzzles, paradoxes, and anomalies), and the paper on RESEARCHERS.ONE (purple font for expected value optimization, green font for client-centric financial planning, and black font for growth optimal selection.)

Note to the Reader: To keep the focus on the key points, the matching presentation slide was edited down from the table on the following page.

Slide 6 – A Practitioner's Focused List of Puzzles, Paradoxes, & Anomalies

Level 2: Puzzles, Paradoxes & Anomalies Headlines	**Financial Economics Conjuring Reality in the Ensemble**	**Ergodicity Economics Restoring Temporal Skepticism**
Data Generation Process: Model of randomness Growth dynamics, dx	Instability of financial risk tolerance	**Can EE's growth dynamics provide directly measurable, and more reliable metrics than psychological risk tolerance as an input for financial planning?**
Microstructure of Data: Stochastic Process: Trajectory histories, x(t) Random Variables: Increments Increments: Returns, Multipliers, growth rates Probability distribution of the increments		
Design of Study: Relevance and coherence		
Descriptive Macrostructure Statistics: Measures of central tendency, μ Measures of variability, σ Time averages, $\bar{g}$ Ergodic transformations, v(x)	Instability of the 4% rule Puzzle of prevalence of certain utility functions	**Can EE stabilize the 4% rule with time average?** **Can EE's growth dynamics explain, and justify the most frequently used utility functions, their matching risk aversion properties, and how they relate to client risk tolerance?**
Inferential Macrostructure Statistics: The meaning of the means $\langle g \rangle = \bar{g} = \Upsilon$ Measures of relationships, r Measures of precision and accuracy	Puzzle regression to the mean	**Can EE's time average provide new signal-from-noise explanations to the puzzle of regression to the mean?**
Interpretation of results Explaining and reframing for understanding	Limits of Asset allocation, Style, & Factor Investing	**Can EE extend from growth optimal selection to growth optimal portfolio construction?**

Slide 7 Speaker's Notes – Vignette for Discussion: Risk Tolerance

Question for discussion:

- Can EE's growth dynamics provide more stable metrics than psychological risk tolerance as an input for financial planning?

Selected puzzle, paradox, and anomaly:

- Instability of financial risk tolerance, starting in 1964 with N. Kogan and M.A. Wallach's book *Risk Taking: A Study in Cognition and Personality*, and followed up by K.R. MacCrimmon and D.A. Wehrung with their 1984 paper "The Risk In-Basket." This work is generally based on classic (psychometric) test theory (CTT). It was institutionalized into practice with standards such as the 2006 definition by the Organization for Standardization as follows: Risk tolerance is ["the extent to which someone is willing to experience a less favorable outcome in the pursuit of an outcome with more favorable attributes"]. However, a person's risk tolerance does not seem to be stable over time or under different circumstances.

Potential direction for resolution:

- Growth dynamics do not rely on unconstrained psychological explanations, and present a discrete number of alternatives that can be empirically matched to specific circumstances.

Slide 7 – Vignette for Discussion: Risk Tolerance

Vignette for Discussion: Risk Tolerance

Question for Discussion:

Can EE's growth dynamics provide directly measurable, and more reliable metrics than psychological risk tolerance as an input for financial planning?

Selected Puzzle, Paradox & Anomaly:

Instability of financial risk tolerance, In 2006, the Organization for Standardization defined risk tolerance as: "the extent to which someone is willing to experience a less favorable outcome in the pursuit of an outcome with more favorable attributes".

However, a person's risk tolerance does not seem to be directly measurable, stable over time, or stable under different circumstances.

Potential Direction for Resolution:

Growth dynamics do not rely on psychological explanations, and present a discrete number of alternatives that can be empirically matched to specific circumstances.

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Slide 8 Speaker's Notes – Vignette for Discussion: Utility Functions

Question for discussion:

- Can EE's growth dynamics explain, and justify the most frequently used utility functions, their matching risk aversion properties, and how they relate to client risk tolerance?

Selected puzzle, paradox, and anomaly:

- The puzzle of the prevalence of certain utility functions: What client characteristics explain the frequent use of utility functions with their matching risk aversion formula, including:
  - Exponential, $u(x) = 1 - e^{-ax}$, $a > 0$, CARA
  - Power, $u(x) = (x^a - 1) / \Upsilon$, $\Upsilon < 1$
  - Iso-elastic, $u(x) = [(x^\lambda) - 1] / \lambda$, $0 < \lambda < 1$, CRRA
  - Logarithmic, $u(x) = \ln x$ or $\log(x)$, DARA
- Risk aversion properties of utility functions:
  - Absolute Risk Aversion = $- u''(x) / u'(x)$ [concavity divided by slope), Relative Risk Aversion = $- x [u''(x) / u'(x)]$
  - CARA (constant absolute risk aversion), CRRA (constant relative risk aversion),
  - IARA (increasing absolute risk aversion), IRRA (increasing relative risk aversion),
  - DARA (decreasing absolute risk aversion), DRRA (decreasing relative risk aversion) in financial economics?

Potential direction for resolution:

- EE connects the dots between growth dynamics, utility functions, and their matching risk aversion properties in ways that could provide explanations, and justifications based on empirical measurements of client behavior, and circumstances.

Slide 8 – Vignette for Discussion: Utility Function

Vignette for Discussion: Utility Functions

Question for Discussion:

Can EE's growth dynamics explain, and justify the most frequently used utility functions, their matching risk aversion properties, and how they relate to client risk tolerance?

Selected Puzzle, Paradox & Anomaly:

The puzzle of the prevalence of certain utility functions.

Potential Direction for Resolution:

EE connects the dots between growth dynamics, utility functions, and their matching risk aversion properties in ways that could provide new explanations, and justifications based on empirical measurements of client behavior, and circumstances.

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Slide 9 Speaker's Notes – Vignette for Discussion: 4% Rule

Question for discussion:

- Can EE stabilize the 4% rule with time average?

Selected puzzle, paradox, and anomaly:

- Puzzle of the instability of the 4% rule, starting in 1994 with a paper "Determining Withdrawal Rates Using Historical Data," written by William P. Bengen, who was seeking a "safemax" rate of withdrawal from a retirement portfolio exposed to market risk and recommending a 4% first year withdrawal followed by inflation-adjusted withdrawals, assuming a minimum portfolio longevity of 30 years. This number was changed over time by Bengen and others. Changes ranged from nearly half to nearly twice the original estimate.

Potential direction for resolution:

- A growing system with a measure of randomness will show two distinct and unequal growth rates.
- EE's time average, instead of a range of ensemble averages, could be the first order, upper-bound estimate for a sustainable withdrawal rate over the retirement horizon.

Slide 9 – Vignette for Discussion: 4% Rule

Vignette for Discussion: The 4% Rule

Question for Discussion:

Can EE stabilize the 4% rule with time average?

Selected Puzzle, Paradox & Anomaly:

Puzzle of the instability of the 4% rule. The "rule" recommends a 4% first year withdrawal followed by inflation-adjusted withdrawals, assuming a minimum portfolio longevity of 30 years.

This number was changed over time by Bengen and others. Changes ranged from nearly half to nearly twice the original estimate.

Potential Direction for Resolution:

EE's time average, instead of a range of ensemble averages, could be the first order, upper-bound estimate for a sustainable withdrawal rate over the retirement horizon.

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Slide 10 Speaker's Notes – Vignette for Discussion: Regression to the Mean

Question for discussion:

- Can EE' time average provide new signal-from-noise explanations to the puzzle of regression to the mean?

Selected puzzle, paradox, and anomaly:

- Puzzle of correlation-based regression to the mean, a statistical phenomenon that invites causal explanations for random fluctuations, starts in 1886 with Francis Galton's paper "Regression Towards Mediocrity in Hereditary Stature," and continues in 1933 with Horace Secrist's "The Triumph of Mediocrity in Business," and Harold Hotelling's paper, "Review of the Triumph of Mediocrity in Business, by Horace Secrist" to correct the error of confusing a regression analysis signal with random noise from imperfectly correlated variables.

  Mauboussin's "shrinking factor" formula: $r(t+1) = \langle r \rangle + \lambda (r(t) - \langle r \rangle)$, $\lambda = 0$ means total randomness with visible and fast regression to the mean, $\lambda = 1$ means total skillfulness with no regression to the mean]. "The Success Equation" Michael Mauboussin (2012)

  The error persists to the current time as documented by Gary Smith in his 2016 paper "A Fallacy that Will Not Die."

Potential direction for resolution:

- What is the impact of time average on regression to the (expected value) mean?

- Can this RTM equation $r(t+1) = \langle r \rangle + \lambda (r(t) - \langle r \rangle)$ become $g(t+1) = \bar{g} + \lambda (g(t) - \bar{g})$?

- Does time average change regression to the ensemble average by factoring variance, and thus quantifies randomness into the equation?

- Is (arithmetic) mean reversion a statement about ensemble averages (comparing population samples) as opposed to a statement about time averages (comparing timeline segments)?

- Do timeline segments move toward the population time average over time and only move toward the population expected value when the observables are ergodic?

- EE's time average could improve our understanding of performance measurement with improved signal-from-noise explanations to the puzzle of regression to the (ensemble average) mean.

Slide 10 – Vignette for Discussion: Regression to the Mean

Vignette for Discussion: Regression to the Mean

Question for Discussion:

Can EE's time average provide new signal-from-noise explanations to the puzzle of regression to the mean?

Selected Puzzle, Paradox & Anomaly:
Puzzle of regression to the mean, a statistical phenomenon that invites causal explanations for random fluctuations.

Potential Direction for Resolution:
EE's time average could improve our understanding of performance measurement with improved signal-from-noise explanations to the puzzle of regression to the (ensemble average) mean.

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Slide 11 Speaker's Notes – Portfolio Construction

Question for discussion:

- Can EE extend from growth optimal selection to growth optimal portfolio construction?

Selected puzzle, paradox, and anomaly:

- The limits of asset allocation, starting in 1986 with the paper by Gary P. Brinson, L. Randolph Hood, and Gilbert L. Beebower "Determinants of Portfolio Performance," seeking to quantify "… which investment decisions had the greatest impact on the magnitude of total returns" but providing an answer of a 94.6% impact on the variance of total returns. James X. Xiong, Roger G. Ibbotson, Thomas M. Idzorek, and Peng Chen (2010), in their paper, "The Equal Importance of Asset Allocation and Active Management," show that asset allocation may account for 12.5% of total return variation. Asset allocation may be a confounded variable with 75%+ of the effect coming from market exposure.
- Style investing anomalies, starting in 1949 with Benjamin Graham's book *The Intelligent Investor*, focusing on the buy-and-hold, value style, and differentiating Mr. Market's daily price from the investor's empirical analysis of the value of the stock to determine the presence of a "margin of safety." Style investing developed beyond value investing to include Morningstar's "style box," factor investing, alternative investments, and ESG/SRI/impact investing.
- Factor investing anomalies, including Fama-French Three-Factor CAPM, starting in 1992 with the paper by Eugene Fama and Kenneth French in "The Cross-Section of Expected Stock Returns," combining (i) market risk, (ii) small vs. large companies, and (iii) high vs. small book/market ratios to explain 90% of portfolio returns, as contrasted with the original CAPM single market risk factor (beta), may only explain 70% of portfolio returns. Extensions include the Carhart Four-Factor CAPM, starting in 1997 with Mark Carhart's paper "On Persistence in Mutual Funds Performance," and the Fama-French Five-Factor CAPM, starting in 2015 with the paper by Eugene Fama & Kenneth French "Dissecting anomalies with a five-factor model."

Potential direction for resolution:

- EE may show that one cannot improve time average with asset allocation, style investing, and factor investing.
    - Growth optimal leverage, starting in 2011 with Ole Peters' paper "Optimal leverage from non-ergodicity," and establishing that one cannot improve time average growth rates through asset allocations that depart from optimal leverage: 100% of the growth optimal selection.
- EE's portfolio construction solution may, or may not, be similar to Thomas Cover's universal portfolio.
    - Universal portfolio management, starting in 1980 with the paper by Robert Bell and Thomas Cover "Competitive Optimality of Logarithmic Investment" and formalized in 1991 with Thomas M. Cover's paper "Universal Portfolios." This is an algorithmic approach to portfolio rebalancing. It learns adaptively from past data, and maximizes the log-optimal growth rate by reallocating daily across all holdings based on the previous day's price changes.

Slide 11 – Portfolio Construction

Vignette for Discussion: Portfolio Construction

Question for Discussion:

Can EE extend from growth optimal selection to growth optimal portfolio construction?

Selected Puzzle, Paradox & Anomaly:

The limits of asset allocation, style, and factor investing. For instance, asset allocation may be a confounded variable with 75%+ of the effect coming from market exposure.

Potential Directions for Resolution:

EE may show that one cannot improve time average with asset allocation, style investing, and factor investing.

EE's portfolio construction solution may, or may not be similar to Thomas Cover's universal portfolio.

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Speaker's Notes – Thank You & Next Steps

About the Author

François Gadenne is a serial fintech entrepreneur with 40 years of experience. He emigrated from France to the United States in his early twenties after completing a Baccalaureate in Mathematics, Latin, and Greek, and receiving the diploma from the Ecole Superieure the Commerce de Paris. He also completed an MBA from the J. L. Kellogg Graduate School of Management at Northwestern University. In the 2000s and for nearly a decade, he was a lecturer at Boston University, teaching FI728 "Principles of Finance," Professor Zvi Bodie's class in the Master of Science in Investment Management (MSIM) program.

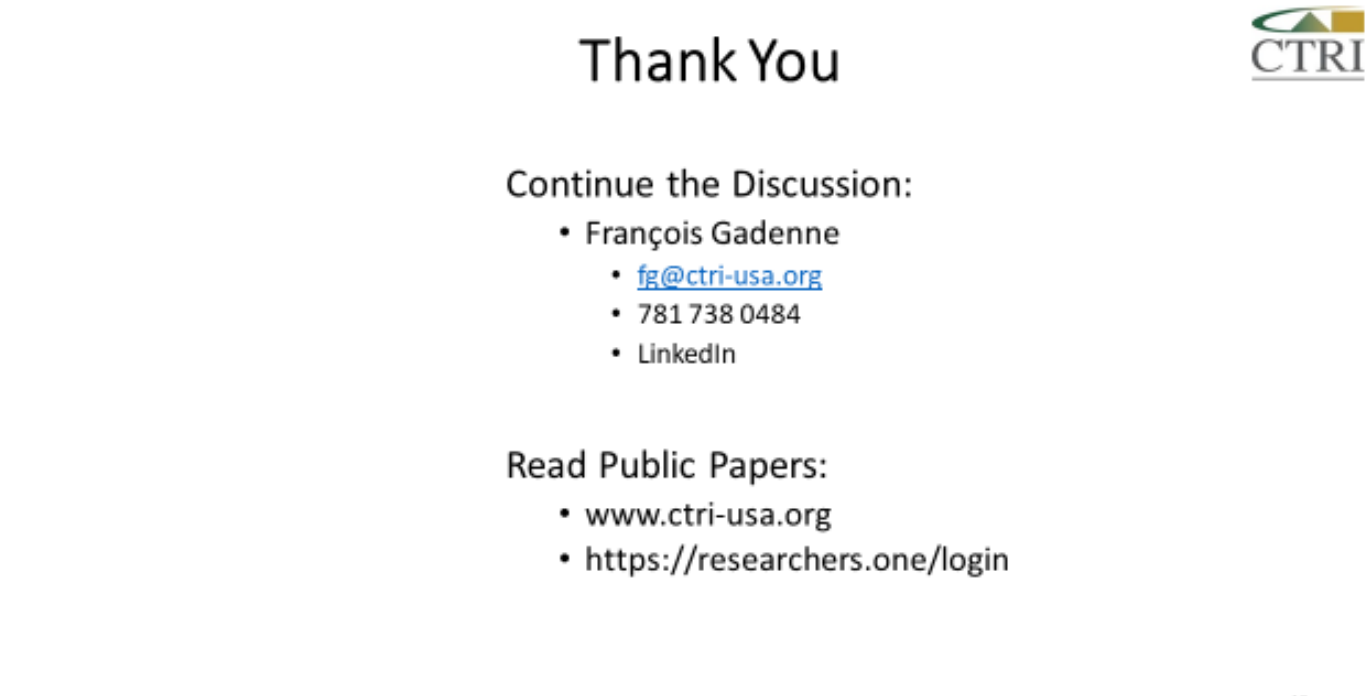
François' current venture, the Curve, Triangle & Rectangle Institute℠ (CTRI℠), is a not-for-profit, membership-based research & development institute that was founded in 2018. CTRI extends the research findings of a previous venture, the Retirement Income Industry Association (RIIA), that are documented in the published paper "The Shapes of Retirement: Are you a Curve, a Triangle, or a Rectangle?" (Collins/Gadenne 2017). In 2017, the Investments & Wealth Institute® (formerly IMCA) acquired RIIA's educational assets, including the Retirement Management Advisor® designation (RMA®), and the peer-reviewed *Retirement Management Journal* (*RMJ*).

CTRI explores the unseen connections in research, and translates research findings in plain English in order to (i) find the powerful ideas that are not widely known, (ii) flag the false positives, and (iii) validate the insights that can be transformed into differentiated business practices, or new ventures.

Membership is by invitation only, exclusive by industry, and includes leaders from an investment company, a life company, a financial distributor, a professional education provider, and a fintech start-up. Invitations have been extended to a Bank, and an RIA based on the personal recommendations from existing members. Members prioritize research based on the issues that keep them up at night. Findings are shared with members in membership notes, personal calls, and group meetings. Members have the option to customize the development of shared findings, under NDA.

From time to time, members approve a limited number of these findings for general publication. This paper was written for CTRI members who decided to share its content with the public based on the importance of the topic for the financial industry and its clients. François can be reached at fg@ctri-usa.org.

Slide 12: Thank You & Next Steps



Appendix I - Ergodicity Economics Papers – Reference List

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